F_ω^{CAL} TYPE SYSTEM

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We define the System F_ω^{CAL}. System F_ω^{CAL} is system F_ω^{AL} extended with a comptime kind, in an attempt to type Zig's notion of comptime. That is, we have have kinds $\star$ for unrestricted types, $\bullet$ for affine types, and $\circ$ for linear types, with the following subkinding relation: $\star \leq \bullet \leq \circ$. We then add kind $\mathcal{C}$. Values of kind $\mathcal{C}$ are known at compile time. We have the following basic rules: $\star, \bullet, \circ : \mathcal{C}$. That is, types are values known at compile time. Additionally, an unrestricted type may be considered a comptime type.

$\kappa ::= \star \mid \bullet \mid \circ \mid \mathcal{C} \mid \kappa \Rightarrow \kappa$	kinds
$\tau ::= \alpha \mid \tau \xrightarrow{\kappa} \tau \mid \forall \alpha{:}\kappa.\, \tau \mid \lambda x : A : \mathcal{C}.\, \tau \mid \tau\ \tau$	types
$e ::= x \mid \lambda^\kappa (x : \tau).\, e \mid e\ e \mid \Lambda(\alpha : \kappa).\, v \mid e[\tau]$	expressions
$v ::= \lambda^\kappa (x : \tau).\, e \mid \Lambda(\alpha : \kappa).\, v$	values

We have contexts $\Psi; \Gamma; \Delta; \Xi$ in the following rules. Context Ψ consists of variables of kind $\mathcal{C}$; Γ consists of variables of kind $\star$; Δ consists of variables of kind $\bullet$, and Ξ consists of variables of kind $\circ$.

We may introduce a type $B : \star$ dependent on a variable $x : A : \mathcal{C}$. That is, the type B may be dependent on a compile time known value. Typically, when we have $A : \mathcal{C}$, A will be one of $\star, \bullet, \circ$. In this case B is simply a type dependent on another type x. However, A can vary, and gives compile time dependent types. Suppose we have $\mathbb{N} : \star$. We allow unrestricted types to be *lifted*, giving $\mathbb{N} : \mathcal{C}$. Then if the above A is $\mathbb{N}$, B is now a family of types depending on a compile time known natural number.

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$$\frac{\Psi;\Gamma \vdash \tau : \star}{\Psi;\Gamma \vdash \tau : \bullet}\ [\textsf{K-Sub}_1] \qquad \frac{\Psi;\Gamma \vdash \tau : \bullet}{\Psi;\Gamma \vdash \tau : \circ}\ [\textsf{K-Sub}_2] \qquad \frac{}{\star, \bullet, \circ : \mathcal{C}}$$

$$\frac{\Psi \vdash A : \star}{\Psi \vdash A : \mathcal{C}}\ [\textsc{Lifting}] \qquad \frac{}{\vdash \cdot}\ [\textsf{Empty}]$$

$$\frac{\vdash \Psi \qquad \alpha \notin \Psi \qquad \Psi \vdash \kappa : \mathcal{C}}{\vdash \Psi, \alpha : \kappa}\ [\textsf{TExt}] \qquad \frac{\alpha : \kappa \in \Gamma}{\Gamma \vdash \alpha : \kappa}\ [\textsf{U-TVar}]$$

$$\frac{\Gamma, \alpha : \kappa \vdash \tau : \kappa'}{\Gamma \vdash \forall \alpha{:}\kappa.\, \tau : \kappa'}\ [\textsf{All}] \qquad \frac{\Gamma \vdash \tau_1 : \kappa_1 \qquad \Gamma \vdash \tau_2 : \kappa_2}{\Gamma \vdash \tau_1 \xrightarrow{\kappa} \tau_2 : \kappa}\ [\textsf{K-Arr}]$$

$$\frac{\Psi \vdash \kappa_1 : \mathcal{C} \qquad \Psi, A : \kappa_1; \Gamma \vdash \tau : \kappa_2}{\Psi;\Gamma \vdash \lambda(A{:}\kappa_1).\, \tau : \kappa_1 \Rightarrow \kappa_2}\ [\textsf{K-Lam}]$$

$$\frac{\Gamma \vdash \tau_1 : \kappa_2 \Rightarrow \kappa_1 \qquad \Gamma \vdash \tau_2 : \kappa_2}{\Gamma \vdash \tau_1 \tau_2 : \kappa_1}\ [\textsf{K-App}]$$

$$\frac{\Gamma_1, \Gamma_2; \Delta; \Xi \vdash e : \tau' \qquad \Gamma_1 \vdash \tau : \star}{\Gamma_1, x : \tau, \Gamma_2; \Delta; \Xi \vdash e : \tau'}\ [\textsf{U-Wkg}]$$

$$\frac{\Gamma; \Delta_1, \Delta_2; \Xi \vdash e : \tau' \qquad \Gamma; \Delta_1 \vdash \tau : \bullet}{\Gamma; \Delta_1, x : \tau, \Delta_2; \Xi \vdash e : \tau'}\ [\textsf{A-Wkg}]$$

$$\cdot \Cup \cdot = \cdot\ [\textsf{AU-Empty}] \qquad \frac{\Delta_1 \Cup \Delta_2 = \Delta \qquad x \notin \Delta}{\Delta_1, x : \tau \Cup \Delta_2 = \Delta, x : \tau}\ [\textsf{AU-Left}]$$

$$\frac{\Delta_1 \Cup \Delta_2 = \Delta \qquad x \notin \Delta}{\Delta_1 \Cup \Delta_2, x : \tau = \Delta, x : \tau}\ [\textsf{AU-Right}] \qquad \cdot \Cup \cdot = \cdot\ [\textsf{LU-Empty}]$$

$$\frac{\Xi_1 \Cup \Xi_2 = \Xi \qquad x \notin \Xi}{\Xi_1, x : \tau \Cup \Xi_2 = \Xi, x : \tau}\ [\textsf{LU-Left}] \qquad \frac{\Xi_1 \Cup \Xi_2 = \Xi \qquad x \notin \Xi}{\Xi_1 \Cup \Xi_2, x : \tau = \Xi, x : \tau}\ [\textsf{LU-Right}]$$

$$\frac{x : \tau \in \Gamma}{\Gamma; \cdot; \cdot \vdash x : \tau}\ [\textsf{U-Var}] \qquad \Gamma; x : \tau; \Xi \vdash x : \tau\ [\textsf{A-Var}]$$

$$\Gamma; \Delta; x : \tau \vdash x : \tau\ [\textsf{L-Var}]$$

$$\frac{\Gamma \vdash \tau : \star \qquad x \notin \Gamma; \Delta; \Xi}{[\Gamma; \Delta; \Xi], x : \tau \Supset \Gamma, x : \tau; \Delta; \Xi}\ [\textsf{U-Ext}]$$

$$\frac{\Gamma \vdash \tau : \bullet \qquad x \notin \Gamma; \Delta; \Xi}{[\Gamma; \Delta; \Xi], x : \tau \Supset \Gamma; \Delta, x : \tau; \Xi}\ [\textsf{A-Ext}]$$

$$\frac{\Gamma \vdash \tau : \circ \qquad x \notin \Gamma; \Delta; \Xi}{[\Gamma; \Delta; \Xi], x : \tau \Supset \Gamma; \Delta; \Xi, x : \tau}\ [\textsf{L-Ext}]$$

$$\frac{[\Gamma;\cdot;\cdot], x : \tau_1 \Supset \Gamma';\Delta';\Xi' \qquad \Gamma';\Delta';\Xi' \vdash e : \tau_2}{\Gamma \vdash \lambda^\star(x : \tau_1).\, e : \tau_1 \xrightarrow{\star} \tau_2}\ [\textsf{U-Lam}]$$

$$\frac{[\Gamma;\Delta;\cdot], x : \tau_1 \Supset \Gamma';\Delta';\Xi' \qquad \Gamma';\Delta';\Xi' \vdash e : \tau_2}{\Gamma;\Delta \vdash \lambda^\bullet(x : \tau_1).\, e : \tau_1 \xrightarrow{\bullet} \tau_2}\ [\textsf{A-Lam}]$$

$$\frac{[\Gamma;\Delta;\Xi], x : \tau_1 \Supset \Gamma';\Delta';\Xi' \qquad \Gamma';\Delta';\Xi' \vdash e : \tau_2}{\Gamma;\Delta,\Xi \vdash \lambda^\circ(x : \tau_1).\, e : \tau_1 \xrightarrow{\circ} \tau_2}\ [\textsf{L-Lam}]$$

$$\frac{\Gamma;\Delta_1;\Xi_1 \vdash e_1 : \tau_1 \xrightarrow{\kappa} \tau_2 \qquad \Gamma;\Delta_2;\Xi_2 \vdash e_2 : \tau_1 \qquad \Delta_1 \Cup \Delta_2 = \Delta \qquad \Xi_1 \Cup \Xi_2 = \Xi}{\Gamma;\Delta;\Xi \vdash e_1 e_2 : \tau_2}\ [\textsf{App}]$$

$$\frac{\Gamma, \alpha : \kappa;\Delta;\Xi \vdash v : \tau}{\Gamma;\Delta;\Xi \vdash \Lambda(\alpha : \kappa).\, v : \forall \alpha{:}\kappa.\, \tau}\ [\textsf{TLam}]$$

$$\frac{\Gamma;\Delta;\Xi \vdash e : \forall \alpha{:}\kappa.\, \tau' \qquad \Gamma \vdash \tau : \kappa}{\Gamma;\Delta;\Xi \vdash e[\tau] : \tau'[\tau/\alpha]}\ [\textsf{TApp}]$$

Evaluation rules:

$$(\lambda^\kappa(x : \tau).\, e)v \mapsto e[v/x] \qquad (\Lambda(\alpha : \kappa).\, v)[\tau] \mapsto v[\tau/\alpha] \qquad \frac{e_1 \mapsto e_1'}{e_1 e_2 \mapsto e_1' e_2}$$

$$\frac{e \mapsto e'}{ve \mapsto ve'} \qquad \frac{e \mapsto e'}{e[\tau] \mapsto e'[\tau]}$$