Elemental equations

Interpolation Matrix [N]

MATLAB was used to develop the interpolation matrix [N] which was then copied here from the command window of MATLAB. A lot of effort was made to export it from the MATLAB in a readble format for mathematica but interpolation matrix was of symbolic class due to which it was not being read by mathematica. It is the reason it was copied directly here. So we will use this matrix to develop the strain interpolation matrix [B].

```
intMAT = {{((a - x) * (b - y)) / (a * b), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, (x * (b - y)) / (a * b), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, (x * y) / (a * b), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, (y * (a - x)) / (a * b), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, ((a - x) * (b - y)) / (a * b), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, (x * (b - y)) / (a * b), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, (x * y) / (a * b), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    (y * (a - x)) / (a * b), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, ((a - x) * (b - y) * (a^2 * b^2 + a^2 * b * y - 2 * a^2 * y^2 + a * b^2 * x - 2 * b^2 * x^2)) /
      (a^3 * b^3), (x * (a - x)^2 * (b - y)) / (a^2 * b),
    (y * (a - x) * (b - y)^2) / (a * b^2), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    - (x * (b - y) * (-a^2 * b * y + 2 * a^2 * y^2 - 3 * a * b^2 * x + 2 * b^2 * x^2)) / (a^3 * b^3),
    - (x^2 * (a - x) * (b - y)) / (a^2 * b), (x * y * (b - y)^2) / (a * b^2),
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    - (x * y * (a^2 * b^2 - 3 * a^2 * b * y + 2 * a^2 * y^2 - 3 * a * b^2 * x + 2 * b^2 * x^2)) /
      (a^3 * b^3), - (x^2 * y * (a - x)) / (a^2 * b),
    - (x * y^2 * (b - y)) / (a * b^2), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    - (y * (a - x) * (-3 * a^2 * b * y + 2 * a^2 * y^2 - a * b^2 * x + 2 * b^2 * x^2)) / (a^3 * b^3),
    (x * y * (a - x)^2) / (a^2 * b), - (y^2 * (a - x) * (b - y)) / (a * b^2),
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, ((a - x) * (b - y)) / (a * b), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, (x * (b - y)) / (a * b), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, (x * y) / (a * b), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, (y * (a - x)) / (a * b), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, ((a - x) * (b - y)) / (a * b), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, (x * (b - y)) / (a * b), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, (x * y) / (a * b), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, (y * (a - x)) / (a * b), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, ((a - x) * (b - y) * (a^2 * b^2 + a^2 * b * y - 2 * a^2 * y^2 +
```

```
        a*b^2*x-2*b^2*x^2))/(a^3*b^3), (x*(a-x)^2*(b-y))/(a^2*b),
    (y*(a-x)*(b-y)^2)/(a*b^2), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    -(x*(b-y)*(-a^2*b*y+2*a^2*y^2-3*a*b^2*x+2*b^2*x^2))/(a^3*b^3),
    -(x^2*(a-x)*(b-y))/(a^2*b), (x*y*(b-y)^2)/(a*b^2),
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    -(x*y*(a^2*b^2-3*a^2*b*y+2*a^2*y^2-3*a*b^2*x+2*b^2*x^2))/
     (a^3*b^3), -(x^2*y*(a-x))/(a^2*b),
    -(x*y^2*(b-y))/(a*b^2), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    -(y*(a-x)*(-3*a^2*b*y+2*a^2*y^2-a*b^2*x+2*b^2*x^2))/(a^3*b^3),
    (x*y*(a-x)^2)/(a^2*b),
    -(y^2*(a-x)*(b-y))/(a*b^2), 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, ((a-x)*(b-y))/(a*b), 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, (x*(b-y))/(a*b), 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, (x*y)/(a*b), 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, (y*(a-x))/(a*b), 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, ((a-x)*(b-y))/(a*b), 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, (x*(b-y))/(a*b), 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, (x*y)/(a*b), 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, (y*(a-x))/(a*b), 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    ((a-x)*(b-y)*(a^2*b^2+a^2*b*y-2*a^2*y^2+a*b^2*x-2*b^2*x^2))/
     (a^3*b^3), (x*(a-x)^2*(b-y))/(a^2*b),
    (y*(a-x)*(b-y)^2)/(a*b^2), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    -(x*(b-y)*(-a^2*b*y+2*a^2*y^2-3*a*b^2*x+2*b^2*x^2))/(a^3*b^3),
    -(x^2*(a-x)*(b-y))/(a^2*b), (x*y*(b-y)^2)/(a*b^2),
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    -(x*y*(a^2*b^2-3*a^2*b*y+2*a^2*y^2-3*a*b^2*x+2*b^2*x^2))/
     (a^3*b^3), -(x^2*y*(a-x))/(a^2*b),
    -(x*y^2*(b-y))/(a*b^2), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    -(y*(a-x)*(-3*a^2*b*y+2*a^2*y^2-a*b^2*x+2*b^2*x^2))/(a^3*b^3),
    (x*y*(a-x)^2)/(a^2*b), -(y^2*(a-x)*(b-y))/(a*b^2), 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, ((a-x)*(b-y))/(a*b), 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, (x*(b-y))/(a*b), 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, (x*y)/(a*b), 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, (y*(a-x))/(a*b), 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, ((a-x)*(b-y))/(a*b),
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, (x*(b-y))/(a*b),
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, (x*y)/(a*b), 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, (y*(a-x))/(a*b)}};
intMAT // MatrixForm
```

Differential operator matrix [L]

$$\zeta_t = z - \left(c + \frac{ft}{2}\right);$$

$$\zeta_b = z + \left(c + \frac{fb}{2}\right);$$

Following are the differential operator matrices for top & bottom sheets and core.

$$\begin{array}{l}
\mathbf{Lt} = \{\{\mathrm{D}[\#, x]\ \&,\ 0\,\#\ \&,\ -\zeta_t * \mathrm{D}[\#, \{x, 2\}]\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&, \\
\quad 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&\},\ \{0\,\#\ \&,\ \mathrm{D}[\#, y]\ \&,\ -\zeta_t * \mathrm{D}[\#, \{y, 2\}]\ \&, \\
\quad 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&\},\ \{\mathrm{D}[\#, y]\ \&,\ \mathrm{D}[\#, x]\ \&, \\
\quad -2 * \zeta_t * \partial_{x,y}\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&\}\};
\end{array}$$

$$\begin{array}{l}
\mathbf{Lb} = \{\{0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ \partial_x\#\ \&,\ 0\,\#\ \&,\ -\zeta_b * \partial_{x,x}\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&\}, \\
\quad \{0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ \partial_y\#\ \&,\ -\zeta_b * \partial_{y,y}\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&\}, \\
\quad \{0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ \partial_y\#\ \&,\ \partial_x\#\ \&,\ -2 * \zeta_b * \partial_{x,y}\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&\}\};
\end{array}$$

$$\begin{array}{l}
\mathbf{Lc} = \Big\{\Big\{\left(\frac{z^3}{2\,c^3} + \frac{z^2}{2\,c^2}\right)\partial_x\#\ \&,\ 0\,\#\ \&,\ \left(\frac{z^3}{4\,c^3} + \frac{z^2}{4\,c^2}\right) \mathrm{ft}\,\partial_{x,x}\#\ \&,\ \left(-\frac{z^3}{2\,c^3} + \frac{z^2}{2\,c^2}\right)\partial_x\#\ \&,\ 0\,\#\ \&, \\
\quad \left(\frac{z^3}{4\,c^3} - \frac{z^2}{4\,c^2}\right) \mathrm{fb}\,\partial_{x,x}\#\ \&,\ \left(1 - \frac{z^2}{c^2}\right)\partial_x\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ \left(z - \frac{z^3}{c^2}\right)\partial_x\#\ \&,\ 0\,\#\ \&\Big\}, \\
\quad \Big\{0\,\#\ \&,\ \left(\frac{z^3}{2\,c^3} + \frac{z^2}{2\,c^2}\right)\partial_y\#\ \&,\ \left(\frac{z^3}{4\,c^3} + \frac{z^2}{4\,c^2}\right) \mathrm{ft}\,\partial_{y,y}\#\ \&,\ 0\,\#\ \&,\ \left(-\frac{z^3}{2\,c^3} + \frac{z^2}{2\,c^2}\right)\partial_y\#\ \&, \\
\quad \left(\frac{z^3}{4\,c^3} - \frac{z^2}{4\,c^2}\right) \mathrm{fb}\,\partial_{y,y}\#\ \&,\ 0\,\#\ \&,\ \left(1 - \frac{z^2}{c^2}\right)\partial_y\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ \left(-z + \frac{z^3}{c^2}\right)\partial_y\#\ \&\Big\}, \\
\quad \Big\{0\,\#\ \&,\ 0\,\#\ \&,\ \left(\frac{z}{c^2} + \frac{1}{2\,c}\right)\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&,\ \left(\frac{z}{c^2} - \frac{1}{2\,c}\right)\#\ \&, \\
\quad 0\,\#\ \&,\ 0\,\#\ \&,\ \left(-\frac{2\,z}{c^2}\right)\#\ \&,\ 0\,\#\ \&,\ 0\,\#\ \&\Big\}, \\
\quad \Big\{0\,\#\ \&,\ \left(\frac{3\,z^2}{2\,c^3} + \frac{z}{c^2}\right)\#\ \&,\ \left(\frac{z^2}{2\,c^2} + \frac{z}{2\,c} + \frac{3\,z^2}{4\,c^3}\,\mathrm{ft} + \frac{z}{2\,c^2}\,\mathrm{ft}\right)\partial_y\#\ \&,\ 0\,\#\ \&, \\
\quad \left(-\frac{3\,z^2}{2\,c^3} + \frac{z}{c^2}\right)\#\ \&,\ \left(\frac{z^2}{2\,c^2} - \frac{z}{2\,c} + \frac{3\,z^2}{4\,c^3}\,\mathrm{fb} - \frac{z}{2\,c^2}\,\mathrm{fb}\right)\partial_y\#\ \&, \\
\quad 0\,\#\ \&,\ \left(-\frac{2\,z}{c^2}\right)\#\ \&,\ \left(1 - \frac{z^2}{c^2}\right)\partial_y\#\ \&,\ 0\,\#\ \&,\ \left(-1 + \frac{3\,z^2}{c^2}\right)\#\ \&\Big\}, \\
\quad \Big\{\left(\frac{3\,z^2}{2\,c^3} + \frac{z}{c^2}\right)\#\ \&,\ 0\,\#\ \&,\ \left(\frac{z^2}{2\,c^2} + \frac{z}{2\,c} + \frac{3\,z^2}{4\,c^3}\,\mathrm{ft} + \frac{z}{2\,c^2}\,\mathrm{ft}\right)\partial_x\#\ \&,\ \left(-\frac{3\,z^2}{2\,c^3} + \frac{z}{c^2}\right)\#\ \&, \\
\quad 0\,\#\ \&,\ \left(\frac{z^2}{2\,c^2} - \frac{z}{2\,c} + \frac{3\,z^2}{4\,c^3}\,\mathrm{fb} - \frac{z}{2\,c^2}\,\mathrm{fb}\right)\partial_x\#\ \&,\ \left(-2 * \frac{z}{c^2}\right)\#\ \&,\ 0\,\#\ \&,\ \left(1 - \frac{z^2}{c^2}\right)\partial_x\#\ \&, \\
\quad \left(1 - \frac{3\,z^2}{c^2}\right)\#\ \&,\ 0\,\#\ \&\Big\},\ \Big\{\left(\frac{z^3}{2\,c^3} + \frac{z^2}{2\,c^2}\right)\partial_y\#\ \&,\ \left(\frac{z^3}{2\,c^3} + \frac{z^2}{2\,c^2}\right)\partial_x\#\ \&,\ \left(\frac{z^3}{2\,c^3} + \frac{z^2}{2\,c^2}\right)\mathrm{ft}\,\partial_{x,y}\#\ \&, \\
\quad \left(-\frac{z^3}{2\,c^3} + \frac{z^2}{2\,c^2}\right)\partial_y\#\ \&,\ \left(-\frac{z^3}{2\,c^3} + \frac{z^2}{2\,c^2}\right)\partial_x\#\ \&,\ \left(\frac{z^3}{2\,c^3} - \frac{z^2}{2\,c^2}\right)\mathrm{fb}\,\partial_{x,y}\#\ \&, \\
\quad \left(1 - \frac{z^2}{c^2}\right)\partial_y\#\ \&,\ \left(1 - \frac{z^2}{c^2}\right)\partial_x\#\ \&,\ 0\,\#\ \&,\ \left(z - \frac{z^3}{c^2}\right)\partial_y\#\ \&,\ \left(-z + \frac{z^3}{c^2}\right)\partial_x\#\ \&\Big\}\Big\};
\end{array}$$

Lc // MatrixForm

Strain interpolation matrix [B]

Following is a user defined function which applies the differential operator matrix on the required matrix.

```
apply[aa_, bb_] := Inner[#1[#2] &, aa, bb]
```

Following are the strain interpolation matrices obtained after applying the differential operators on the interpolation matrices for respective layers.

```
Bt = apply[Lt, intMAT];

Bb = apply[Lb, intMAT];

Bc = apply[Lc, intMAT];
Bc // MatrixForm
```

Material property matrix [C]

Following is the material property matrices for top & bottom sheets and core.

```
Ct = {{c11t, c12t, c16t}, {c12t, c22t, c26t}, {c16t, c26t, c66t}};

Cb = {{c11b, c12b, c16b}, {c12b, c22b, c26b}, {c16b, c26b, c66b}};

Cc = {{c11c, c12c, c13c, 0, 0, 0},
   {c12c, c22c, c23c, 0, 0, 0}, {c13c, c23c, c33c, 0, 0, 0},
   {0, 0, 0, c44c, 0, 0}, {0, 0, 0, 0, c55c, 0}, {0, 0, 0, 0, 0, c66c}};
```

Stiffness matrix [K_{el}]

```
Kt = Integrate[Integrate[Integrate[(Btᵀ.Ct.Bt), {z, c, c + ft}], {x, 0, a}], {y, 0, b}];

Kb =
  Integrate[Integrate[Integrate[(Bbᵀ.Cb.Bb), {z, -c - fb, -c}], {x, 0, a}], {y, 0, b}];

Kc = Integrate[Integrate[Integrate[integrand, {z, -c, c}], {x, 0, a}], {y, 0, b}];

<< ToMatlab`
```

$[K]^{t,b}$ were calculated here (in around 7-10 minutes each) and then were exported to MATLAB using following two commands.

```
WriteMatlab[Kt, "Kt.m", "K_t"]

WriteMatlab[Kb, "Kb.m", "K_b"]
```

But, $[K]^c$ was taking very long to be calculated here. I almost gave it 08 hours but even then it did'nt complete. So I exported the $[B]^c$ and $[C]^c$ matrix to MATLAB using following commands. $[C]^c$ was exported seamlessly but there was a problem exporting $[B]^c$ so I used the command “PrintMatlab” and then copied the output in MATLAB. In MATLAB, $[K]^c$ was calculated in almost 75 minutes without parallel pool.

```
WriteMatlab[Bc, "Bc.m", "B_c"]

WriteMatlab[Bb, "Bb.m", "B_b"]

WriteMatlab[Bt, "Bt.m", "B_t"]

WriteMatlab[Cb, "Cb.m", "C_b"]

WriteMatlab[Ct, "Ct.m", "C_t"]

WriteMatlab[Cc, "Cc.m", "C_c"]

PrintMatlab[Bc, "B_c"]

Kel = Kt + Kb + Kc
```

Therefore final elemental stiffness matrix is calculated in the MATLAB.