Trajectories

1. General function for straight-line trajectories (simulink block)

$$x(t) = x_0 + \alpha x(t)$$
$$y(t) = y_0 + \alpha y(t)$$
$$z(t) = z_0 + \alpha z(t)$$

Input

- x_0, y_0, z_0
- x_f, y_f, z_f
- t_d
- q_{init} (for inverse position kinematics algorithm)

Thus

$$\alpha_x = \frac{x_f - x_0}{t_d}, \quad \alpha_y = \frac{y_f - y_0}{t_d}, \quad \alpha_z = \frac{z_f - z_0}{t_d}$$

~~Output~~ Function

- → use input time (clock) to get $x_{cur}, y_{cur}, z_{cur}$
- → take & put into inverse position kinematics to get q_{cur}

Output

q_1, q_2, q_3, q_4 @ this time instant

2. Joint Space trajectory (for $l_4 = 0$)

Note:
- $l_1 = 0.044$
- $l_2 = 0.14$
- $l_3 = 0.134$
- $l_4 = 0$

Consider

initial pos: $x_0 = -0.1266$, $y_0 = -0.0535$, $z_0 = 0.2711$

final pos: $x_f = -0.17$, $y_f = -0.1$, $z_f = 0.01$

initial joints: $\begin{bmatrix} 0.4 & 0.3 & 0.5 & 0 \end{bmatrix}$

final joints: $\begin{bmatrix} 0.5317 & 1.099 & 1.5041 & 0 \end{bmatrix}$

Thus

- $q_1(0) = 0.4 \qquad q_1(10) = 0.5317$
- $\dot{q}_1(0) = \dot{q}_1(10) = 0$

① 3rd order cubic polynomial for $q_1(t)$

$$q_1(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3$$

① $q_1(0) = 0.4$

$q_1(0) = 0.4 = C_0 \qquad$ Thus $\boxed{C_0 = 0.4}$

② $\dot{q}_1(0) = 0$

$\dot{q}_1(t) = C_1 + 2C_2 t + 3C_3 t^2$

$\dot{q}_1(0) = 0 = C_1$ Thus $C_1 = 0$

③ $q_1(10) = 0.5317$

$0.5317 = 0.4 + C_2(10)^2 + C_3(10^3)$

$\boxed{100 C_2 + 1000 C_3 = 0.1317}$

$\begin{cases} C_0 = 0.4 \\ C_1 = 0 \\ C_2 = 3.951 \times 10^{-3} \\ C_3 = -2.634 \times 10^{-4} \end{cases}$

④ $\dot{q}_1(10) = 6$

$\boxed{0 = 20 C_2 + 300 C_3 = 0}$

Thus

$$\boxed{q_1(t) = 0.4 + 3.951 \times 10^{-3} t^2 - 2.634 \times 10^{-4} t^3}$$

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② 3rd order cubic polynomial for $q_2(t)$

$q_2(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3$ | $q_2(0) = 0.3$, $q_2(10) = 1.099$

$\dot{q}_2(t) = C_1 + 2C_2 t + 3C_3 t^2$ | $\dot{q}_2(0) = \dot{q}_2(10) = 0$

$q_2(0) = 0.3$

Thus $\boxed{C_0 = 0.3}$

$\dot{q}_2(0) = 0$

$\boxed{C_1 = 0}$

$q_2(10) = 1.099$

$1.099 = 0.3 + 100 C_2 + 1000 C_3$

$\boxed{100 C_2 + 1000 C_3 = 0.799}$

$\dot{q}_2(10) = 0$

$\boxed{20 C_2 + 300 C_3 = 0}$

Thus $\boxed{C_2 = 0.02397}$ $\boxed{C_3 = -1.598 \times 10^{-3}}$

Thus

$$\boxed{q_2(t) = 0.3 + 0.02397 t^2 - 1.598 \times 10^{-3} t^3}$$

③ 3rd order cubic polynomial for $q_3(t)$

- $q_3(0) = 0.5$ $q_3(10) = 1.5041$
- $\dot{q}_3(0) = \dot{q}_3(10) = 0$

- $q_3(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3$
- $\dot{q}_3(t) = C_1 + 2C_2 t + 3C_3 t^2$

$\underline{q_3(0) = 0.5}$ $\underline{\dot{q}_3(0) = 0}$

$C_0 = 0.5$ $C_1 = 0$

$\underline{q_3(10) = 1.5041}$

$1.5041 = 0.5 + C_2(100) + C_3(1000)$

$\boxed{C_2(100) + C_3(1000) = 1.0041}$

$\boxed{\begin{array}{l} C_2 = 0.030123 \\ C_3 = -2.0082 \times 10^{-3} \end{array}}$

$\underline{\dot{q}_3(10) = 0}$

$\boxed{0 = 20C_2 + 300C_3}$

$\boxed{q_3(t) = 0.5 + 0.030123 t^2 - 2.0082 \times 10^{-3} t^3}$

$\boxed{\begin{array}{l} q_1(t) = 0.4 + 3.951 \times 10^{-3} t^2 - 2.634 \times 10^{-4} t^3 \\ q_2(t) = 0.3 + 0.02397 t^2 - 1.598 \times 10^{-3} t^3 \\ q_3(t) = 0.5 + 0.030123 t^2 - 2.0082 \times 10^{-3} t^3 \\ q_4(t) = 0 \end{array}}$

Final position of Joint space trajectory

$x_f = -161.8$ mm

$y_f = -95.17$ mm

$z_f = 2.464$ mm

Final position of task space trajectory

$x_f = -164.7$ mm

$y_f = -96.86$ mm

$z_f = 4.47$ mm

3. Joint Space Trajectories for $l_4 = 0.09m$ (gen)

Consider:

$$x_0 = -0.1266, \quad y_0 = -0.05354, \quad z_0 = 0.2711$$
$$x_f = -0.17, \quad y_f = -0.11, \quad z_f = 0.01$$

Initial Joints: $[0.4001, -0.4318, 1.3774, 0.3837]$

Final Joints: $[0.5317, 0.3852, 1.854, 0.4599]$

For joint q_1

$$q_1(0) = 0.4001, \quad q_1(10) = 0.5317$$
$$\dot{q}_1(0) = 0, \quad \dot{q}_1(10) = 0$$

$$q_1(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$$
$$\dot{q}_1(t) = c_1 + 2 c_2 t + 3 c_3 t^2$$

$q_1(0) = 0.4001$	$\dot{q}_1(0) = 0$
$c_0 = 0.4001$	$c_1 = 0$

$q_1(10) = 0.5317$

$$0.5317 = 0.4001 + 100 c_2 + 1000 c_3$$

$$\boxed{100 c_2 + 1000 c_3 = 0.1316}$$

$\dot{q}_1(10) = 0$

$$\boxed{0 = 20 c_2 + 300 c_3}$$

$$\begin{cases} c_0 = 0.4001 \\ c_1 = 0 \\ c_2 = 3.948 \times 10^{-3} \\ c_3 = -2.632 \times 10^{-4} \end{cases}$$

$$\boxed{q_1(t) = 0.4001 + 3.948 \times 10^{-3} t^2 - 2.632 \times 10^{-4} t^3}$$

For joint q_2

$$q_2(0) = -0.4318, \quad q_2(10) = 0.3852$$

$$\dot{q}_2(0) = \dot{q}_2(10) = 0$$

$$q_2(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$$
$$\dot{q}_2(t) = c_1 + 2c_2 t + 3c_3 t^2$$

$\underline{q_2(0) = -0.4318}$ \qquad $\underline{\dot{q}_2(0) = 0}$

$$\boxed{c_0 = -0.4318} \qquad \boxed{c_1 = 0}$$

$\underline{q_2(10) = 0.3852}$

$$0.3852 = -0.4318 + 100 c_2 + 1000 c_3$$

$$\boxed{100 c_2 + 1000 c_3 = 0.8967}$$

$$\boxed{\begin{array}{l} c_2 = 0.02451 \\ c_3 = -1.634 \times 10^{-3} \end{array}}$$

$\underline{\dot{q}_2(10) = 0}$

$$\boxed{0 = 20 c_2 + 300 c_3}$$

~~$c_2 = 0.027204$~~
~~$c_3 = -1.8136 \times 10^{-3}$~~

$$\boxed{q_2(t) = -0.4318 + 0.02451 t^2 - 1.634 \times 10^{-3} t^3}$$

For joint q_3

$$q_3(0) = 1.3774 \quad q_3(10) = 1.8574$$
$$\dot{q}_3(0) = \dot{q}_3(10) = 0$$

$\underline{q_3(0) = 1.3774}$ \qquad $\underline{\dot{q}_3(0) = 0}$

$$\boxed{c_0 = 1.3774} \qquad \boxed{c_1 = 0}$$

$\underline{q_3(10) = 1.8574}$

$$1.8574 = 1.3774 + 100 c_2 + 1000 c_3$$

$$\boxed{100 c_2 + 1000 c_3 = 0.48}$$

$\underline{\dot{q}_3(10) = 0}$

$$\boxed{20 c_2 + 300 c_3 = 0}$$

~~$c_2 = 0.96 \times 10^{-3}$~~
~~$c_3 = -5.464 \times 10^{-4}$~~

~~$c_2 - c_3 =$~~ $\boxed{\begin{array}{l} c_2 = 0.0144 \\ c_3 = -9.6 \times 10^{-4} \end{array}}$

$$q_3(t) = 1.3774 + 0.0144t^2 - 9.6 \times 10^{-4}t^3$$

For joint q_4

$$q_4(0) = 0.3837, \quad q_4(10) = 0.4544$$
$$\dot{q}_4(0) = \dot{q}_4(10) = 0$$

$$q_4(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3$$
$$\dot{q}_4(t) = C_1 + 2C_2 t + 3C_3 t^2$$

$$\underline{q_4(0) = 0.3837} \qquad\qquad \underline{\dot{q}_4(0) = 0}$$
$$C_0 = 0.3837 \qquad\qquad\qquad C_1 = 0$$

$$\underline{q_4(10) = 0.4544} \qquad\qquad\qquad \underline{\dot{q}_4(10) = 0}$$
$$0.4544 = 0.3837 + 100C_2 + 1000C_3 \qquad\qquad \boxed{0 = 20C_2 + 300C_3}$$
$$\boxed{100C_2 + 1000C_3 = 0.0707}$$

$$C_2 = 2.121 \times 10^{-3}$$
$$C_3 = -1.414 \times 10^{-4}$$

$$\boxed{q_4(t) = 0.3837 + 2.121 \times 10^{-3}t^2 - 1.414 \times 10^{-4}t^3}$$

$$\boxed{\begin{aligned}
q_1(t) &= 0.4001 + 3.948 \times 10^{-3}t^2 - 2.632 \times 10^{-4}t^3 \\
q_2(t) &= -0.4318 + 0.027204t^2 - 1.8136 \times 10^{-3}t^3 \\
q_3(t) &= 1.3774 + 8.196 \times 10^{-3}t^2 - 5.464 \times 10^{-4}t^3 \\
q_4(t) &= 0.3837 + 0.012483t^2 - 8.322 \times 10^{-4}t^3
\end{aligned}}$$

$$\boxed{\begin{aligned}
q_1(t) &= 0.4001 + 3.948 \times 10^{-3}t^2 - 2.632 \times 10^{-4}t^3 \\
q_2(t) &= -0.4318 + 0.02451t^2 - 1.634 \times 10^{-3}t^3 \\
q_3(t) &= 1.3774 + 0.0144t^2 - 9.6 \times 10^{-4}t^3 \\
q_4(t) &= 0.3837 + 2.121 \times 10^{-3}t^2 - 1.414 \times 10^{-4}t^3
\end{aligned}}$$