Projections

Computer Graphics and Visualization

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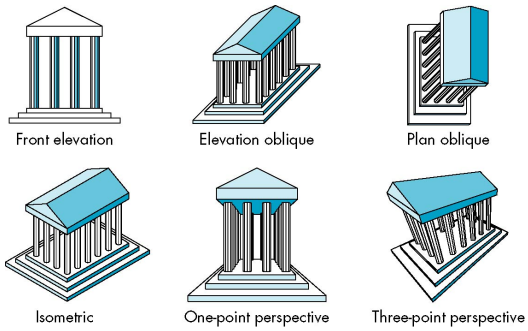
Planar Geometric Projections

- Standard projections project onto a plane
- Projectors are lines that either
  - converge at a center of projection
  - are parallel
- Nonplanar projections are needed for applications such as map construction

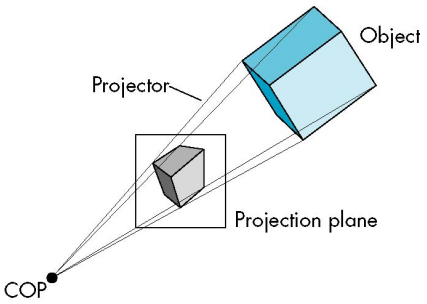
Classical Projections



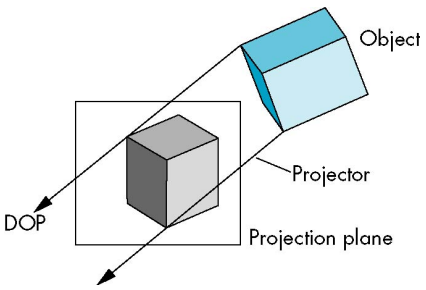
Perspective vs Parallel

- Computer graphics treats all projections the same and implements them with a single pipeline
- Classical viewing developed different techniques for drawing each type of projection
- Fundamental distinction is between parallel and perspective viewing even though mathematically parallel viewing is the limit of perspective viewing

Perspective Projection

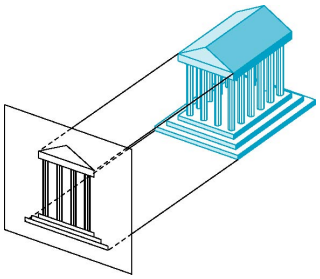


Parallel Projection



Orthographic Projection

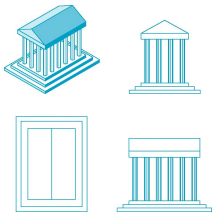
Projectors are orthogonal to projection surface



Multiview Orthographic Projection

- Projection plane parallel to principal face
- Usually form front, top, side views

In CAD and architecture, we often display three multiviews plus isometric (see below)



Advantages and Disadvantages

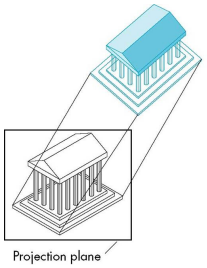
- Preserves both distances and angles
  - Shapes preserved
  - Can be used for measurements
    - Building plans
    - Manuals
- Cannot see what object really looks like because many surfaces hidden from view
  - Often we add the isometric

Axonometric Projections

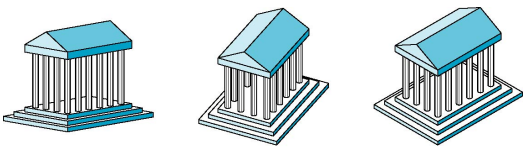
- Allow projection plane to move relative to object
- Still Orthographic!

Classify by how many angles of a corner of a projected cube are the same

none: trimetric
two: dimetric
three: isometric



Types of Axonometric Projections

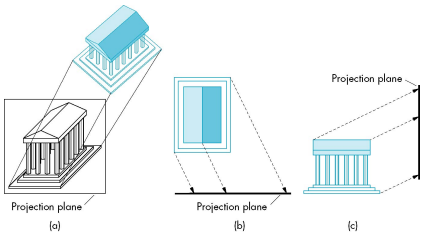


Advantages and Disadvantages

- Lines are scaled (foreshortened) but can find scaling factors
- Lines preserved but angles are not
  - Projection of a circle in a plane not parallel to the projection plane is an ellipse
- Can see three principal faces of a box-like object
- Some optical illusions possible
  - Parallel lines appear to diverge
- Does not look real because far objects are scaled the same as near objects
- Used in CAD applications

Oblique Projection

Arbitrary relationship between projectors and projection plane

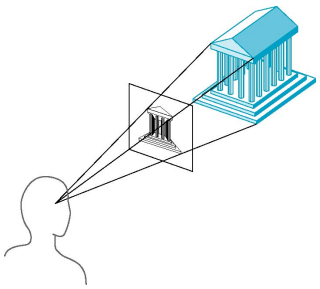


Advantages and Disadvantages

- Can pick the angles to emphasize a particular face
- Architecture: plan oblique, elevation oblique
- Angles in faces parallel to projection plane are preserved while we can still see ”around” a side
- In physical world, cannot create with simple camera; possible with special lens

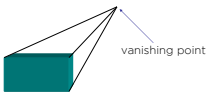
Perspective Projection

Projectors coverge at center of projection



Vanishing Points

- Parallel lines (not parallel to the projection plan) on the object converge at a single point in the projection (the vanishing point)
- Drawing simple perspectives by hand uses these vanishing point(s)



One-Point Perspective

- One principal face parallel to projection plane
- One vanishing point for cube

One-Point Perspective

Two-Point Perspective

- On principal direction parallel to projection plane
- Two vanishing points for cube

Two-Point Perspective

Three-Point Perspective

- No principal face parallel to projection plane
- Three vanishing points for cube

Three-Point Perspective

Advantages and Disadvantages

- Objects further from viewer are projected smaller than the same sized objects closer to the viewer (diminution)
  - Looks realistic
- Equal distances along a line are not projected into equal distances (nonuniform foreshortening)
- Angles preserved only in planes parallel to the projection plane
- More difficult to construct by hand than parallel projections (but not more difficult by computer)

Taxonomy of Planar Geometric Projections

Parallel Projection

- Orthographic
  - Top
  - Front
  - Side
  - Axonometric
    - Isometric
- Oblique
  - Cabinet
  - Cavalier

Perspective Projections

- One point
- Two point
- Three point
- Camera model

Homogeneous Coordinate Representation

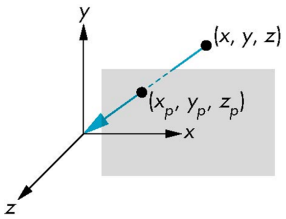
- Orthographic projection

$$p_p = \mathbf{M}p$$

$$x_p = x \quad y_p = y \quad z_p = 0 \quad w_p = 1 \qquad \mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

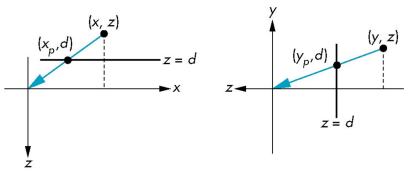
Simple Perspective

- Center of projection at the origin
- Projection plane $z = d$, $d < 0$



Perspective Equations

Consider top and side views



$$x_p = \frac{x}{z/d} \qquad y_p = \frac{y}{z/d} \qquad z_p = d$$

Homogeneous Coordinate Form

consider $q = \mathbf{M}p$ where $\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$

$$q = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad \Rightarrow p = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

Perspective Division

- However $w \neq 1$, so we must divide by w to return from homogeneous coordinates
- This perspective division yields

$$x_p = \frac{x}{z/d} \qquad y_p = \frac{y}{z/d} \qquad z_p = d$$

the desired perspective equations