CS 33

Data Representation (Part 3)

Floating-Point Operations: Basic Idea

- `x` $+_f$ `y = Round(x + y)`
- `x` $\times_f$ `y = Round(x` $\times$ `y)`
- Basic idea
  - first compute exact result
  - make it fit into desired precision
    - » possibly overflow if exponent too large
    - » possibly round to fit into `frac`

Rounding

- Rounding modes (illustrated with $ rounding)

	$1.40	$1.60	$1.50	$2.50	–$1.50
towards zero	$1	$1	$1	$2	–$1
round down (−∞)	$1	$1	$1	$2	–$2
round up (+∞)	$2	$2	$2	$3	–$1
nearest integer	$1	$2	?	?	?
nearest even (default)	$1	$2	$2	$2	–$2

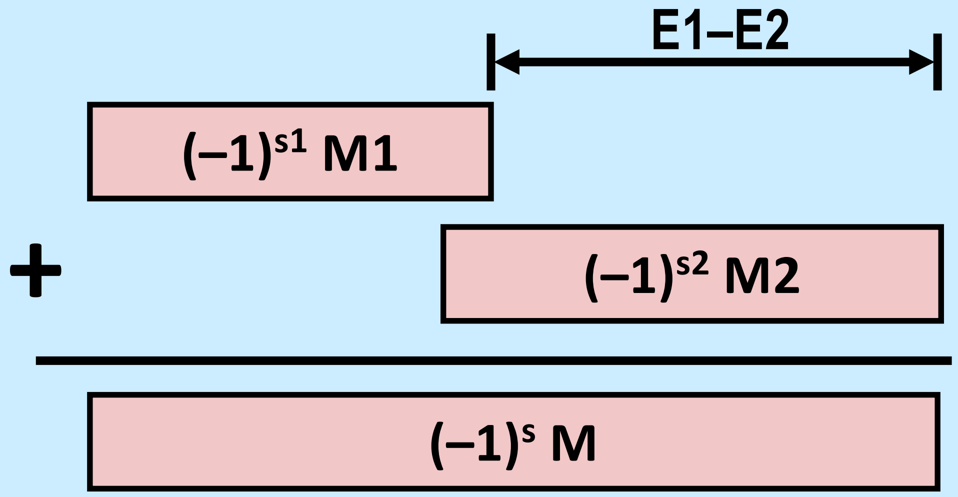
Floating-Point Multiplication

- $(-1)^{s1}$ M1 2^{E1} x $(-1)^{s2}$ M2 2^{E2}
- Exact result: $(-1)^{s}$ M 2^{E}
  - sign s: s1 ^ s2
  - significand M: M1 x M2
  - exponent E: E1 + E2

- Fixing
  - if M ≥ 2, shift M right, increment E
  - if E out of range, overflow (or underflow)
  - round M to fit `frac` precision
- Implementation
  - biggest chore is multiplying significands

Floating-Point Addition

- $(-1)^{s1}$ M1 2^{E1} + $(-1)^{s2}$ M2 2^{E2}
  - assume E1 > E2

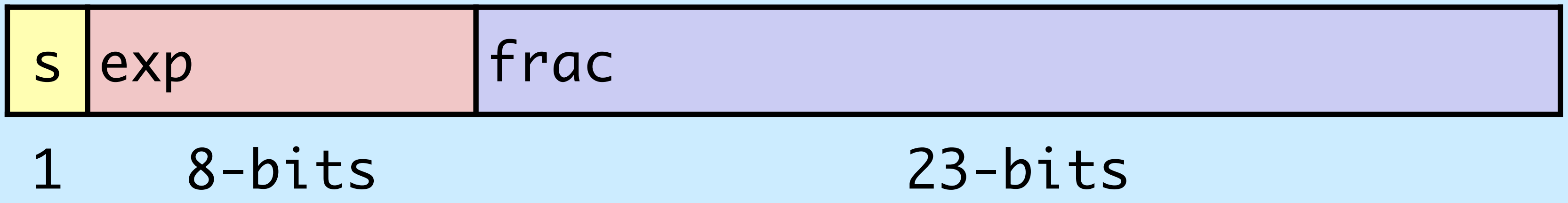


- **Exact result:** $(-1)^{s}$ M 2^{E}
  - sign s, significand M:
    - » result of signed align & add
  - exponent E: E1

- **Fixing**
  - if M ≥ 2, shift M right, increment E
  - if M < 1, shift M left k positions, decrement E by k
  - overflow if E out of range
  - round M to fit `frac` precision

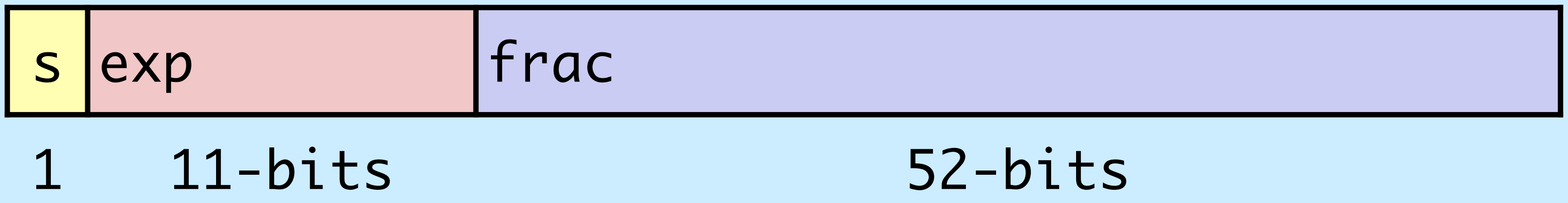
Floating Point

- **Single precision (float)**



  - **range: ±1.8×10^{-38} – ±3.4×10^{38}, ~7 decimal digits**

- **Double Precision (double)**



  - **range: ±2.23×10^{-308} – ±1.8×10^{308}, ~16 decimal digits**

Floating Point in C

- **Conversions/casting**
  - casting between `int`, `float`, and `double` changes bit representation
  - `double`/`float` → `int`
    - » truncates fractional part
    - » like rounding toward zero
    - » not defined when out of range or NaN: generally sets to TMin
  - `int` → `double`
    - » exact conversion, as long as `int` has ≤ 53-bit word size
  - `int` → `float`
    - » will round according to rounding mode

Quiz 1

Suppose *f*, declared to be a `float`, is assigned the largest possible floating-point positive value (other than +∞). What is the value of *g = f+1.0*?

a) f

b) +∞

c) NaN

d) 0

Float is not Rational …

- **Floating addition**
  - commutative: $a +_f b = b +_f a$
    - » yes!
  - associative: $a +_f (b +_f c) = (a +_f b) +_f c$
    - » no!
      - $2 +_f (1e38 +_f -1e38) = 2$
      - $(2 +_f 1e38) +_f -1e38 = 0$

Float is not Rational …

- **Multiplication**
  - commutative: $a *_f b = b *_f a$
    - » yes!
  - associative: $a *_f (b *_f c) = (a *_f b) *_f c$
    - » no!
      - $1e37 *_f (1e37 *_f 1e\text{-}37) = 1e37$
      - $(1e37 *_f 1e37) *_f 1e\text{-}37 = +\infty$

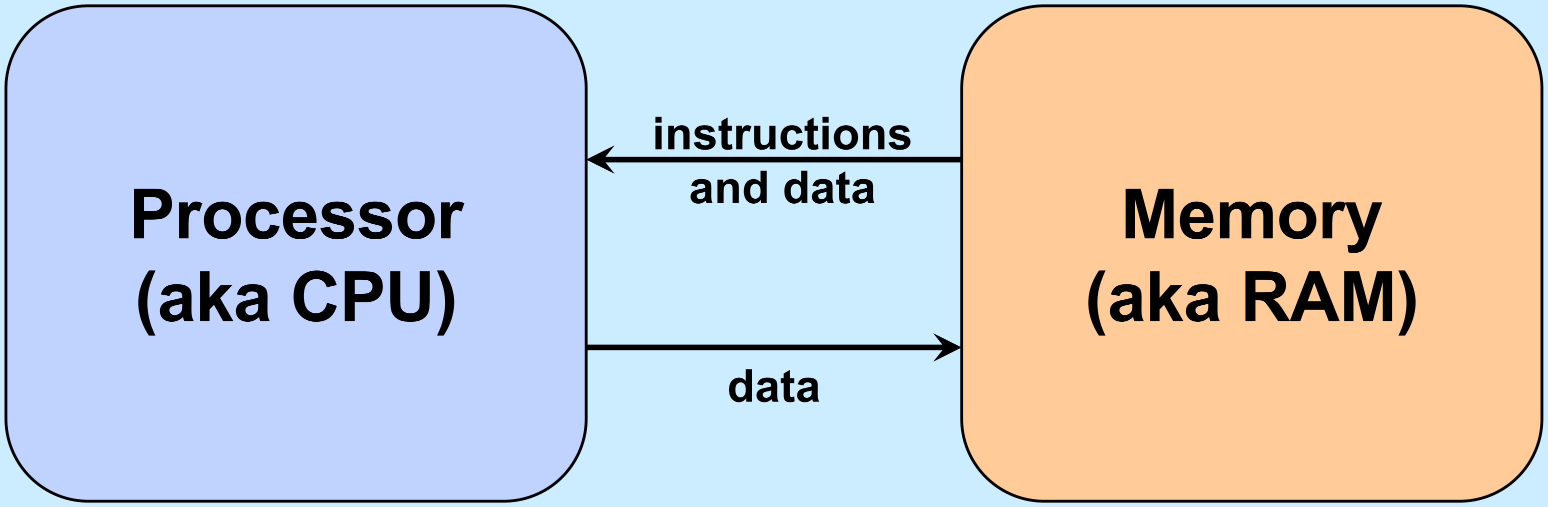
Float is not Rational …

- More …
  - multiplication distributes over addition:
    $a *_f (b +_f c) = (a *_f b) +_f (a *_f c)$
    » no!
    » $1e38 *_f (1e38 +_f -1e38) = 0$
    » $(1e38 *_f 1e38) +_f (1e38 *_f -1e38) = NaN$
  - insignificance:
    $x=y +_f 1$
    $z=2 /_f (x -_f y)$
    $z==2?$
    » not necessarily!
      - consider y = 1e38

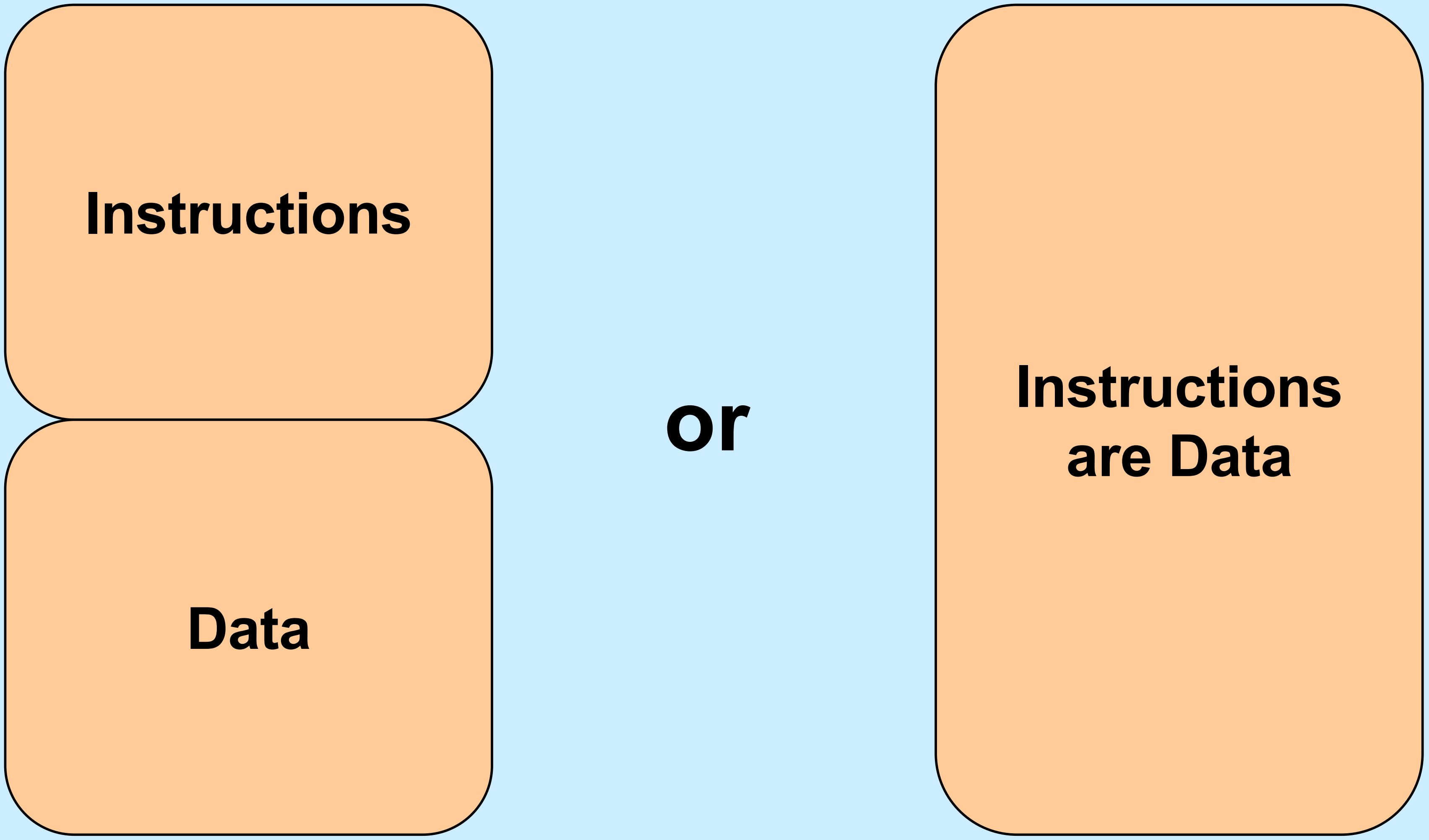
CS 33

Intro to Machine Programming

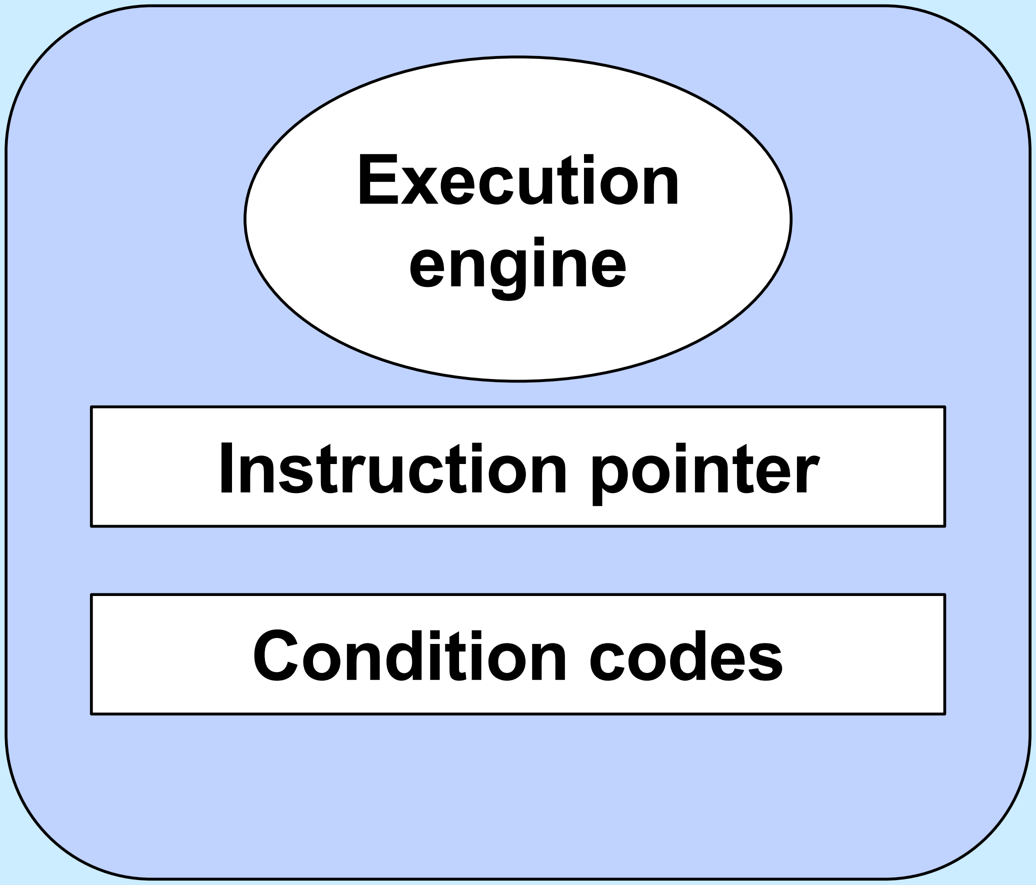
Machine Model



Memory



Processor: Some Details



Processor: Basic Operation

```
while (forever) {
  fetch instruction IP points at
  decode instruction
  fetch operands
  execute
  store results
  update IP and condition code
}
```

Instructions ...

Op code	**Operand1**	**Operand2**	**...**

Operands

- Form
  - immediate vs. reference
    - » value vs. address
- How many?
  - 3
    - » add a,b,c
      - c = a + b
  - 2
    - » add a,b
      - b += a

Operands (continued)

- **Accumulator**
  - special memory in the processor
    - known as a *register*
    - fast access
  - allows single-operand instructions
    - add a
      - acc += a
    - add b
      - acc += b

From C to Assembler ...

```
a = (b + c) * d;

   mov   b,%acc
   add   c,%acc
   mul   d,%acc
   mov   %acc,a
```

```
if (a<b)
   c = 1;
else
   d = 1;

   cmp   a,b
   jge   .L1
   mov   $1,c
   jmp   .L2
.L1
   mov   $1,d
.L2
```

immediate operand

immediate operand

Condition Codes

- Set of flags giving status of most recent operation:
  - zero flag
    - » result was zero
  - sign flag
    - » for signed arithmetic interpretation: sign bit is set
  - overflow flag
    - » for signed arithmetic interpretation
  - carry flag (generated by carry or borrow out of most-significant bit)
    - » for unsigned arithmetic interpretation
- Set implicitly by arithmetic instructions
- Set explicitly by compare instruction
  - cmp a,b
    - » sets flags based on result of b-a

Examples (1)

- Assume 32-bit arithmetic

- x is 0x80000000
  - TMIN if interpreted as two's-complement
  - 2^{31} if interpreted as unsigned
- x-1 (0x7fffffff)
  - TMAX if interpreted as two's-complement
  - 2^{31}-1 if interpreted as unsigned
  - zero flag is not set
  - sign flag is not set
  - overflow flag is set
  - carry flag is not set

Examples (2)

- x is 0xffffffff
  - -1 if interpreted as two's-complement
  - UMAX (2^{32}-1) if interpreted as unsigned
- x+1 (0x00000000)
  - zero under either interpretation
  - zero flag is set
  - sign flag is not set
  - overflow flag is not set
  - carry flag is set

Examples (3)

- x is 0xffffffff
  - -1 if interpreted as two's-complement
  - UMAX (2^{32}-1) if interpreted as unsigned
- x+2 (0x00000001)
  - (+)1 under either interpretation
  - zero flag is not set
  - sign flag is not set
  - overflow flag is not set
  - carry flag is set

Quiz 2

- Set of flags giving status of most recent operation:
  - zero flag
    - » result was zero
  - sign flag
    - » for signed arithmetic interpretation: sign bit is set
  - overflow flag
    - » for signed arithmetic interpretation
  - carry flag (generated by carry or borrow out of most-significant bit)
    - » for unsigned arithmetic interpretation
- Set explicitly by compare instruction
  - cmp a,b
    - » sets flags based on result of b-a

Which flags are set to one by “cmp 2,1”?

a) overflow flag only
b) carry flag only
c) sign and carry flags only
d) sign and overflow flags only
e) sign, overflow, and carry flags

Jump Instructions

- Unconditional jump
  - just do it
- Conditional jump
  - to jump or not to jump determined by condition-code flags
  - field in the op code indicates how this is computed
  - in assembler language, simply say
    - » je
      - jump on equal
    - » jne
      - jump on not equal
    - » jg
      - jump on greater than (signed)
    - » etc.

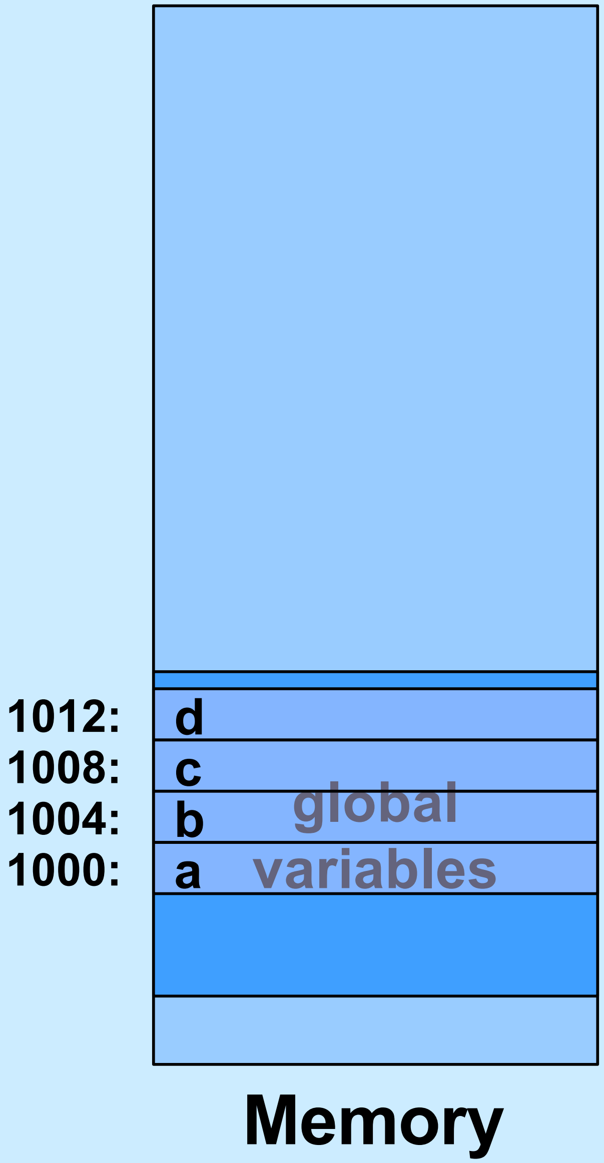
Addresses

```
int a, b, c, d;

int main() {
    a = (b + c) * d;
    ...
}
```

```
mov    b,%acc
add    c,%acc
mul    d,%acc
mov    %acc,a
```

```
mov    1004,%acc
add    1008,%acc
mul    1012,%acc
mov    %acc,1000
```



Addresses

```
int b;

int func(int c, int d) {
    int a;
    a = (b + c) * d;
    ...
}
```

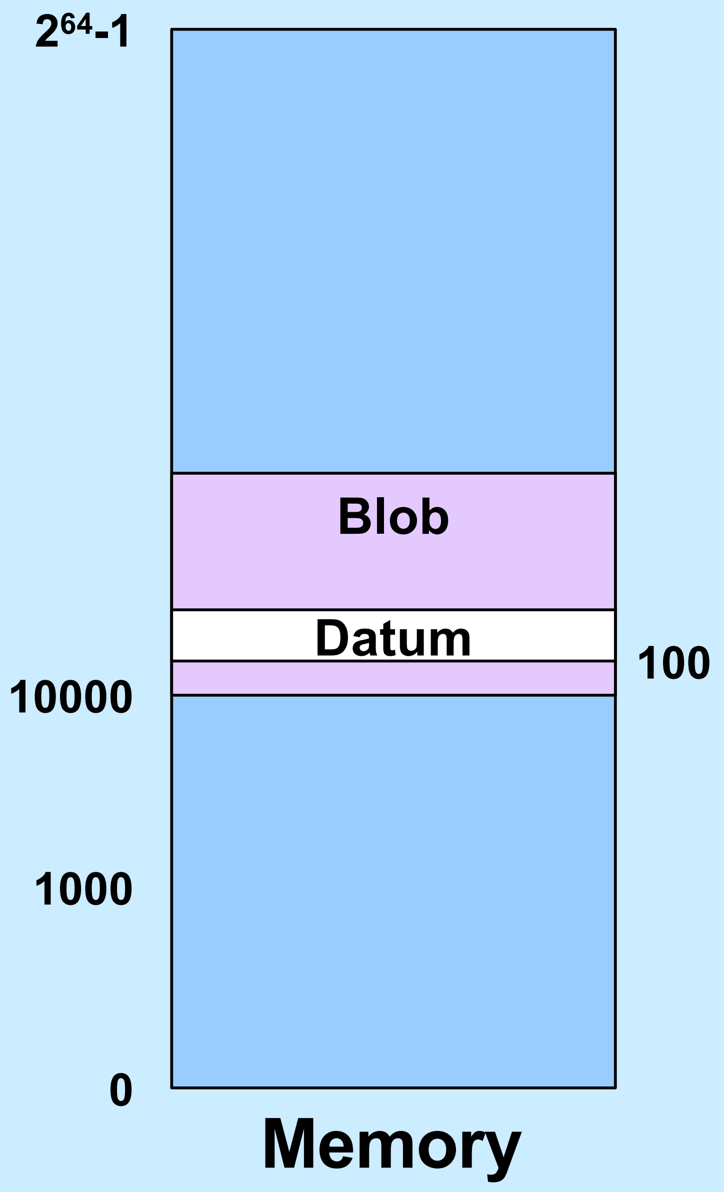
```
    mov    ?,%acc
    add    ?,%acc
    mul    ?,%acc
    mov    %acc,?
```

- One copy of *b* for duration of program's execution
  - *b*'s address is the same for each call to *func*
- Different copies of *a*, *c*, and *d* for each call to *func*
  - addresses are different in each call

Relative Addresses

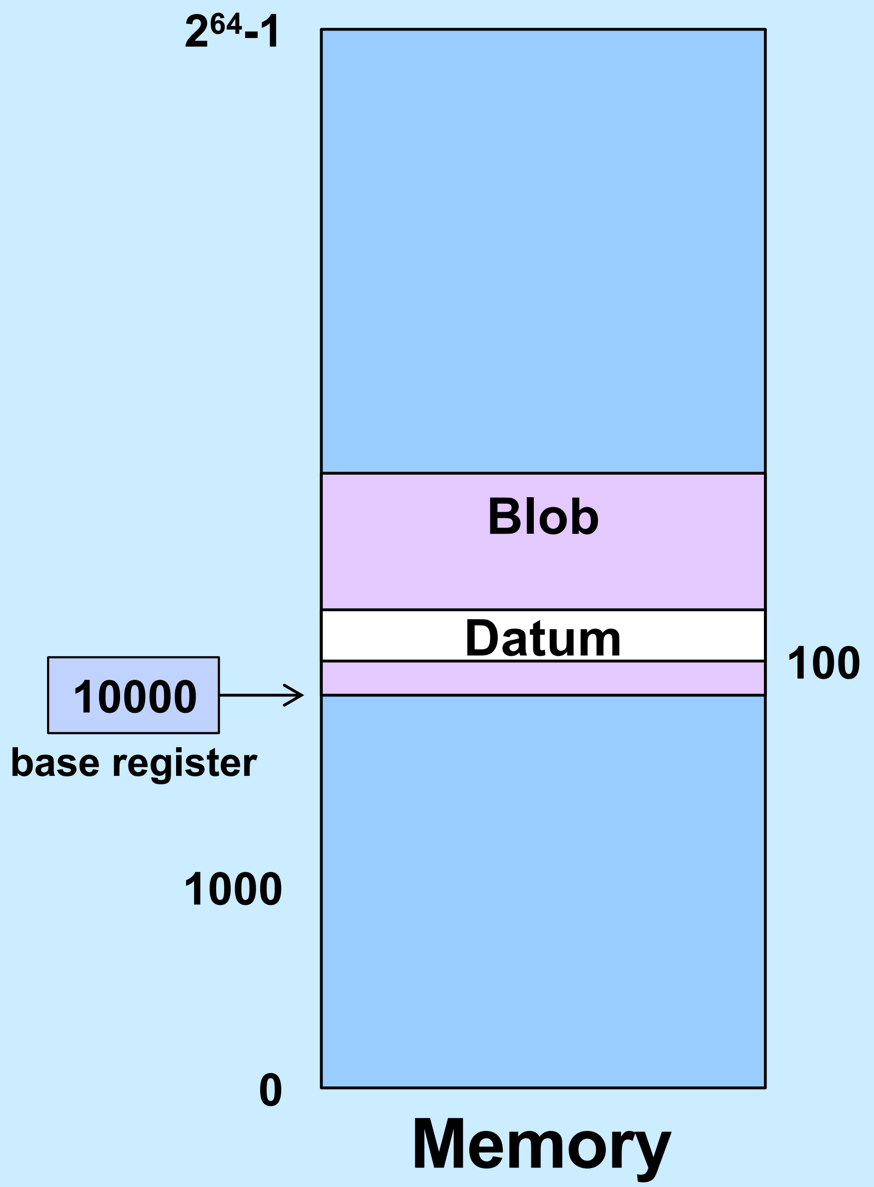
- **Absolute address**
  - actual location in memory
- **Relative address**
  - offset from some other location

- Blob's absolute address is 10000
- Datum's relative address (to Blob) is 100
  - its absolute address is 10100



Base Registers

```
mov $10000, %base
mov $10, 100(%base)
```

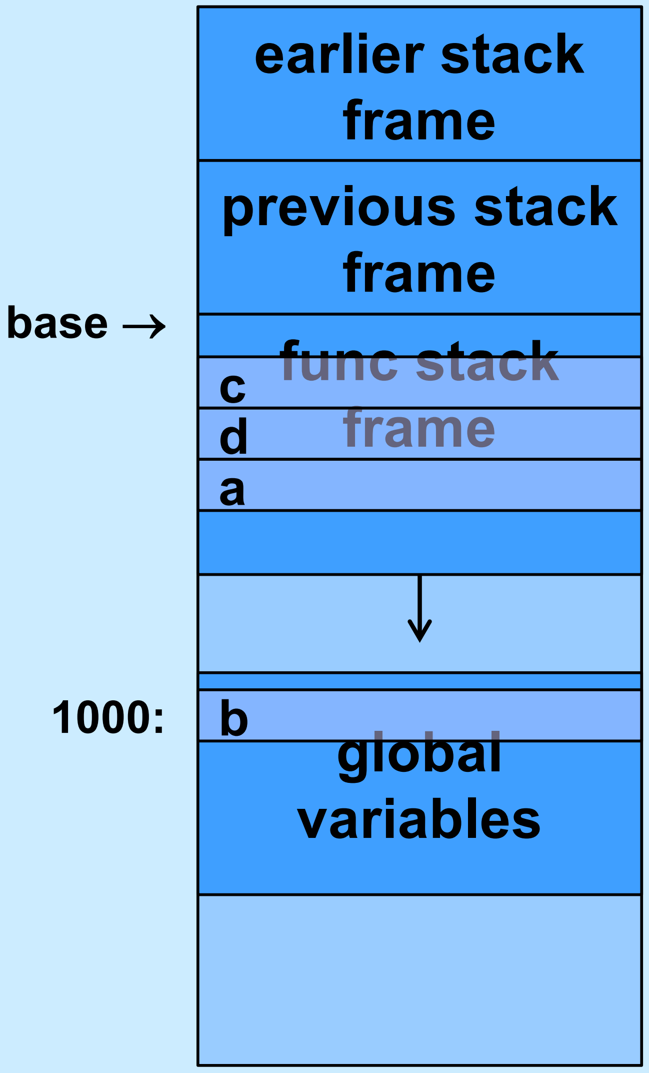


Addresses

```
long b;

int func(long c, long d) {
    long a;
    a = (b + c) * d;
    ...
}
```

```
    mov    1000,%acc
    add    -8(%base),%acc
    mul    -12(%base),%acc
    mov    %acc,-16(%base)
```



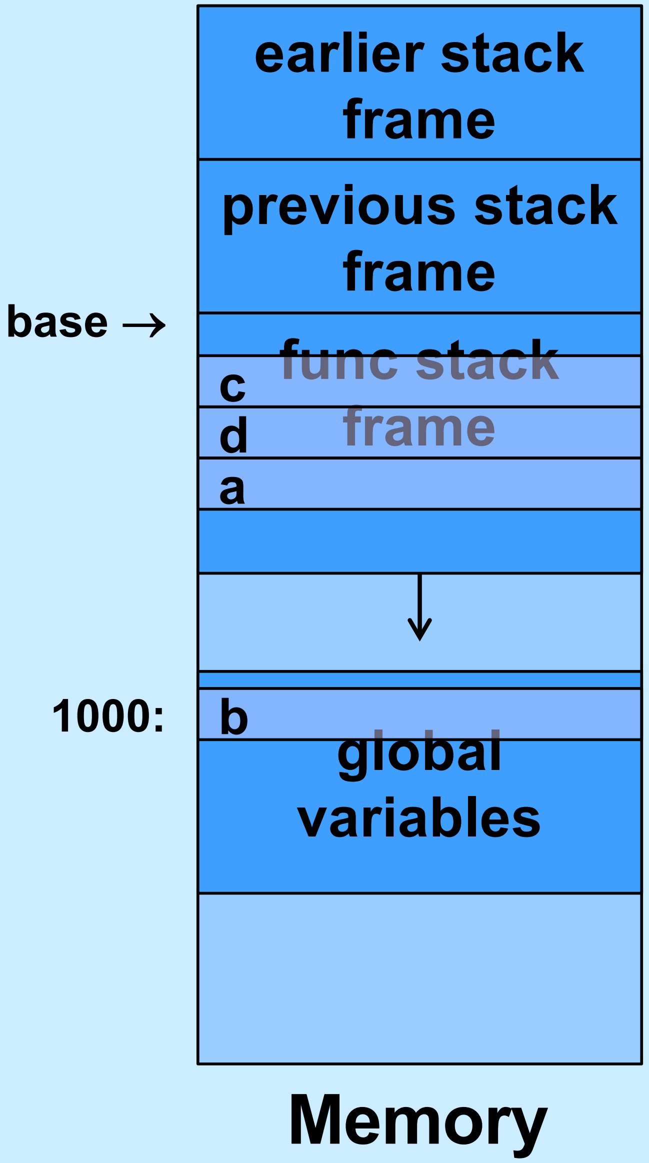
Memory

Quiz 3

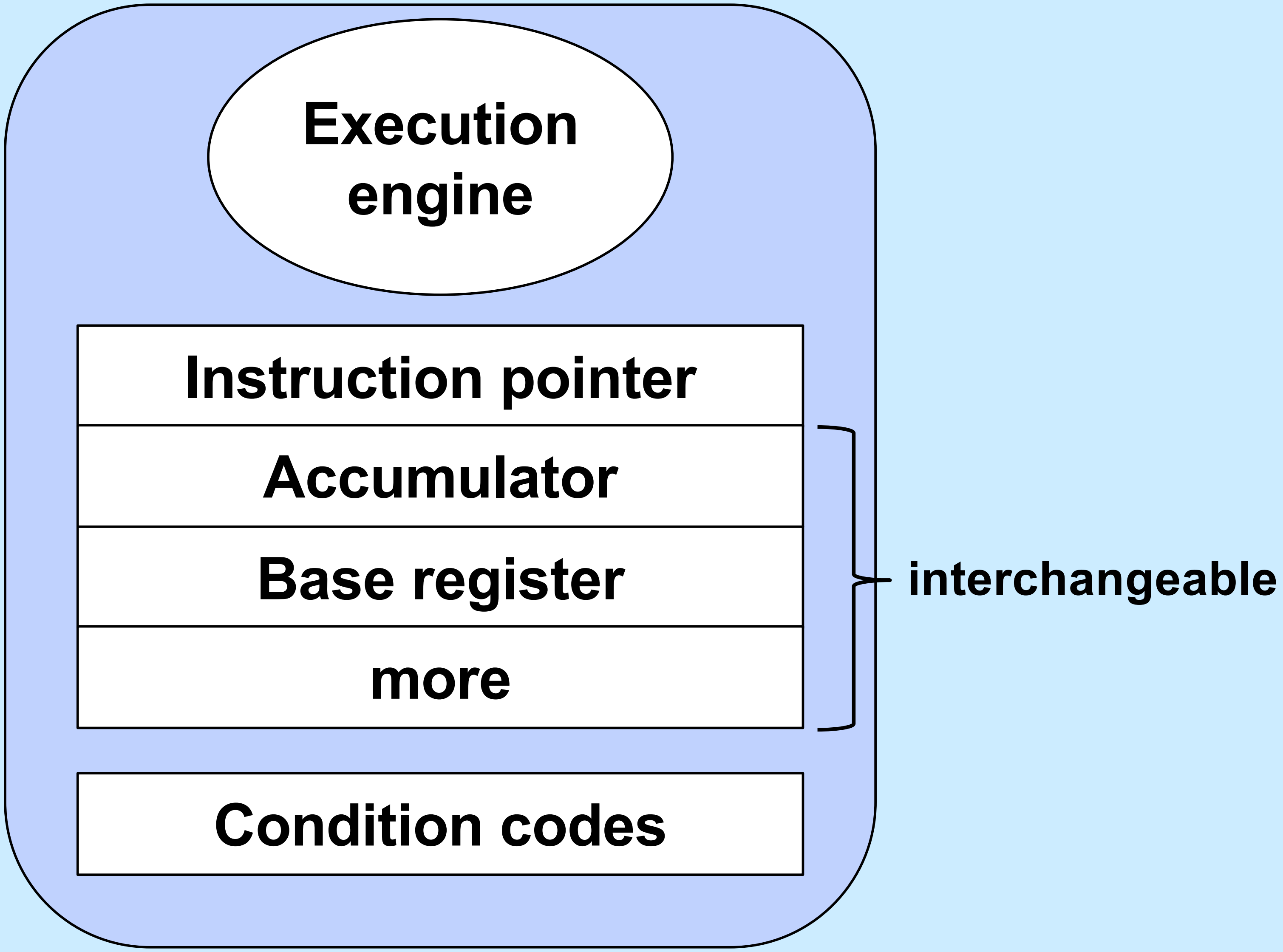
Suppose the value in *base* is 10,000. What is the address of *c*?

a) 9992
b) 9996
c) 10,004
d) 10,008

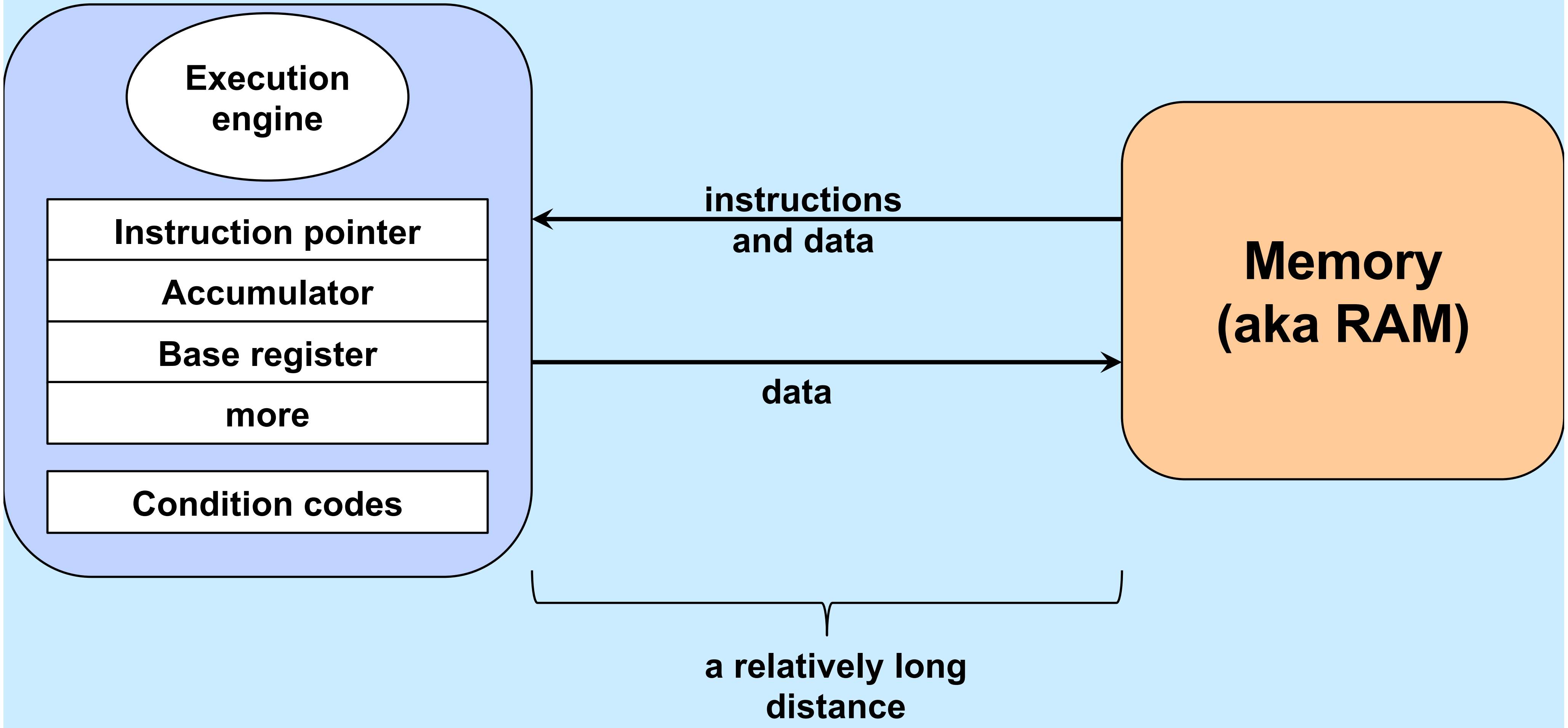
```
mov     1000,%acc
add     -8(%base),%acc
mul     -12(%base),%acc
mov     %acc,-16(%base)
```



Registers


Execution engine
Instruction pointer
Accumulator
Base register
more
interchangeable
Condition codes

Registers vs. Memory


Execution engine
Instruction pointer
Accumulator
Base register
more
Condition codes
instructions and data
data
Memory (aka RAM)
a relatively long distance