AIRLINE OVERBOOKING POLICY: DYNAMIC PROGRAMMING

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The goal of this paper is to determine the optimal overbooking policy for an airline. This information is important because it helps determine how we should price our tickets. We will be determining daily sales price for first-class and coach tickets, as well as the tickets sold throughout the year. At the end we'll calculate the cost resulting from bumping/upgrading customers and compare the expected profit across different overbooking policies.

OVERBOOKING

Overbooking is when a company will sell more than what they have to offer. This is because the company assumes some of the people who purchased their product will not claim it. The practice of overbooking is very common in the airline industry for many reasons. One reason is that people tend to purchase flight tickets far in advance, but circumstances change, and people can no longer make their flight. Companies can use this practice to create higher profits. For example, a plane has 100 coach seats, and the airline sells 103 coach tickets to the flight. If exactly 100 people show up, then the airline will not have to pay an overbooking penalty, and the airline will receive revenue from the individuals who did not show up. However, if more people show up than seats available, then the airline will need to pay an overbooking penalty. The overbooking penalty can be either bumping a customer to first class if a first-class seat is available or paying the customer to voluntarily change to a different flight. If 103 people showed up for a flight that seats 100 individuals, then the airline must compensate the 3 individuals who will be bumped, possibly lowering profits beyond the profits if the company had not used overbooking.

REPORT GOALS

We want to compare expected profits different levels of overbooking. We have a plane that can seat 120 individuals. 20 seats are reserved for first-class tickets and 100 seats are reserved for coach tickets. We will be trying to overbook our coach tickets. If more than 100 individuals show up with coach tickets, then we will try to give them a spot in first-class if possible. If first-class is full, then we will have to bump them from the plane. Can we maximize our profits by selling 109 coach tickets compared to 106 coach tickets? That is the question we are looking to answer. The first step is to calculate expected cost of different combination of tickets sold, then calculating the expected revenue based on different daily pricing strategies, and finally calculating the expected profit across many overbooking policies.

ASSUMPTIONS

These are the following assumptions we've determined for our analysis. They include the high/low ticket pricing, the probability of a sale given the ticket pricing, and the arrival probability of a customer arriving differentiated on the type of ticket they are purchasing. Our discount rate is used to discount future sales, so a ticket sold on day 200 is not worth the same as a ticket sold today. If we had the revenue today, we could use it towards other projects or invest it. We have one year until the plane takes off.

	Coach	First-Class
Low Ticket Price	$300	$425
High Ticket Price	$350	$500
Low Ticket Price Sale Probability	65%	8%
High Ticket Price Sale Probabilty	30%	4%
Arrival Probability	95%	97%
Seats Available	100	20

Discount Rate	99.96%
Days Until Takeoff	365
Bumped to First-Class Cost	$50
Bumped off the Plane Cost	$425

POLICY OPTIONS

We are going to examine two different types of overbooking policies. One requires the airline to offer tickets to coach customer at either a high or low price. The other gives the airline an option to sell a coach ticket at a high or low price, or not offer a coach ticket at all. For the second option we'll set a cap of possible coach tickets sold at 120, because the program will stop selling tickets before the expected profit declines.

<u>POLICY 1</u>
OVERBOOKING 5 INDIVIDUALS

The first step in determining expected profit is determining the expected cost of bumping customers based on combinations of tickets sold for coach and first-class.

```python
for c in range(cN):
    for f in range(fN):
        total_cost = 0
        for i in range(c+1):
            for j in range(f+1):
                if i > 100:
                    extra_space = 20-j
                    extra_passenger =  i-100
                    if extra_space < extra_passenger:
                        cost = (extra_passenger-extra_space)*bumpH + extra_space*bumpL
                    else:
                        cost = extra_passenger*bumpL
                    prob = binom.pmf(i,c,showC)*binom.pmf(j,f,showF)
                    total_cost += cost*prob
        V[c,f,tN-1] = -(total_cost)
```

We use four for loops in the process as we go through every possible value of coach and first-class ticket sold. If there are less than 100 coach tickets sold, then there is no expected cost because nobody will be bumped. If there are more than 100 coach tickets sold, we need to determine how many spaces we have available in first class so we can determine how many customers we can bump to first class and how many customers we will have to bump from the plane. We will bump as many customers to first class as possible and then bump the rest from the plane. If we have 109 individuals arrive to the terminal with coach tickets and 12 individuals arrive with first-class tickets, then we will bump 8 individuals to firs class and bump one individual off the plane. Once we allocate the customers, we calculate cost by taking the sum of allocation multiplied by their cost. Here is an example of the cost calculation:

*8 individuals bumped to first class * $50 = $400*
*1 individual bumped off the plane * $425 = $425*
Total cost = $425+$400 = $825

From here we must factor in that every customer will not show up to a flight. We calculate the probability that a customer will show up using the binomial distribution. We multiply the probability by the cost to get the expected cost. The expected cost will be the terminal boundary for our program since it is only realized on the day of the flight.

Next, we will calculate the expected profit. We will work backwards to be able to factor in future earnings into our current calculations. Our pricing choice daily will be influenced by the future earnings based on our choice. The ticket options we present will be different depending on the number of tickets remaining, we have displayed the scenarios below:

Scenarios	Ticket Sales Options
Scenario 1: First-class and coach are sold out	No tickets for sale
Scenario 2: Coach is sold out	First-class tickets at a high price First-class tickets at a low price
Scenario 3: First-class is sold out	Coach tickets at a high price Coach tickets a low price
Scenario 4: First-class and coach have seats available	First-class and coach tickets available at a high price
	First-class and coach tickets available at a low price
	First-class tickets available at a high price and coach tickets available at a low price
	First-class tickets available at a low price and coach tickets available at a high price

In each scenario we will choose the best outcome based on today's profit and future earnings. We do this calculation using the bellman equation. We have presented a code chunk of our calculation below:

```python
elif c == cN-1 and f < fN-1:
    FL = (priceFL*pFL[1]) + delta*(pFL[1]*V[c,f+1,t+1] + pFL[0]*V[c,f,t+1])
    FH = (priceFH*pFH[1]) + delta*(pFH[1]*V[c,f+1,t+1] + pFH[0]*V[c,f,t+1])
    V[c,f,t] = max(FL, FH)
    U[c,f,t] = np.argmax([FL, FH]) + 1 # choice of price: 1 means (0, L), 2 means (0, H)
```

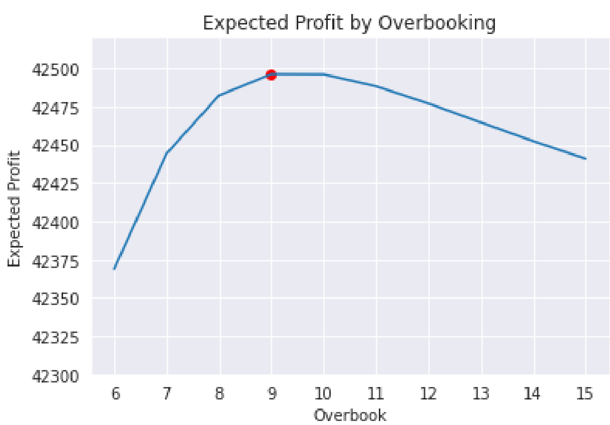
This code chunk evaluates the best option considering coach is sold out. The value of charging a low price for a first-class ticket is the expected revenue today, which is the probability of a sale given the price multiplied by the price of the ticket, plus the value of future earnings. The value of future earnings is discounted by probability of a sale tomorrow multiplied by the value of that sale, plus the probability of no sale tomorrow multiplied by the value of no sale tomorrow. We do this for the high price as well and then choose the best option. This is calculated for every option depending on the scenario.

The results from overbooking 5 individuals using this policy are below:

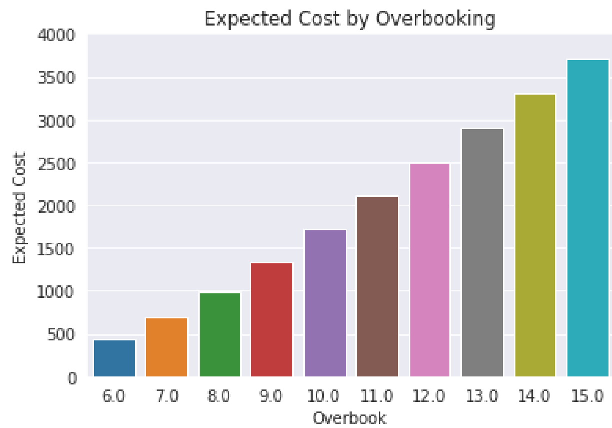
Policy 1	
Overbooked = 5	
Expected Profit	$42,242.86
Expected Cost	$240.93

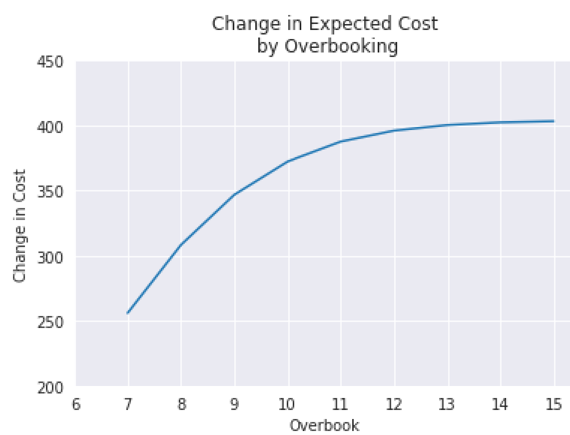
OPTIMAL OVERBOOKED

Next, we want to repeat this process for a different number of overbooked amounts to determine which provides us the highest expected profit. We will be looking at overbooking between 5 and 15 individuals. Another way to look at it is selling between 105 and 115 coach tickets. The optimal overbooking amount is overbooking 9 seats, which means selling 109 coach tickets. The expected profit for selling 109 coach tickets is $42,496.11



We can also see how much the expected costs rise as we increase the number of seats sold. After overbooking by 9 individuals, the cost of selling a ticket outweighs the revenue, that must mean that we are starting to bump customers off the plane instead of bumping them to first class.





POLICY 2

The code for policy 2 is the same as the code for policy 1, but with one tweak. The ticket sales option now include the option to not make a sale. The program will not sell a seat on a specific day if not selling a seat is the best decision to maximize expected profit. We are allowing for 20 overbookings because the flight capacity is 120 seats, including coach and first-class.

Policy 2	
Overbooking Allowed = 20	
Expected Profit	$42,502.67
Expected Cost	$5,725.00

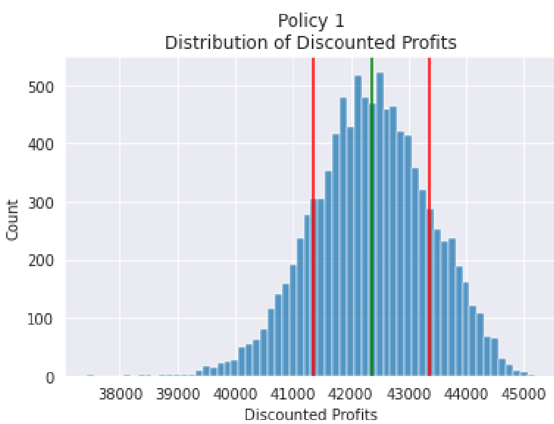
The expected profit using policy 2 is higher than the expected profit using policy 1.

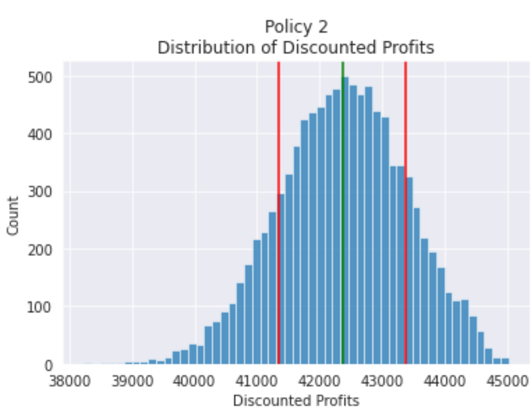
COMPARING POLICIES

To compare the two policies, we ran 10,000 simulations of each policy. Here are the results:

	Policy 1	Policy 2
How often is coach overbooked?	100%	100%
How often are passengers kicked off the plane?	82.5%	82.7%
Average overbooking cost	$965.73	$1,005.31
Average discounted profits	$42,355.16	$42,361.95
Standard deviation of discounted profits	$1,004.99	$1,018.09

The two policies have a similar likelihood of overbooking coach and kicking a passenger off the plane. Both policies will overbook coach every time and will kick a passenger off the plane around 82.5% of the time. The average overbooking cost is $40 higher for policy 2, but the average discounted profit is $6 higher for policy 2.





The distribution of discounted profits is normal due to the number of simulations, so we can confidently say that about 68% of the time the expected profit for policy 1 will be between $41,350 and 43,360. The expected profit for policy 2 will be between $41,344 and $43,380 around 68% of the time. So, policy 2 results will sometimes result in loss profits than average 2, but on average it will produce better results.

CONCLUSION

We recommend using policy 2 to determine your daily offerings to customers. We understand it is more volatile, but the average profits are higher. One thing that worries us is kicking off a passenger around 82% of the time. That is bad optics from a public relations perspective, because a customers might be less likely to purchase from us if they think there is a chance, they will be asked to leave the plane. This booking policy is very common in the industry, so we think customers understand why it is done. Customers also don’t have another option because the chances of finding an airline that doesn’t overbook will be low. In conclusion, we believe we should overbook flights and use policy 2 when determining the ticket offerings for the day.