University of Jaffna, Sri Lanka

General Degree Examination in Science – Level 1G, 2010

End of Course Examination

Physics

PHY102GC3 : Mechanics, Relativity and Structure of matter

By selecting at least ***one*** question from each section answer any ***five*** questions.

Time: ***Three*** hours.

Velocity of light in vacuum (c)	=	$3 \times 10^{8}\ ms^{-1}$
Charge of electron (e)	=	1.60×10^{-19} C
Planck's constant (h)	=	6.63×10^{-34} Js
Permeability of vacuum (ε_0)	=	$8.85 \times 10^{-12}\ N^{-1}m^{-2}C^{2}$
Mass of electron (m_e)	=	9.11×10^{-31} kg
Mass of Hydrogen atom (m_H)	=	1.67×10^{-27} kg

SECTION – A

1. State Newton's second law of motion.

   (a) Neglecting the effects of air resistance, show that the equation of motion of a rocket, which is subject to a continuous force derived from the ejection of fuel is given by $m\frac{dv}{dt} + u\frac{dm}{dt} = -mg$, where m is the mass of the rocket at time t, u is the velocity of the escaping burnt fuel relative to the rocket and v is the velocity of the rocket with respect to the Earth at time t.

   (b) Show that the final velocity of the rocket at time t is given by $v_f = u\ln(m_i/m_f) - gt$, where m_i is the initial mass of the rocket.

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(c) If the mass of the rocket (including fuel) at a time t is $M_0(1-\frac{1}{5}t)$ then,

(i) find the upthrust of the rocket.

(ii) obtain an expression for the initial acceleration of the rocket.

(iii) When the mass of the rocket become half of its initial value show that the speed of the rocket is given by $v = u\ln 2 - \frac{5}{2}g$. What is the height of the rocket above the launching pad when the mass is halved?

You may find the following information useful: $\int \ln x.dx = x(\ln x - 1)$

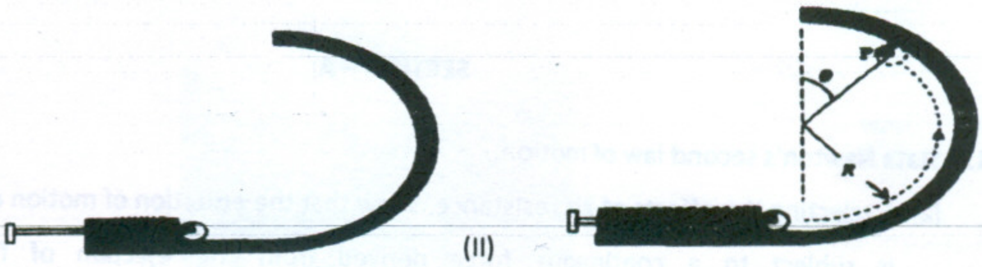
2. State the principle of conservation of mechanical energy.

A spring of negligible mass exerts a restoring force given by $F = k_1 - k_2 x$, where k_1 and k_2 are spring constants.

(a) Calculate the potential energy U stored in the spring for an extension x of the spring, taking $U = 0$ at $x = 0$.

(b) It is found that the stored energy when x = -b is twice the stored energy when x = b. How is k_2 related to k_1 and b?

(c) Sketch the potential energy versus x diagram for the spring.



(d) This spring and a plunger device is used to propel a ball of mass m around a vertical semi-circular track as shown in the diagram above. When the plunger is pulled to the left, the spring is compressed (Figure I). When the plunger is released (Figure II), the spring returns back to its natural length and the ball is propelled around the track. The radius of the curved portion of the path traversed by the ball is R. Treat the ball as a particle and assume that the track is frictionless. Ignore air resistance.

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(i) Write down the equation of motion in the radial direction when the ball is at P.

(ii) If the ball leaves the track at point P, show that the speed of the ball at P is given by $v = \sqrt{Rg\cos\theta}$.

(iii) Using the principle of conservation of mechanical energy, show that the initial compression x of the spring required for the ball to leave the track at P is given by

$$x(3x-4b) = \frac{8mgbR}{k_1}\left(1+\frac{3}{2}\cos\theta\right).$$

(iv) Find the initial compression x of the spring, required to bring the particle to the highest point of the arc.

3. (I)

(a) State the theorem of parallel axis regarding the moment of inertia of a rigid body.

(b) A uniform semi circular lamina of radius a has AOB as its diameter, and OC as the radius perpendicular to AOB. Show that the radius of gyration of the lamina about an axis passing through C and normal to its plane is $a\sqrt{\frac{3}{2}-\frac{8}{3\pi}}$.

(c) Find the period of small oscillations of the lamina about the axis passing through C and normal to its plane.

(II)

(a) State the law of conservation of angular momentum.

(b) Assume that the lamina described above in part (I) is at stable equilibrium with AOB horizontal when it is hung from the point C so that it is free to oscillate in its own plane about the axis through C normal to the lamina. A bullet of mass m is fired vertically with speed v towards the point B. The bullet strikes the diameter AOB normally at the point B and becomes embedded. Find the angular speed with which the lamina with the embedded bullet begins to move.

You may find the following information useful:

The moment of inertia of a semi circular lamina about an axis passing through the centre O and perpendicular to the plane of the lamina is given by $I = \frac{1}{2}Ma^2$ where M is the mass and a is the radius of the lamina. The centre of gravity of the semi circular lamina is at a distance $\left(\frac{4a}{3\pi}\right)$ from the centre O of the lamina.

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4. State and prove the Bernoulli's theorem in fluid dynamics.

Explain the working principle of a Venturimeter.

(a) A Venturimeter of throat diameter 0.076m is fitted to a vertical pipe of diameter 0.152m in which a liquid of density 800 kg/m^3 flows downwards. The throat and the portion of the pipe 0.914m above the throat are connected to the two ends of a mercury U-tube manometer. Take the coefficient of discharge of the Venturimeter as 0.97. If the manometer shows a pressure difference of 15170 N/m^2, find the volume of liquid flowing through the pipe per unit time.

(b) If the velocity (v) of flow along the pipe is related to the difference in level H of the mercury columns of the manometer by the equation $v=25\sqrt{H}$. Calculate the value of H. Density of the mercury is 13 600 kg/m^3.

Section B

5. (a) The coordinates of an event when measured in inertial frames Σ' and Σ are (x', y', z', t') and (x, y, z, t) respectively. The frame Σ' travels relative to the frame Σ with speed v along the common $x - x'$ axis. Write down the Lorentz transformation equations to relate the coordinates (x', y', z', t') with (x, y, z, t).

(b) Show that two simultaneous events in Σ' frame are not simultaneous events in Σ frame.

(c) Obtain the Lorentz velocity transformation equations.

(d) Two spaceships A and B are approaching each other in perpendicular directions as observed by an inertial observer. Suppose A moves with speed 0.8c along the x-axis and B moves with speed 0.8 c along the $(-y)$ axis, estimate the speed and the direction of the ship B as observed by an observer at rest in the ship A.

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6. The relativistic equation for the energy E of a particle having rest mass m_0 and momentum p is given by $E^2 = p^2c^2 + m_0^2c^4$, where c is the speed of light.

(a) Write down the expression for the kinetic energy T.

(b) Show that $m_0 = (p^2c^2 - T^2)/2Tc^2$.

(c) An unstable particle A at rest decays into particles B and C having kinetic energies 200 MeV and 50 MeV respectively. Each particle carries momentum of magnitude 300MeV/c. Determine the following:

(i) Rest masses of the particles B and C.

(ii) Energies of the particles B and C.

(iii) Energy and rest mass of particle A.

PART – C

7. (a) State the postulates of the Bohr theory of a hydrogen like atom.

(b) Assuming that the nucleus is at rest, derive an expression for the total energy of the electron in the n[th] Bohr orbit of a hydrogen like atom.

(c) Derive an expression for the difference in wavelength between the main lines of the Balmer and Lyman series of the atomic spectra of a hydrogen like atom?

(d) What is the atomic number of the hydrogen like atom which has a difference in wavelength of 666 Å between the main lines of the Balmer and Lyman series?

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8. (a) What is meant by the isotopes of an element?

(b) Describe briefly, with the help of a clearly labelled diagram, the operating principle of a Bainbridge mass spectrograph.

(c) The velocity selector in a Bainbridge mass spectrograph has an electric field of 10^3 *V/m* and a magnetic field of 0.5 Wb m^{-2}. Singly charged neon ions that pass through the velocity selector enter the circular portion of the spectrograph which has a magnetic field of the same strength.

(i) What is the speed of the ion that enters the circular portion of the spectrograph?

(ii) Find the distance between the centres of the curved lines formed on the photographic plate of the spectrograph by the isotopes of neon having mass numbers 20, 21 and 22.

Derive any formula you may use.

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