University of Jaffna, Sri Lanka
General Degree Examination in Science – Level 1G -2008
End of Course Examination

Physics
PHY102GC3 : Mechanics, Relativity and Structure of Matter

By selecting at least ***one*** question from each section answer any ***five*** Questions.

Time: ***Three*** hours.

Acceleration due to gravity at the Earth's surface (g)	=	$10.0\ ms^{-2}$
Velocity of light in vacuum (c)	=	$3x10^{8}\ ms^{-1}$
Electronic charge (e)	=	1.6×10^{-19} C
Avagadro number (N_A)	=	$6.02 \times 10^{23}\ mol^{-1}$
Permittivity of vacuum (ε_0)	=	$8.85 \times 10^{-12}\ Fm^{-1}$

SECTION – A

1. State Newton's second law of motion.
   (a) Neglecting the effects of gravitation and air resistance, show that the equation of motion of a rocket, which is subject to a continuous force derived from the ejection of fuel is given by $m\frac{dv}{dt}+u\frac{dm}{dt}=0$, where *m* is the mass of the rocket at time *t*, *u* is the velocity of the escaping burnt fuel relative to the rocket and *v* is the velocity of the rocket with respect to the Earth at time *t*.
   (b) Show that the final velocity of the rocket is given by $v_f = u\ln(m_i/(m_i-\lambda t_0))$, where m_i is the initial mass of the rocket and t_0 is time taken to burn out the fuel. The fuel burns at a constant rate of λ kgs^{-1}.
   (c) Show that the height reached by the rocket at the burn out of the fuel is given by,
   $h=ut_0\ln[m_i/(m_i-\lambda t_0)]+\frac{u}{\lambda}\{m_i\ln[(m_i-\lambda t_0)/m_i]\}+ut_0$.

→ *Question 1 Continued*

A rocket of initial mass M_O, carrying a payload of mass M_p, is to be launched vertically form rest at the surface of the Earth in the absence of gravity and other resisting forces. The rocket is designed as a two stage rocket, with 75% of its initial mass of each stage as its fuel, but with the second stage being half the mass of the first stage. The rocket burns the fuel at a constant rate in both stages.

(d) Show that the speed of the rocket at the end of the first stage of the travelling is

$$u \ln\left[\frac{M_P + M_0}{M_P + \frac{1}{2}M_0}\right]$$, where u velocity of the escaping burnt fuel relative to the rocket.

(e) Find the time taken to complete the first stage of the travelling.

(f) Find the height of the rocket at the end of the first stage travelling of the rocket.

(g) Show that the maximum speed attained by the payload is given by,

$$u \ln\left[\frac{M_P + M_0}{M_P + \frac{1}{2}M_0} \cdot \frac{M_P + \frac{1}{3}M_0}{M_P + \frac{1}{12}M_0}\right]$$

You may find the following information useful:

$$\int \ln x . dx = x.(\ln x - 1)$$

→ *Continued*

2. Explain what is meant by "*centre of mass frame*" of a system of particles.

Consider a system of N non interacting particles each subjected to an external force and show that the centre of mass of this system of particles moves as if the total mass of the particles is concentrated at that point and all external forces are applied to it.

A particle (*Particle* 1) of mass m and initial ***velocity*** $2u$ collides elastically with another particle (*Particle* 2) of mass M with initial ***speed*** u and moves opposite to the first particle. As a result of the collision, the *particle* 1 is deflected through 90°. (All speeds and angles are those observed in the laboratory system.)

(a) Find the ***velocity*** of the centre of mass frame,

(b) Find the ***velocity*** of the particle 1 in the centre of mass frame before the collision,

(c) Find the ***velocity*** of the particle 2 in the centre of mass frame before the collision,

(d) Show that the ***magnitude of momentum*** of the particles are the same for before and after the collision in the centre of mass frame,

(e) Find the ***velocities*** of the particle 1 and particle 2 in the centre of mass frame after the collision,

(f) Find the possible range for the ratio of m/M for the above result of the collision,

(g) Find the ***speed*** of the particle 1 in the laboratory frame after the collision,

(h) What fraction of initial kinetic energy is transferred from one particle to the other?

→ *Continued*

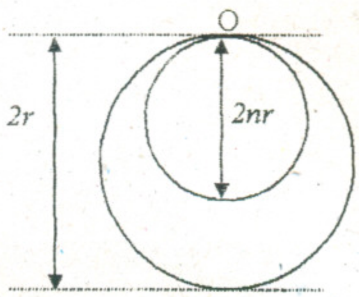
3.

(a) Explain the term "Moment of Inertia".

(b) State and prove the theorem of parallel axis.

(c) Show that the moment of inertia of a uniform circular disc of mass M and radius a about an axis perpendicular to plane of the disc and passing through a point on the edge of the disc is given by $I = \frac{3}{2}Ma^2$.

(d) An ear ring has a shape as shown in the figure below. It was designed by removing a circular disc of the radius nr (n<1) from a uniform circular disc of radius r. Show that the moment of inertia of the ear ring about an axis passing through O and normal to its plane is given by $\frac{3r^2}{2}M(1+n^2)$, where M is the mass of the ear ring.



(e) Find the position of the centre of mass of the ear ring from O.

(f) Find the period of small oscillations of the ear ring about an axis passing through O and normal to its plane.

(g) If $r = 3cm$ and $n = 0.8$ then, find the period of small oscillations of the ear ring about an axis passing through O and normal to its plane.

→ *Continued*

4. State and prove the Bernoulli's theorem in fluid dynamics.

Explain the working principle of the following devices:

i. Venturi meter.

ii. Pitot tube.

iii. Aerofoil.

Two vertical cylindrical tanks of cross sectional areas A_1 and A_2 containing water are joined at their bases by a pipe of cross sectional area A_0. The pipe is short enough to treat as an orifice with a unity coefficient of discharge.

(a) Using the Bernoulli's equation, show that the discharge through an orifice is given by $A_0\sqrt{2gh}$, where h is difference of the water levels in the tanks above the pipe.

(b) Initially, tank 1 has higher water level than in the tank 2. Show that the time taken for the water levels difference to drop from $h_{initial}$ to h_{final} is given by, $t = \dfrac{2A_1A_2}{A_0(A_1+A_2)\sqrt{2g}}\left[\sqrt{h_{initial}} - \sqrt{h_{final}}\right]$.

(c) Suppose that the tank 2 is disconnected from tank 1 and the water in the tank 1 is allowed to discharge through the orifice at the bottom of the tank. The water level in the tank 1 drops from h_1 to h_2 above the orifice in tank 1. Using the equation given in part (b), show that the time taken for the water level to drop from h_1 to h_2 is given by, $t = \dfrac{2A_1}{A_0\sqrt{2g}}\left[\sqrt{h_1} - \sqrt{h_2}\right]$.

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SECTION – B

5. The S and S' are two inertial frames of reference. Their origins coincide when $t = t' = 0$ and S' moves at a uniform velocity V relative to S along their common X-axis.

(a) Write down the Lorentz transformation equations for the spatial and time coordinates of an event described in both inertial frames S and S'.

(b) The velocity components of a particle measured by observers in S and S' respectively are (u_x, u_y, u_z) and (u'_x, u'_y, u'_z). Obtain velocity transformation equations.

(c) The velocity of a particle in the S frame is u. Show that the velocity u' of the particle in the S' frame is given by $u'^2 = [(u_x - V)^2 + (u^2 - u_x^2)(1 - V^2/c^2)]\left[1 - \frac{Vu_x}{c^2}\right]^{-2}$, where other symbols have their usual meaning.

(d) A radio active nucleus moving at a velocity 0.2c relative to the laboratory emits a β particle with speed 0.5c with respect to the nucleus. What is the velocity and direction of the β particle in the laboratory frame if it is emitted?

(i) in the direction of the motion of the nucleus ?

(ii) in the direction perpendicular to the direction of motion of the nucleus ?

→ Continued

6. Write down the Lorentz transformation equations for momentum P and energy E of a particle of rest mass m_0.

Show that E^2-P^2c^2 is an invariant and equal to $m_0^2c^4$.

Particle A of rest mass m_A and is at rest in the laboratory decays into particles B and C with rest masses m_B and m_C respectively. Show that the kinetic energy T_B of the particle B is given by

$$T_B = \frac{(m_A - m_B)^2c^4 - m_C^2c^4}{2m_Ac^2}.$$

A Σ^+ particle at rest in a laboratory decays into a π^0 particle and a proton(p). Estimate

(a) the kinetic energy of the π^0 particle.

(b) the velocity of the π^0 particle.

(c) the distance travelled by the π^0 particle in the laboratory before it disintegrates, if its mean life time is 8.7×10^{-17} seconds.

You may assume the following rest masses:

m_Σ=1190 MeV/c^2, m_π= 135 MeV/c^2 and m_p=940MeV/c^2.

→ *Continued*

SECTION – C

7. A beam of positively charged particles each of charge ze and kinetic energy E approaches a stationary nucleus of atomic number Z.

   (a) Obtain an expression for the distance of closest approach of the charge particles that are likely to take part in a head-on collision with the nucleus.

   (b) Stating your assumptions clearly, derive the following relationship between impact parameter b and the scattering angle ϕ: $b = \dfrac{Zze^2}{8\pi\varepsilon_0 E}\cot\left(\dfrac{\phi}{2}\right)$.

   (c) Discuss with the help of a diagram the path followed by the particles that have different impact parameters.

   (d) If the beam is incident normally on a foil of thickness t, show that the fraction of the particles scattered through an angle greater than ϕ is $\left(\dfrac{Zze^2}{8\pi\varepsilon_0 E}\right)^2 \pi nt \cot^2\left(\dfrac{\phi}{2}\right)$, where n is the number of atoms per unit volume in the foil. State any assumption you made to derive the expression

   (e) A beam of 7.9 MeV α–particles is incident normally on a gold foil of thickness 5 μm. What is the fraction of the particles that will be scattered by more than 90^0? Mass number, atomic number and density of gold are 197, 79, 1.93×10^4 kgm^{-3}, respectively.

8. (a) What is meant by *Bravais lattice* in the theory of solids?

   (b) Draw the three *Bravais* lattices of the cubic crystals.

   (c) Find the number of atoms in the unit cell of each cubic system.

   (d) Show that for a cubic crystal with lattice constant a is given by $a = \left[\dfrac{nM}{N_A \rho}\right]^{1/3}$, where n is the number of atoms in the unit cell, M is the atomic mass of the atom in the crystal, ρ is the density of the crystal and N_A is the Avagadro's number.

   (e) Copper crystallizes into a cubical lattice with lattice parameter 3.61 Å. If the density of copper is 8930 kg/m^3, identify the type of the *Bravais* lattice of copper. The atomic mass of the copper is 64.

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