Induction

Strong Induction & Weak Induction

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What is Induction?

- Proof technique

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- Simple yet efficient method

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- Proof technique
- Simple yet efficient method
- Proves a statement for a set of numbers

Steps & Types

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- Base Step
- Inductive Step

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Types

- Weak Induction (Regular Induction)
- Inductive Step

Principles of *Weak Induction*

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Principles of *Strong Induction*

1. **Base Step**: verify $P(1) \implies P(0)$
2. **Inductive Step**: prove $\forall k \in N : k \leq n, P(k) \implies P(n+1)$

Equivalence of Weak & Strong Induction

- If a statement can be proven by **Weak Induction**, it can be proven by **Strong Induction**
- If a statement can be proven by **Strong Induction**, it can be proven by **Weak Induction**
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If provable by Strong, can be proven by Weak

Let, a statement be P. $\forall n \in N, P(n)$ If P can be proven by **Strong Induction**

1. $P(1)$ is true
2. $\forall k \in N : k \leq P(k) \implies P(n+1)$ We are to prove, P can be proven using *Weak Induction*

If provable by Strong, can be proven by Weak

Define,

$$
\begin{aligned}
Q(n) &= \forall k, n \in N : k \leq n, P(k) \\
&= P(1) \wedge P(2) \wedge P(3) \ldots P(n)
\end{aligned}
$$

It is Obvious, $Q(1)$ is true, as $Q(1) = P(1)$
Rewriting we get, $\forall n \in N, Q(n) \implies P(n+1)$
As, $(A \implies B) \wedge (A \implies C) \implies (A \implies (B \wedge C))$
So, we get,

$$
\begin{aligned}
Q(n) &\implies Q(n) \wedge P(n+1) \\
&\implies P(1) \wedge P(2) \ldots P(n) \wedge P(n+1) \\
Q(n) &\implies Q(n+1)
\end{aligned}
$$

If provable by Strong, can be proven by Weak

Now,

$$Q(1) \wedge (\forall n \in N, Q(n) \implies Q(n+1)) \implies \forall n \in N, Q(n)$$

which is the weak inductive principle
From our assumption, $Q(n)$ can never be true without $P(n)$
So, $Q(n) \implies P(n)$
Therefore, the statement $P(n)$ for all n is just proved by using weak induction.