CSCB63 Assignment 1 – AVL Trees

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Due: February 9th, 2024

Question 1

a)
Let f, g, h : $\mathbb{N} \to \mathbb{R}^+$. Suppose $f \in \mathcal{O}(g)$, $g \in \mathcal{O}(h)$.

$\because$ f(n) $\leq$ $c_1 \cdot g$(n), $g(n) \leq c_2 \cdot h(n)$, for some c_1, $c_2 \in \mathbb{R}^+$. [def. of $\mathcal{O}$]
$\therefore$ f(n) $\leq$ $c_1 \cdot g(n) \leq c_1 \cdot c_2 \cdot h(n)$ [combining inequalities]

Let c = $c_1 \cdot c_2$. $\implies$ f(n) $\leq$ c $\cdot$ h(n).
Hence, $f \in \mathcal{O}(h)$, as wanted. $\square$

b)
Let f_1, f_2, g_1, g_2 : $\mathbb{N} \to \mathbb{R}^+$. Suppose $f_1 \in \mathcal{O}(g_1)$, $f_2 \in \mathcal{O}(g_2)$.

$\because$ $f_1(n) \leq c_1 \cdot g(n)$, $f_2(n) \leq c_2 \cdot g(n)$, for some c_1, $c_2 \in \mathbb{R}^+$. [def. of $\mathcal{O}$]
$\therefore$ $f(n) = f_1(n) \cdot f_2(n) \leq c_1 \cdot c_2 \cdot g_1(n) \cdot g_2(n) = c_1 \cdot c_2 \cdot g(n)$. [combining inequalities]

Let c = $c_1 \cdot c_2$. $\implies$ f(n) $\leq$ c $\cdot$ g(n).
Hence, $f \in \mathcal{O}(g)$, as wanted. $\square$