Bayesian non-parametric models for machine learning

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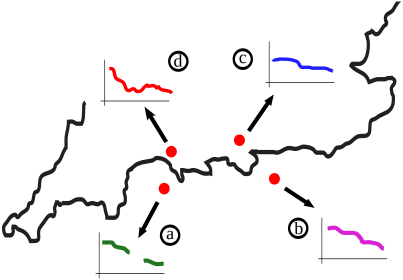
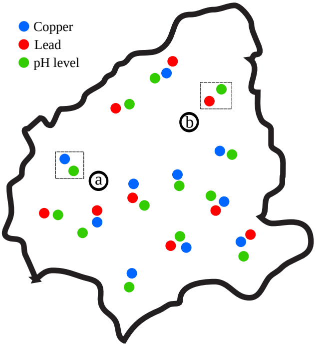
Lecturer in Machine Learning since January 2017.

Contents

Dependencies between related processes



A network of sensors



Pollutant metals concentrations

Latent variable/function models

- Consider a set of processes $\{f_d(\mathbf{x})\}_{d=1}^{D}$, with $\mathbf{x} \in \mathcal{X}$.

- Each function can be expressed as

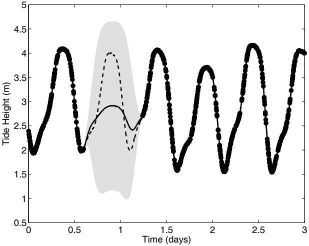
$$f_d(\mathbf{x}) = \int_{\mathcal{X}} G_d(\mathbf{x} - \mathbf{z}) u(\mathbf{z}) \mathrm{d}\mathbf{z} = G_d(\mathbf{x}) * u(\mathbf{x}).$$

- If $u(\mathbf{x})$ is a GP, then $f_d(\mathbf{x})$ is also a GP.

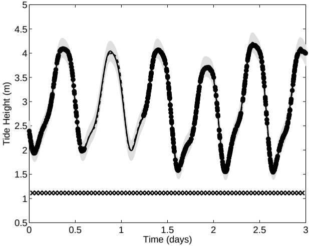
- We could also include more latent processes $u_1(\mathbf{x}), u_2(\mathbf{x}), \ldots, u_Q(\mathbf{x})$

$$f_d(\mathbf{x}) = \sum_{q=1}^{Q} \int_{\mathcal{X}} G_{d,q}(\mathbf{x} - \mathbf{z}) u_q(\mathbf{z}) \mathrm{d}\mathbf{z}.$$

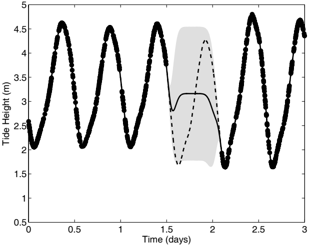
Example: Predicting tide height



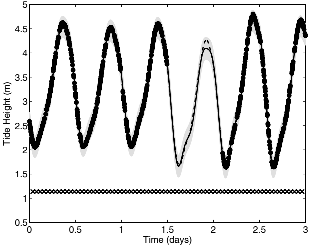
Bramblemet, independent



Bramblemet, multi-task



Cambermet, independent



Cambermet, multi-task

Extensions

- Hierarchical multi-task learning.
- Semi-supervised multi-task learning.
- Multi-resolution multi-task learning.
- Model selection with the Indian Buffet Process.

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Green's functions

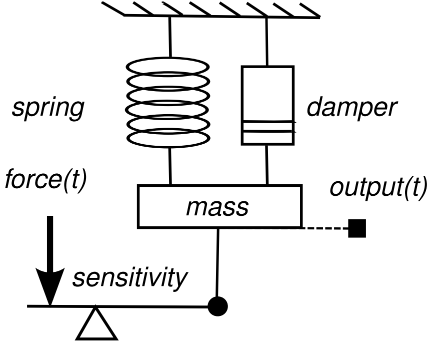
- As we saw before, we can express processes $f_d(\mathbf{x})$ using

$$f_d(\mathbf{x}) = \int_{\mathcal{X}} G_d(\mathbf{x} - \mathbf{z})u(\mathbf{z})\mathrm{d}\mathbf{z}$$

- Function $G_d(\mathbf{x} - \mathbf{z})$ might be related to the so called Green's function of a dynamical system.

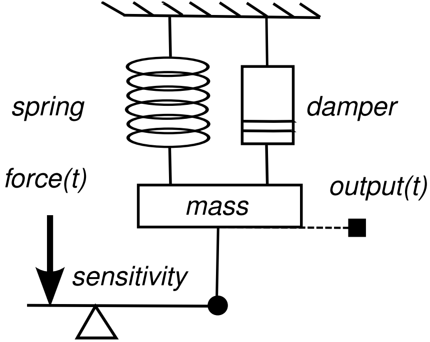
- We can encode mechanistic properties in data-driven models.

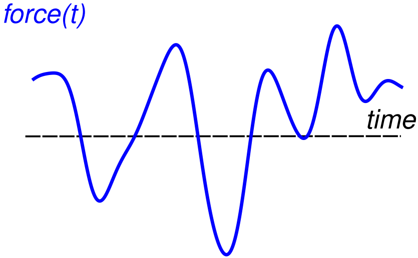
Example: a second order dynamical system



force(t) $\sim \mathcal{GP}$*(0, k(t,t'))*

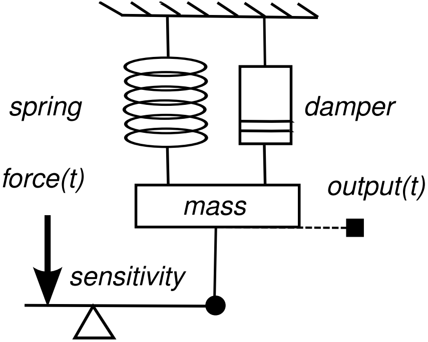
Example: a second order dynamical system



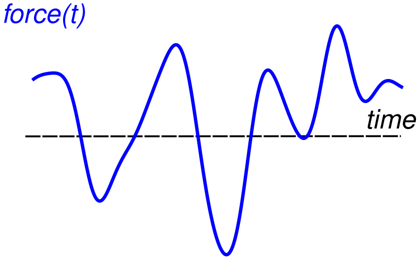


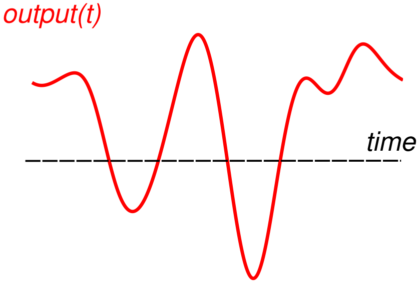
force(t) $\sim \mathcal{GP}(0, k(t,t'))$

Example: a second order dynamical system



$$force(t) \sim \mathcal{GP}(0, k(t,t'))$$





Example: a second order dynamical system

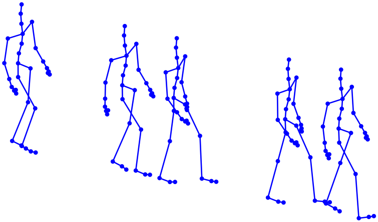
- A second order dynamical system can be described by

$$mass_d \frac{\mathrm{d}^2 f_d(t)}{\mathrm{d}t^2} + damper_d \frac{\mathrm{d}f_d(t)}{\mathrm{d}t} + spring_d f_d(t) = u(t).$$

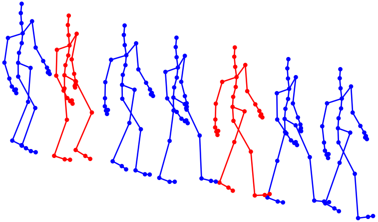
- There is a Green's function associated to this equation.

- We can compute things like $p(u|f)$ (Bayesian inverse problems) or $p(f^*|f)$ (predictive modeling).

Human motion description

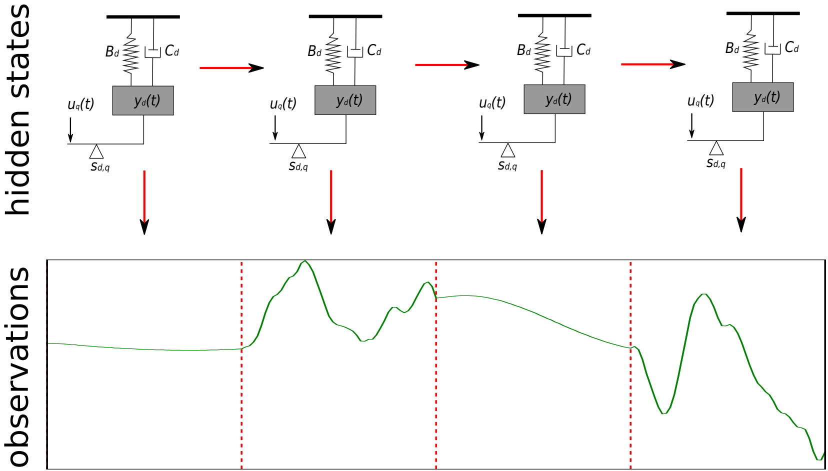


Walking movement with missing poses



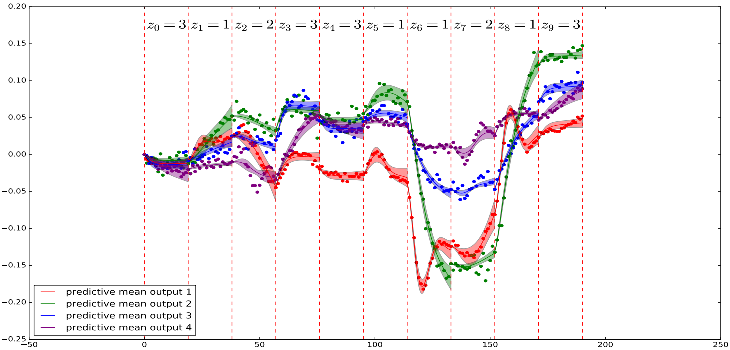
Frames have been filled with plausible poses

Semi-parametric LFM: HMM + LFM



- Motor primitive representation: Latent Force Models (LFM).
- Motor primitives sequential dynamics: Hidden Markov Models (HMM).

Synthetic example



The correct hidden state was recovered with a success rate of 95% failing only in 10 out of 200 validation segments (10/20 trajectories for validation).

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Deep Brain Stimulation for Parkinson's patients

Deep Brain Stimulation

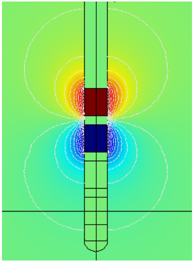
Volume of tissue activated

Stimulation parameters:

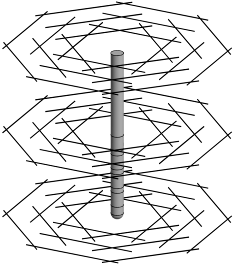
- Voltage Amplitude.
- Pulse width.
- Contacts: cathode, anode or switched-off.

VTA estimation - Gold standard

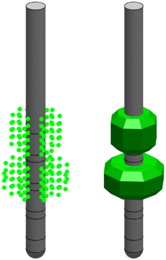
Electric potential (FEM model)



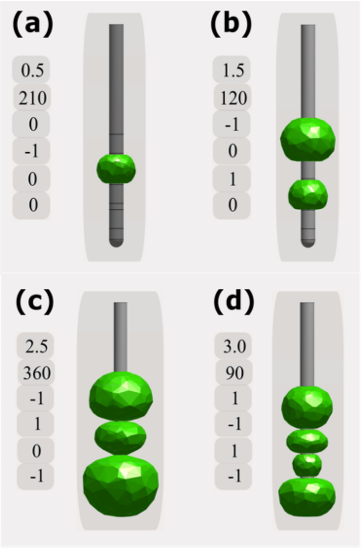
Multicompartment axon model



Volume of tissue activated (VTA)



Machine learning challenges



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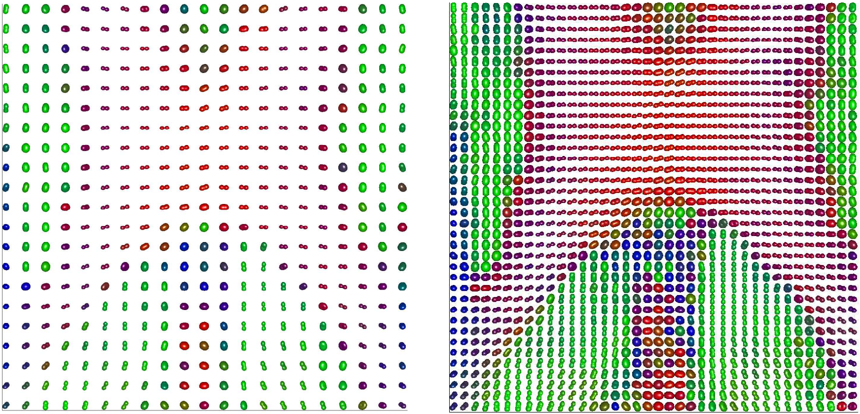
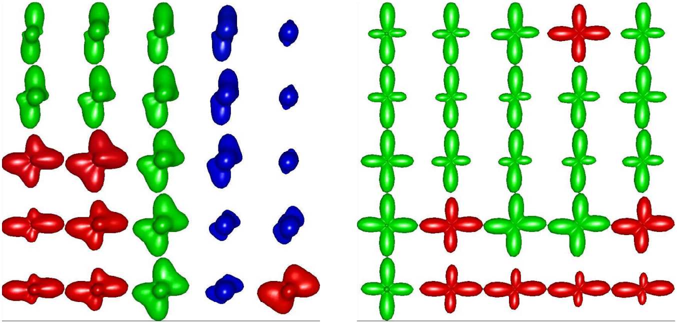
Resolution enhancement for diffusion tensor imaging



Real tensor field (left). Enhanced tensor field (right).

High-order tensor field interpolation



Examples of HOT fields: (left) rank-4; (right) rank-6