Basic Trigonometric Identities and Equations

August 29, 2022

Trigonometric Identities

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad \cot^2\theta + 1 = \csc^2\theta$$

$$\sin^2\theta = 1 - \cos^2\theta \qquad \tan^2\theta + 1 = \sec^2\theta \qquad cot\hat{2}\theta = \csc^2\theta - 1$$

$$\cos^2\theta = 1 - \sin^2\theta$$

Where did our pythagorean identities come from??

Do you remember the Unit Circle?

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- What does x = ? What does y = ?
  (in terms of trig functions)

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- What is the equation for the unit circle?

$$x^2 + y^2 = 1$$

- What does x $=$? What does y $=$?

(in terms of trig functions)

$$\sin^2\theta + \cos^2\theta = 1$$

Using the identities you now know, find the trig value.

1) If $\cos\theta = \frac{3}{4}$, find $\sec\theta$

2) If $\cos\theta = \frac{3}{5}$, find $\csc\theta$

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2) If $\cos\theta = \frac{3}{5}$, find $\csc\theta$

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2) If $\cos\theta = \frac{3}{5}$, find $\csc\theta$

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta + (\tfrac{3}{5})^2 = 1$$
$$\sin^2\theta = \tfrac{25}{25} - \tfrac{9}{25}$$
$$\sqrt{\sin^2\theta} = \sqrt{\tfrac{16}{25}}$$
$$\sin\theta = \pm\tfrac{4}{5}$$
$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\pm\frac{4}{5}} = \pm\frac{5}{4}$$

Simplify each expression.

$\frac{\csc\theta}{\cot\theta}$ | $\cos x \csc x \tan x$ | $\cos x \cot x + \sin x$

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$\frac{\csc\theta}{\cot\theta}$

$\frac{\frac{1}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}}$

$\frac{1}{\sin\theta}\cdot\frac{\sin\theta}{\cos\theta}$

$\frac{1}{\cos\theta}=\sec\theta$

$\cos x \csc x \tan x$

$\cos x \cot x + \sin x$

Simplify each expression.

$\frac{\csc\theta}{\cot\theta}$

$\frac{\frac{1}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}}$

$\frac{1}{\sin\theta}\cdot\frac{\sin\theta}{\cos\theta}$

$\frac{1}{\cos\theta} = \sec\theta$

$\cos x \csc x \tan x$

$= \cos x(\frac{1}{\sin x})(\frac{\sin x}{\cos x})$

$= 1$

$\cos x \cot x + \sin x$

$\cos x(\frac{\cos x}{\sin x}) + \sin x$

$\frac{\cos^2 x}{\sin x} + \frac{\sin^2 x}{\sin x}$

$\frac{\cos^2 x + \sin^2 x}{\sin x}$

$\frac{1}{\sin x} = \csc x$

Practice

$\sec\theta\cot\theta\sin\theta$	$\sin^2\theta(\csc^2\theta-1)$	$\cot\theta\sin\theta$	$\frac{1-\sin^2\theta}{1+\cot^2\theta}$
1	$\cos^2\theta$	$\cos\theta$	$\sin^2\theta\cos^2\theta$
$(1-\cos\theta)(1+\sec\theta)$	$\cot^2\theta(1+\tan^2\theta)$	$\cos\theta\csc\theta$	$\frac{1+\tan^2\theta}{1+\cot^2\theta}$

Practice

$\sec\theta\cot\theta\sin\theta$	$\sin^2\theta(\csc^2\theta-1)$	$\cot\theta\sin\theta$	$\frac{1-\sin^2\theta}{1+\cot^2\theta}$
1	$\cos^2\theta$	$\cos\theta$	$\sin^2\theta\cos^2\theta$
$(1-\cos\theta)(1+\sec\theta)$	$\cot^2\theta(1+\tan^2\theta)$	$\cos\theta\csc\theta$	$\frac{1+\tan^2\theta}{1+\cot^2\theta}$
$\sec\theta-\cos\theta$	$\csc^2\theta$	$\cot\theta$	$\tan^2\theta$