Universitat Politècnica de Catalunya · Barcelona Tech - UPC
Escola d'Enginyeria de Barcelona Est EEBE
Homework 1, Metrics for Evaluating Multi-Class Classification Problems

Sheik, Abdullahi

25 September

Given a confusion matrix:

	A	B	C	D
A	2542	2	1	15
B	7	1231	40	2
C	1	45	1230	4
D	36	6	3	1225

Definitions:

- tp (true positive): Number of correct positive predictions for a class.
- tn (true negative): Number of correct negative predictions for a class.
- fp (false positive): Number of incorrect positive predictions for a class.
- fn (false negative): Number of incorrect negative predictions for a class.

Computations:

1. Average Accuracy ($a\bar{c}c$)

$$a\bar{c}c = \frac{1}{l}\sum_{i=1}^{l}\frac{tp_i + tn_i}{tp_i + fn_i + fp_i + tn_i}$$

2. Micro-Averaged Precision ($p\bar{p}v_{\mu}$)

$$p\bar{p}v_{\mu} = \frac{\sum_{i=1}^{l} tp_i}{\sum_{i=1}^{l}(tp_i + fp_i)}$$

3. Micro-Averaged Recall ($\bar{tpr}_\mu$)

$$\bar{tpr}_\mu = \frac{\sum_{i=1}^{l} tp_i}{\sum_{i=1}^{l} (tp_i + fn_i)}$$

4. Micro-Averaged F1 Score ($F_{1\mu}$)

$$F_{1\mu} = \frac{2 \cdot (\bar{ppv}_\mu \cdot \bar{tpr}_\mu)}{\bar{ppv}_\mu + \bar{tpr}_\mu}$$

5. Macro-Averaged Specificity ($\bar{tnr}_M$)

$$\bar{tnr}_M = \frac{1}{l} \sum_{i=1}^{l} \frac{tn_i}{tn_i + fp_i}$$

6. Micro-Averaged Specificity ($\bar{tnr}_\mu$)

$$\bar{tnr}_\mu = \frac{\sum_{i=1}^{l} tn_i}{\sum_{i=1}^{l} (tn_i + fp_i)}$$

Calculations:

Let's use the given formulas to calculate the metrics:

Class-wise Metrics

Class A:

$$tp_A = 2542$$
$$fn_A = 7 + 1 + 36 = 44$$
$$fp_A = 2 + 1 + 15 = 18$$
$$tn_A = 1231 + 40 + 2 + 45 + 1230 + 4 + 6 + 3 + 1225 = 3786$$

Class B:

$$tp_B = 1231$$
$$fn_B = 2 + 45 + 6 = 53$$
$$fp_B = 7 + 40 + 2 = 49$$
$$tn_B = 2542 + 1 + 15 + 1 + 1230 + 4 + 36 + 3 + 1225 = 5056$$

Class C:

$$tp_C = 1230$$
$$fn_C = 1 + 40 + 3 = 44$$
$$fp_C = 1 + 4 = 5$$
$$tn_C = 2542 + 2 + 15 + 7 + 1231 + 2 + 36 + 6 + 1225 = 5064$$

Class D:

$$tp_D = 1225$$
$$fn_D = 15 + 2 + 4 = 21$$
$$fp_D = 36 + 6 + 3 = 45$$
$$tn_D = 2542 + 2 + 1 + 7 + 1231 + 40 + 1 + 45 + 1230 = 5099$$

1. Average Accuracy ($a\bar{c}c$)

$$a\bar{c}c = \frac{1}{4}\left(\frac{2542+3786}{2542+44+18+3786} + \frac{1231+5056}{1231+53+49+5056} + \frac{1230+5064}{1230+44+5+5064} + \frac{1225+5099}{1225+21+45+509}$$

$$a\bar{c}c \approx \frac{1}{4}\left(0.992 + 0.988 + 0.990 + 0.988\right)$$
$$a\bar{c}c \approx 0.9895$$

2. Micro-Averaged Precision ($p\bar{p}v_\mu$)

$$p\bar{p}v_\mu = \frac{2542+1231+1230+1225}{(2542+18)+(1231+49)+(1230+5)+(1225+45)}$$
$$p\bar{p}v_\mu \approx \frac{6228}{6294}$$
$$p\bar{p}v_\mu \approx 0.9895$$

3. Micro-Averaged Recall ($t\bar{p}r_\mu$)

$$t\bar{p}r_\mu = \frac{2542+1231+1230+1225}{(2542+44)+(1231+53)+(1230+44)+(1225+21)}$$
$$t\bar{p}r_\mu \approx \frac{6228}{6338}$$
$$t\bar{p}r_\mu \approx 0.9827$$

4. Micro-Averaged F1 Score ($F_{1\mu}$)

$$F_{1\mu} = \frac{2 \cdot (p\bar{p}v_\mu \cdot t\bar{p}r_\mu)}{p\bar{p}v_\mu + t\bar{p}r_\mu}$$
$$F_{1\mu} \approx \frac{2 \cdot (0.9724)}{1.9722}$$
$$F_{1\mu} \approx 0.9860$$

5. Macro-Averaged Specificity ($t\bar{n}r_M$)

$$t\bar{n}r_M = \frac{1}{4}\left(\frac{3786}{3786+18} + \frac{5056}{5056+49} + \frac{5064}{5064+5} + \frac{5099}{5099+45}\right)$$

$$t\bar{n}r_M \approx \frac{1}{4}\left(0.995 + 0.990 + 0.999 + 0.991\right)$$

$$t\bar{n}r_M \approx 0.9938$$

6. Micro-Averaged Specificity ($t\bar{n}r_\mu$)

$$t\bar{n}r_\mu = \frac{3786 + 5056 + 5064 + 5099}{(3786+18) + (5056+49) + (5064+5) + (5099+45)}$$

$$t\bar{n}r_\mu \approx \frac{19005}{19026}$$

$$t\bar{n}r_\mu \approx 0.9989$$