Logical Equivalences

Given booleans p, q, and r, the following logical equivalences hold:

1 Commutative Laws:

p & $q == q$ & p

$p \mid q == q \mid p$

2 Associative Laws:

$(p$ & $q)$ & $r == p$ & $(q$ & $r)$

$(p \mid q) \mid r == p \mid (q \mid r)$

3 Distributive Laws:

p & $(q \mid r) == (p$ & $q) \mid (p$ & $r)$

$p \mid (q$ & $r) == (p \mid q)$ & $(p \mid r)$

4 Identity Laws:

p & -1 $== p$

$p \mid 0 == p$

5 Negation Laws:

p & $(\sim p) == 0$

$p \mid (\sim p) ==$ -1

6 Double Negate Law:

$$\sim(\sim p) == p$$

7 Idempotent Laws:

$$p \ \& \ p == p$$

$$p \mid p == p$$

8 Universal Bound Laws:

$$p \ \& \ 0 == 0$$

$$p \mid \text{-1} == \text{-1}$$

9 De Morgan's Laws:

$$\sim(p \ \& \ q) == \sim p \mid \sim q$$

$$\sim(p \mid q) == \sim p \ \& \ \sim q$$

10 Absorption Laws:

$$p \mid (p \ \& \ q) == p$$

$$p \ \& \ (p \mid q) == p$$

11 Logical Negation of 0 and not 0:

$$!\text{<non-zero>} == 0$$

$$!0 == 1$$