Inspire…Educate…Transform.

Foundations of Statistics and Probability for Data Science

Discrete and Continuous Distributions, Sampling Distribution of Means, CLT

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FLASHBACK
I REMEMBER IT LIKE IT WAS YESTERDAY...
CMCOTT. 04/13/12 #138

Distributions - Properties

Var(-X) = ?

Options:

- Var(X) ✓
- -Var(X)
- Variance of negative numbers cannot be calculated
- None of the above (Explain)

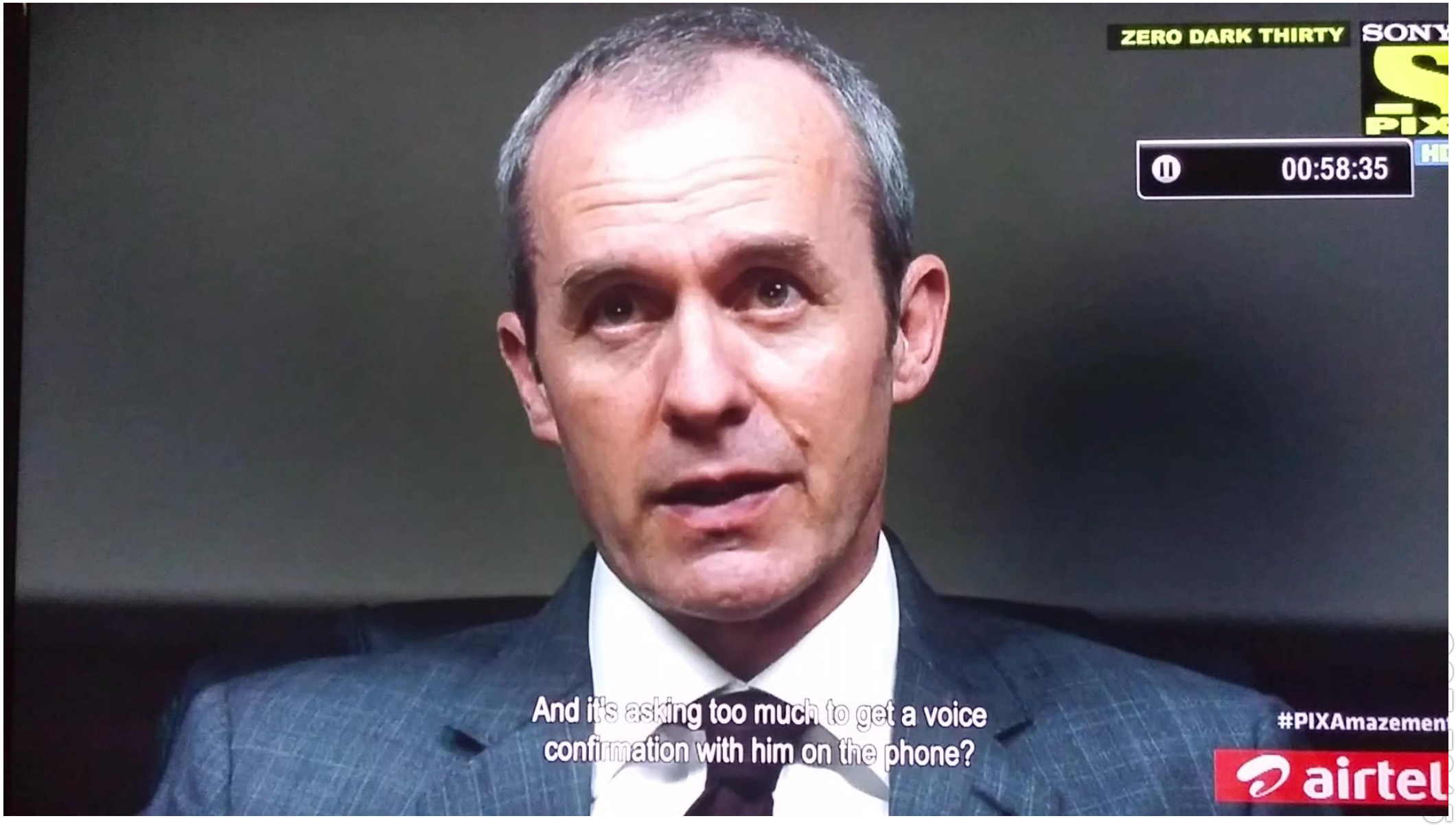
Confusion Matrix – Recent Interview Question

You have been tasked to build a classifier for cancer diagnosis. It is of high importance that patients with cancer can be diagnosed wrongly as negative but patients without cancer should NEVER be diagnosed as positive.

Which of the following classification models would you prefer?
(Assuming: Positives = Cancer, Negatives = Not cancer)

Options:

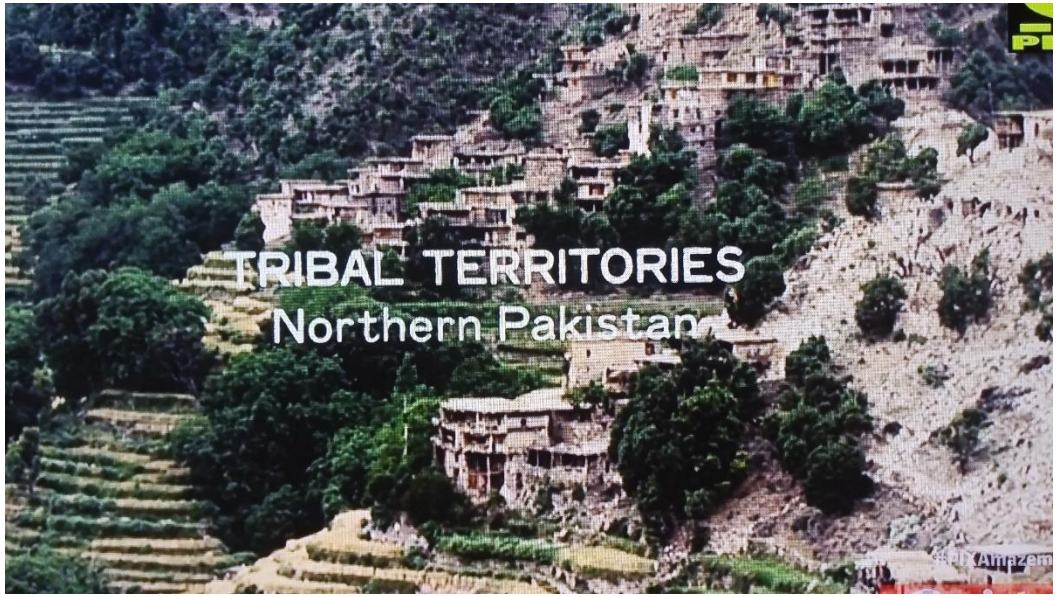
- True Positive Rate [which is = True Positive / Actual Positive]
- True Negative Rate [which is = True Negative / Actual Negative]
- Positive Predictive Value [which is = True Positive / Predicted Positive]
- Total Accuracy [which is = (True Positive + True Negative) / Total Population]


ZERO DARK THIRTY
00:58:35
And it's asking too much to get a voice confirmation with him on the phone?
#PIXAmazemen
airtel


ZERO DARK THIRTY
SONY
PIX
01:06:19
#PIXAmazemen
airtel

Finding Osama Bin Laden Using Bayes' Theorem

Bin Laden was equally likely to be in a compound in a city or in the mountain caves of tribal areas.



Finding Osama Bin Laden Using Bayes' Theorem

The criteria to mark a compound as positive include high walls, inmates excepting a messenger not entering or leaving the compound, inmates carefully avoiding detection including by satellites, etc.

Given a compound meets all the criteria, what is the probability that Bin Laden is in that compound?

Finding Osama Bin Laden Using Bayes' Theorem

P(Bin Laden is in Compound) = 0.5

P(Compound is + | Bin Laden is in Compound) = 0.9

P(Compound is + | Bin Laden is NOT in Compound) = 0.6

P(Bin Laden is in Compound | Compound is +) = ?

$$P(Bin\ Laden\ is\ in\ Compound|Compound\ is\ +) = \frac{P(Bin\ Laden\ is\ in\ Compound) * P(Compound\ is\ +|Bin\ Laden\ is\ in\ Compound)}{P(Bin\ Laden\ is\ in\ Compound) * P(Compound\ is\ +|Bin\ Laden\ is\ in\ Compound) + P(Bin\ Laden\ is\ NOT\ in\ Compound) * P(Compound\ is\ +|Bin\ Laden\ is\ NOT\ in\ Compound)}$$

$$P(Bin\ Laden\ is\ in\ Compound|\text{Compound is } +) = \frac{0.5 * 0.9}{0.5 * 0.9 + 0.5 * 0.6} = 60\%$$

Finding Osama Bin Laden Using Bayes' Theorem

P(Bin Laden is in Compound) = 0.5

P(Compound is + | Bin Laden is in Compound) = 0.95

P(Compound is + | Bin Laden is NOT in Compound) = 0.05

$$P(Bin\ Laden\ is\ in\ Compound | Compound\ is\ +)$$
$$= \frac{0.5 * 0.95}{0.5 * 0.95 + 0.5 * 0.05} = 95\%$$

Confusion Matrix

Buyer or Non-Buyer

A retail store's marketing team uses analytics to predict who is likely to buy a newly introduced high-end (read "expensive") product. Indicate which measure is more important for the business to track and explain why. Calculate other measures also.

Buyer or Not		Actual		Total
		Negative	Positive	
Predicted	Negative	725	158	883
	Positive	75	302	377
Total		800	460	1260

Confusion Matrix

Buyer or Not		Actual		Total
		Negative	Positive	
Predicted	Negative	725	158	883
	Positive	75	302	377
Total		800	460	1260

Buyer or Non-Buyer

State/Calculate:

TP = 302 TN = 725 FP = 75 FN = 158

Should the business be more worried about FP or FN or equally worried about both of them? Why?

FN. If the model predicts that the person will not buy, the product will not be marketed to him/her, and the business will lose…er, business. FP is not such a big worry since only the cost of a phone call, SMS or sending a catalog will be lost.

Confusion Matrix

Buyer or Non-Buyer

Buyer or Not		Actual		Total
		Negative	Positive	
Predicted	Negative	725	158	883
	Positive	75	302	377
Total		800	460	1260

What is more important: Recall, Precision or Accuracy?

$$Recall\ (or\ Sensitivity) = \frac{TP}{TP + FN} = \frac{302}{460} = 65.6\%$$

$$Precision = \frac{TP}{TP + FP} = \frac{302}{377} = 80.1\%$$

$$Accuracy = \frac{TP + TN}{TP + FN + TN + FP} = \frac{1027}{1260} = 81.5\%$$

$$Specificity = \frac{TN}{TN + FP} = \frac{725}{800} = 90.6\%$$

$$F_1 = \frac{2 * Recall * Precision}{Recall + Precision} = \frac{2 * 0.656 * 0.801}{0.656 + 0.801} = 72.1\%$$

Sensitivity - Specificity

TEST NAME	TECHNOLOGY	VALUE	UNITS
ANTI CCP (ACCP)	E.L.I.S.A	0.48	OD Ratio

Reference Range :

Negative : < 0.80
Equivocal: 0.80 - 1.20
Positive : > 1.20

Clinical Significance :

Anti-Cyclic-Citrullinated-Peptide (Anti-CCP) Antibodies hold promise for early and more accurate detection of Rheumatoid Arthritis before the disease proceeds into an irreversible damage.

Analytical Specifications :

Anti-Cyclic-Citrullinated-Peptide (Anti-CCP) antibodies are detected using a solid phase enzyme immuno assay having an analytical sensitivity of 1.0 U/ml. No cross reactivity to other auto antigen is found. Sensitivity of the method is 68% and specificity is 92%.

Method : SOLID PHASE CAPTURE ENZYME IMMUNOASSAY

ANTI NUCLEAR ANTIBODIES (ANA)	E.L.I.S.A	0.29	OD Ratio

Reference Range :

Negative < 0.80
Equivocal 0.8 – 1.20
Positive > 1.20

Sensitivity - Specificity

Rheumatoid arthritis is prevalent among 0.75% of adult Indian population. Assuming a test with a certain sensitivity and specificity gives the following results, what are the values for these metrics?

Rheumatoid Arthritis Diagnostic Test		**Predicted**		**Total**
		Positive	Negative	
Actual	Positive	20	10	30
	Negative	318	3652	3970
Total		338	3662	4000

$$Recall\ (Sensitivity) = \frac{20}{30} = 0.67$$

$$Specificity = \frac{3652}{3970} = 0.92$$

Present Day

SOME COMMON DISTRIBUTIONS

Poisson Distribution

Probability of getting 15 customers requesting for loans in a given day given on average we see 10 customers

$\lambda = 10 \; and \; r = 15$

PMF, $\mathrm{P(X = r)} = \frac{e^{-\lambda}\lambda^{r}}{r!}$

CDF, $\mathrm{P(X \leq r)} = e^{-\lambda}\sum_{i=0}^{r}\frac{\lambda^{i}}{i!}$

Poisson Distribution

$E(X) = \lambda$ Can be equated to *np* of Binomial if *n* is large (>50) and *p* is small (<0.1)

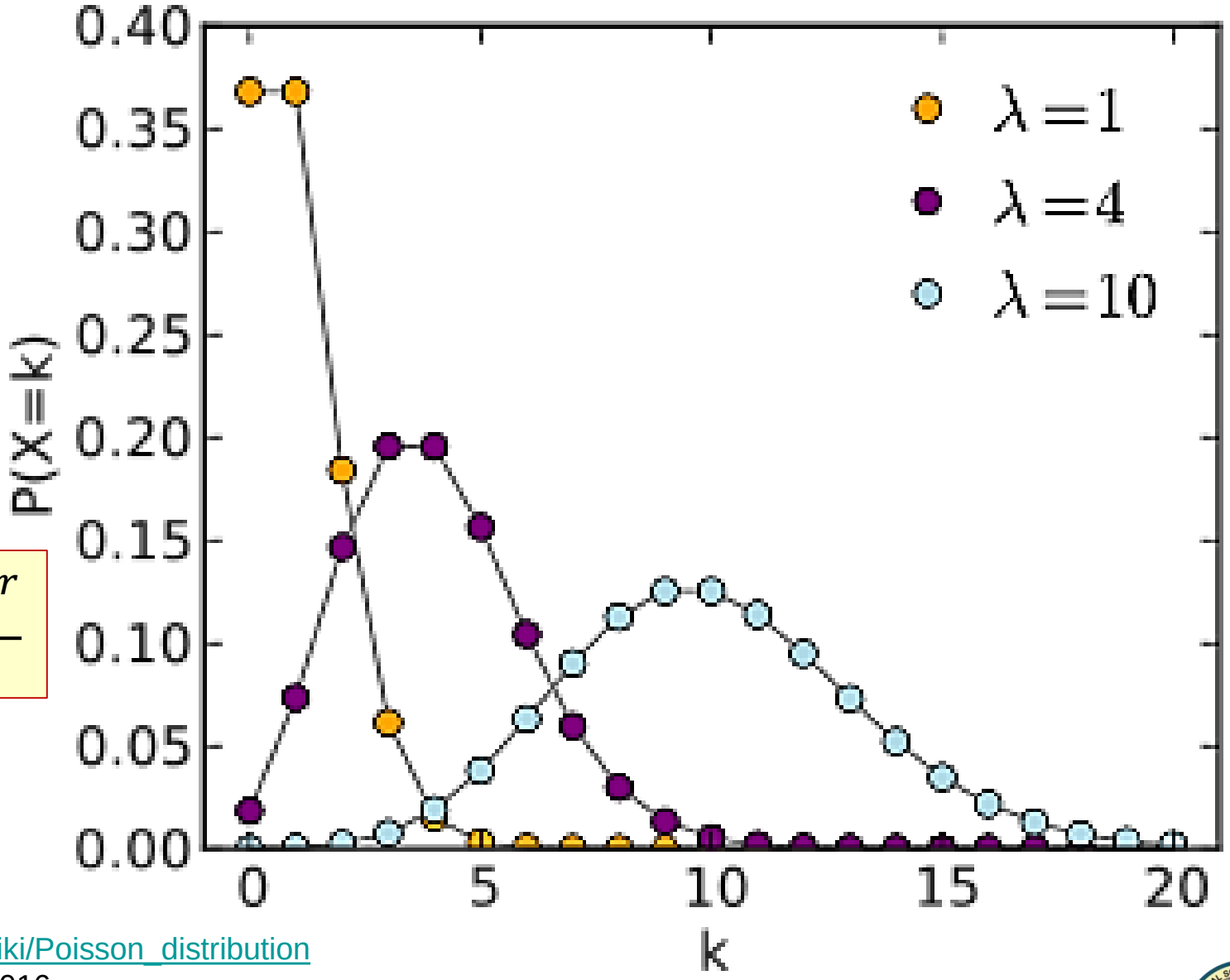
$Var(X) = \lambda$ Can be equated to *npq* of Binomial in the above situation.

When to use?

- Individual events occur at random and independently in a given interval (time or space).
- You know the mean number of occurrences, λ, in the interval or the rate of occurrences, and it is finite.

X~Po(λ)

$$P(X = r) = \frac{e^{-\lambda}\lambda^{r}}{r!}$$



Ref: https://en.wikipedia.org/wiki/Poisson_distribution
Last accessed: February 12, 2016

Poisson Distribution

The probability that no customer will visit the store in one day

$$P(X=0) = \frac{e^{-\lambda}\lambda^0}{0!} = e^{-\lambda}$$

Probability that she will not have a customer for *n* days

$$e^{-n\lambda}$$

Exponential Distribution

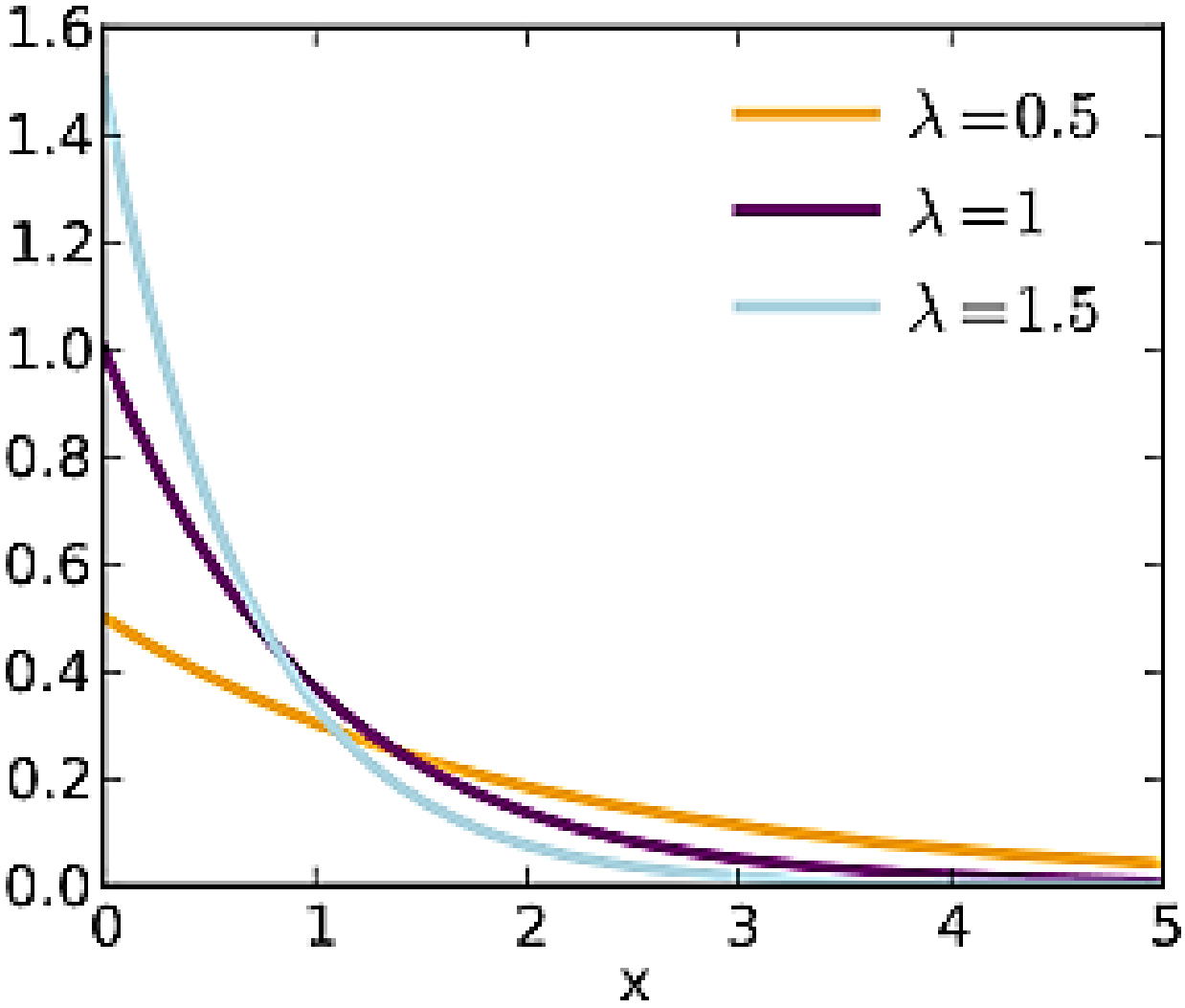
Probability that a customer will visit in n days: $1 - e^{-n\lambda}$

$$CDF = 1 - e^{-n\lambda}, n \geq 0$$

$$PDF = \lambda e^{-n\lambda}, n \geq 0$$

X~Exp(λ)

$$PDF = \lambda e^{-n\lambda}, n \geq 0$$



Ref: http://en.wikipedia.org/wiki/Exponential_distribution
Last accessed: June 12, 2015

Exponential Distribution

- Poisson process
- Continuous analog of Geometric distribution

$$E(X) = \frac{1}{\lambda} \qquad Var(X) = \frac{1}{\lambda^2}$$

Probability Distributions

Geometric:	For estimating number of attempts before first success
Binomial:	For estimating number of successes in *n* attempts
Poisson:	For estimating *n* number of events in a given time period when on average we see *m* events
Exponential:	Time between events

Probability Distributions - Scenarios

Identify the distribution and calculate expectation, variance and the required probabilities.

Q1. A man is bowling. The probability of him knocking all the pins over is 0.3. If he has 10 shots, what is the probability he will knock all the pins over less than 3 times?

Probability Distributions - Scenarios

X ~ B(10,0.3); n=10, p=0.3, q=1-0.3=0.7, r=0, 1, 2 (< 3)

E(X) = np = 3

Var(X) = npq = 2.1

$P(X = r) = {}^{n}C_r p^r q^{n-r}$

P(X=0) = 0.028; P(X=1) = 0.121; P(X=2) = 0.233

∴ P(X<3) = 0.028 + 0.121 + 0.233 = 0.382

Probability Distributions - Scenarios

Identify the distribution and calculate expectation, variance and the required probabilities.

Q2. On average, 1 bus stops at a certain point every 15 minutes. What is the probability that no buses will turn up in a single 15 minute interval?

Probability Distributions - Scenarios

X ~ Po(1); λ=1, r=0

E(X) = λ = 1

Var(X) = λ = 1

$$P(X = r) = \frac{e^{-\lambda}\lambda^{r}}{r!}$$

P(X=0) = 0.368

Probability Distributions - Scenarios

Identify the distribution and calculate expectation, variance and the required probabilities.

Q3. 20% of cereal packets contain a free toy. What is the probability you will need to open fewer than 4 cereal packets before finding your first toy?

Probability Distributions - Scenarios

X ~ Geo(0.2); p=0.2, q=1-0.2=0.8, r<4 or ≤3

$E(X) = \frac{1}{p} = 5$

$Var(X) = \frac{q}{p^2} = 20$

$P(X \leq r) = 1 - q^r$

$P(X \leq 3) = 0.488$

Poisson Distribution Formula Differences?

$$P(X = r) = \frac{e^{-\lambda}\lambda^{r}}{r!} \; or \; \frac{e^{-\lambda t}(\lambda t)^{r}}{r!} \; ?$$

Suppose births in a hospital occur randomly at an average rate of 1.8 births per hour. What is the probability of 5 births in a given 2 hour interval?

What is λ?

$$P(X = 5) = \frac{e^{-3.6}3.6^{5}}{5!} \; or \; \frac{e^{-1.8*2}(1.8 * 2)^{5}}{5!} \; ?$$

If you use 1.8, use t=2 in the second formula. Alternatively, you could say that since the average is 1.8 per hour, it is 3.6 per 2 hours (the interval of interest).

Poisson Distribution Formula Differences?

$$P(X = r) = \frac{e^{-\lambda}\lambda^r}{r!} \; or \; \frac{e^{-\lambda t}(\lambda t)^r}{r!} \; ?$$

Now suppose head injury patients (due to not wearing helmets) arrive in Hospital A randomly at an average rate of 0.25 patients per hour, and in Hospital B randomly at an average rate of 0.75 per hour. What is the probability of more than 3 such patients arriving in a given 2 hour interval in both hospitals together?

What is the probability distribution?

$X \sim Po(\lambda_1)\ and\ Y \sim Po(\lambda_2)$
$X + Y \sim Po(\lambda_1 + \lambda_2)$

What are λ_1 and λ_2 if we use first formula?

$\lambda_1 = 0.5$ and $\lambda_2 = 1.5$
$\lambda_1 + \lambda_2 = 2$

$$P(X + Y > 3) = P(X + Y = 4) + P(X + Y = 5) + P(X + Y = 6) + \cdots$$
$$= 1 - P(X + Y \leq 3) = 1 - (P(X + Y = 0) + P(X + Y = 1) + P(X + Y = 2) + P(X + Y = 3))$$

Poisson or Exponential?

Given a Poisson process:

- The *number* of events in a given time period — Poisson
- The *time* until the first event — Exponential
- The *time* from now until the next occurrence of the event — Exponential
- The *time interval* between two successive events — Exponential

Poisson or Exponential?

The tech support centre of a computer retailer receives 5 calls per hour on an average. What is the probability that the centre will receive 8 calls in the next hour? What is the probability that more than 30 minutes will elapse between calls?

$$P(X = 8) = \frac{e^{-5}5^8}{8!} = 0.065$$

$$P(Time\ between\ calls > 0.5) = \int_{0.5}^{\infty} \lambda e^{-\lambda T} dT = -e^{-\lambda T}]_{0.5}^{\infty}$$

$$= e^{-5*0.5} = 0.082$$

Probability Distributions

Babyboom Data - Excel

Forty-four babies -- a new record -- were born in one 24-hour period at the Mater Mothers' Hospital in Brisbane, Queensland, Australia, on December 18, 1997. For each of the 44 babies, *The Sunday Mail* recorded the time of birth, the sex of the child, and the birth weight in grams.

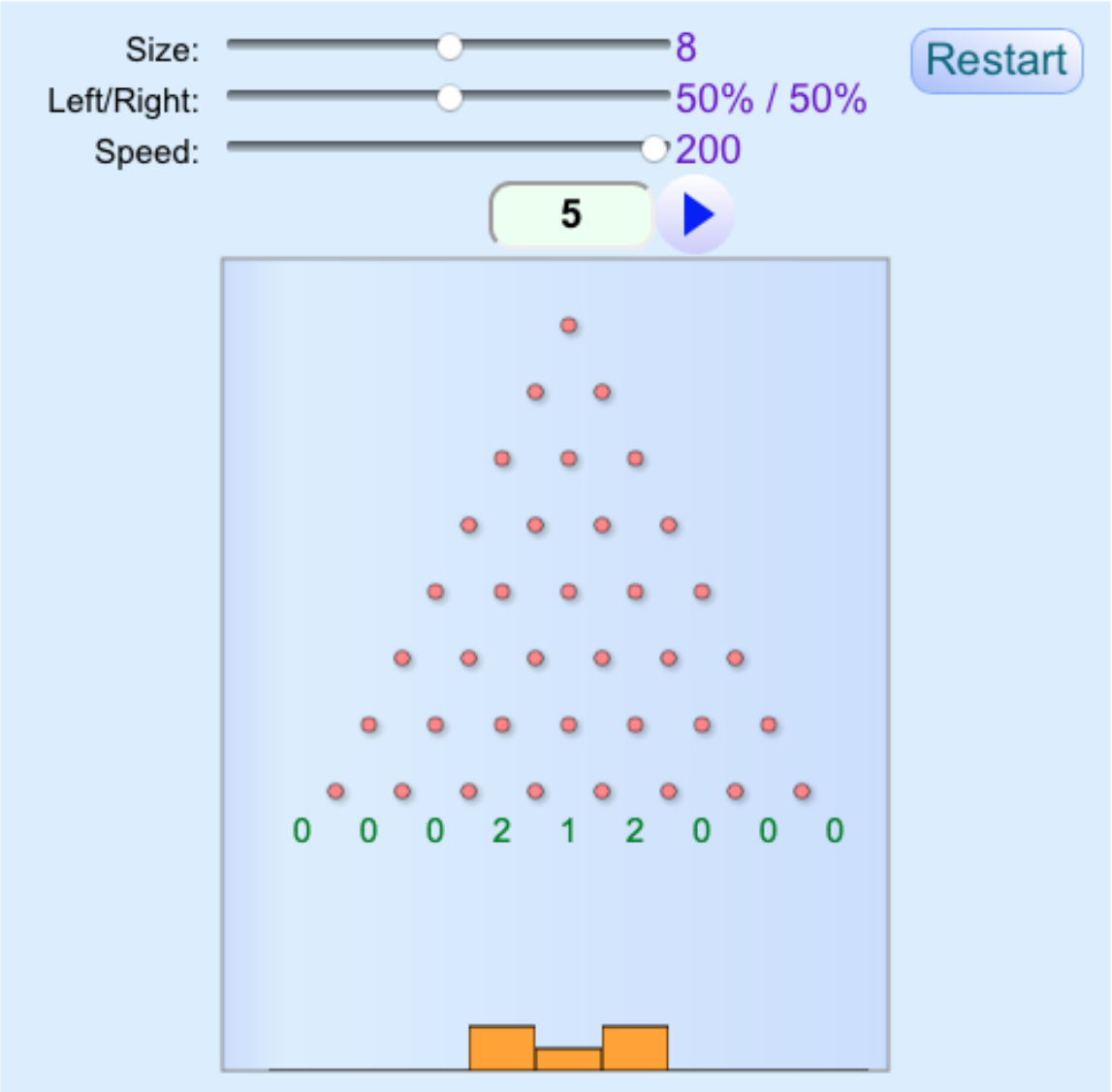
Probability Distributions

Determine the distributions for the following scenarios for this dataset:

1. Probability of observing at least 26 boys in 44 births assuming equal probability of a boy or a girl being born.
2. Probability that 3 births occur before the birth of a girl.
3. Probability of 4 births per hour given 44/24 = 1.83 births per hour on average.
4. Probability that more than 60 minutes will elapse between births.

1. Binomial; 2. Geometric; 3. Poisson; 4. Exponential

Quincunx Demo

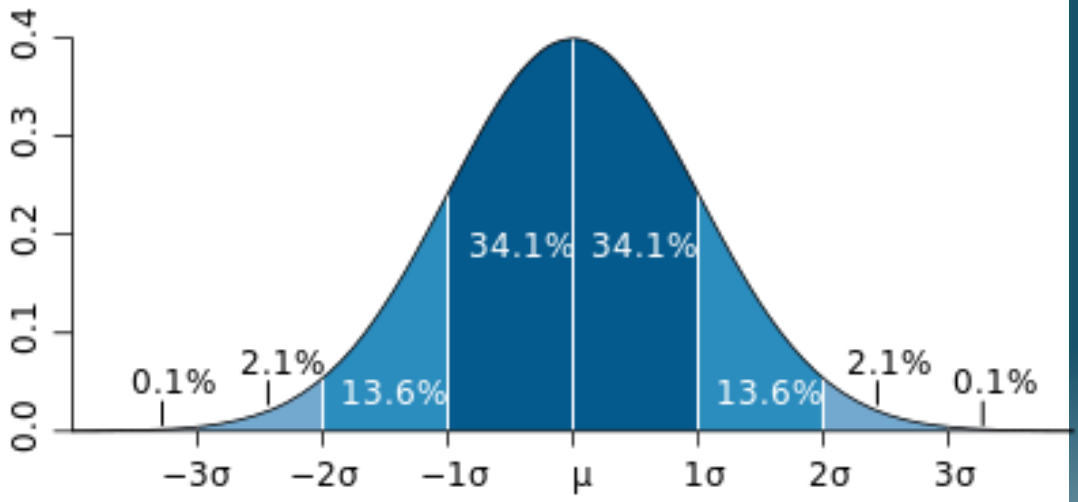


Source: http://www.mathsisfun.com/data/quincunx.html

NORMAL DISTRIBUTION

Normal (Gaussian) Distribution

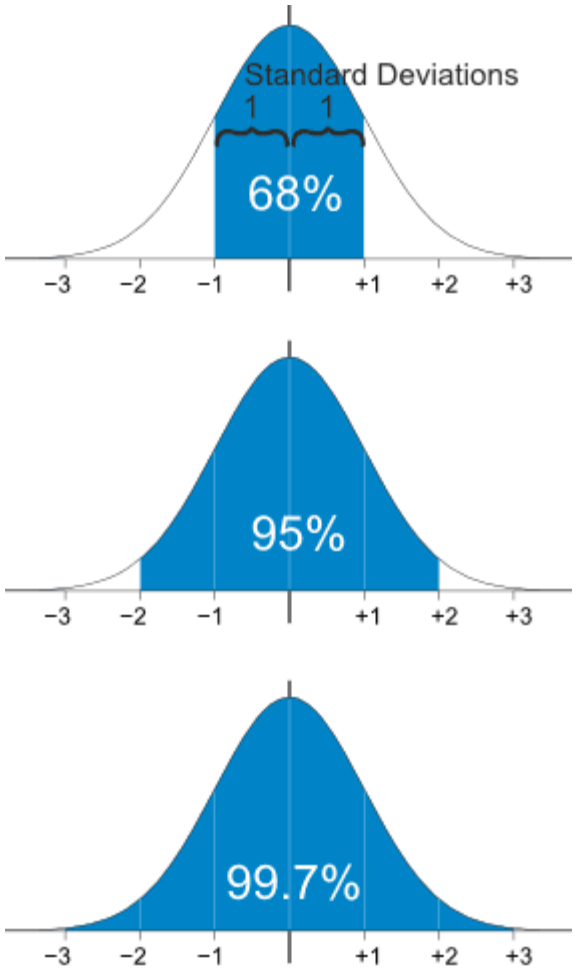
- Mean = Median = Mode
- 68-95-99.7 empirical rule
- Zero Skew and Kurtosis
- X ~ N(μ, σ^2)
- Shaded area gives the probability that X is between the corresponding values



$$f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Measures of Spread (Dispersion)

You know the 68-95-99.7 rule.



A company produces a valve that is specified to weigh 1500g, but there are imperfections in the process. While the mean weight is 1500g, the standard deviation is 300g.

Q1. What is the range of weights within which 95% of the valves will fall?

Q2. Approximately 16% of the weights will be more than what value?

Q3. Approximately 0.15% of the weights will be less than what value?

Image source: http://www.mathsisfun.com/data/standard-normal-distribution.html
Last accessed: December 15, 2017

Sample Software Output

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.717055011
R Square	0.514167888
Adjusted R Square	0.494734604
Standard Error	4.21319131
Observations	27

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	469.6573265	469.6573265	26.4581054	2.57053E-05
Residual	25	443.7745253	17.75098101		
Total	26	913.4318519			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 99.0%*	*Upper 99.0%*
Intercept	-4.154014573	2.447784673	-1.697050651	0.102104456	-9.195321476	0.88729233	-10.97705723	2.669028089
Big Mac Price ($)	3.547427488	0.689658599	5.143744297	2.57053E-05	2.127049014	4.967805962	1.625048409	5.469806567

Sample Software Output

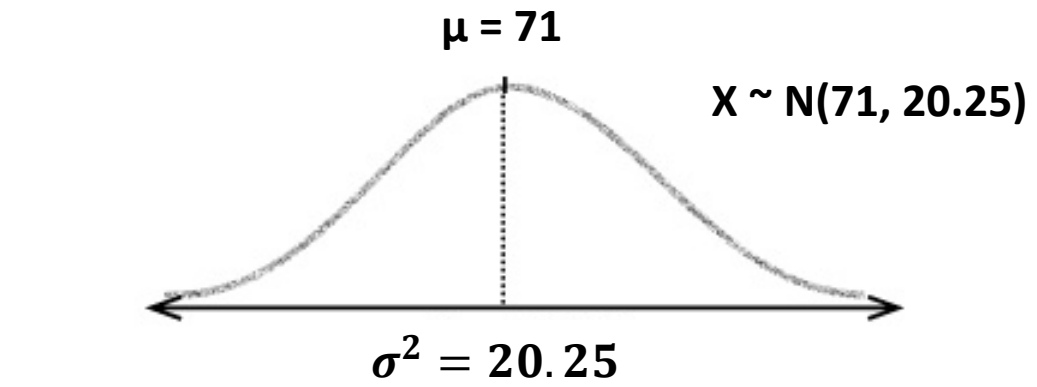
```
Call:
glm(formula = Response ~ Age, family = "binomial", data = flierresponse)

Deviance Residuals:
     Min        1Q    Median        3Q       Max
-1.95015  -0.32016  -0.05335   0.26538   1.72940

Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept) -20.40782    4.52332  -4.512 6.43e-06 ***
Age           0.42592    0.09482   4.492 7.05e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 123.156  on 91  degrees of freedom
Residual deviance:  49.937  on 90  degrees of freedom
AIC: 53.937

Number of Fisher Scoring iterations: 7
```

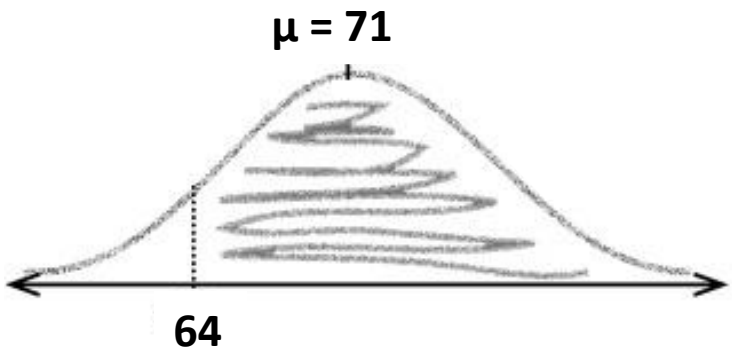
Calculating Normal Probabilities

Step 1: Determine the distribution

Julie wants to marry a person taller than her and is going on blind dates. The mean height of the 'available' guys is 71" and the variance is 20.25 inch^2 (yuck!).

Oh! By the way, Julie is 64" tall.





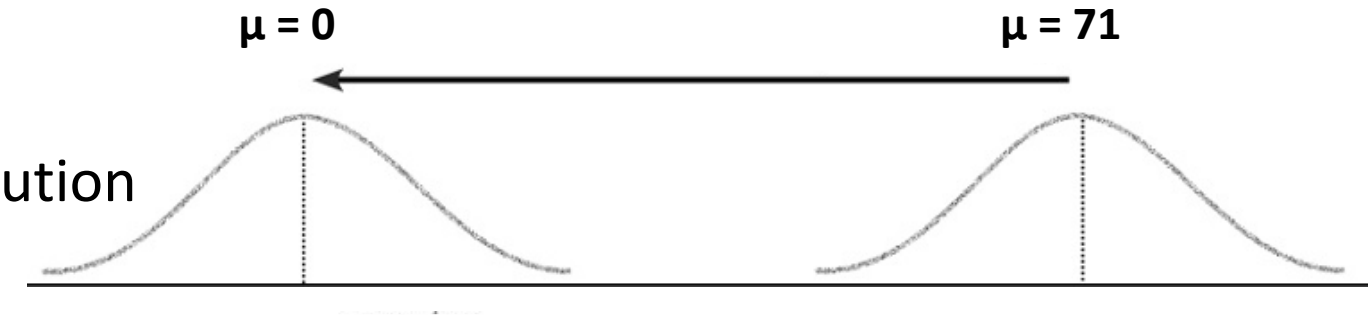
Calculating Normal Probabilities

Step 2: Standardize to Z ~ N(0,1)

1. Move the mean

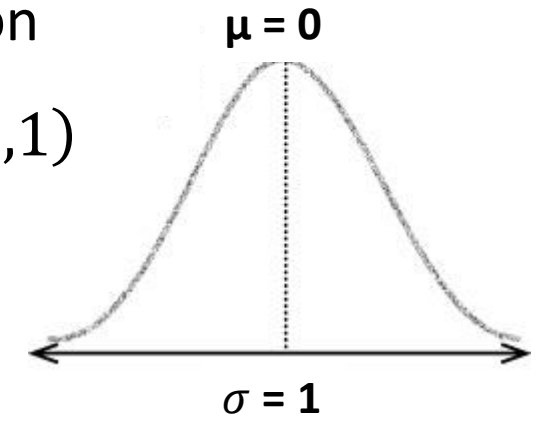
   This gives a new distribution

   X-71 ~ N(0,20.25)



2. Squash the width by dividing by the standard deviation

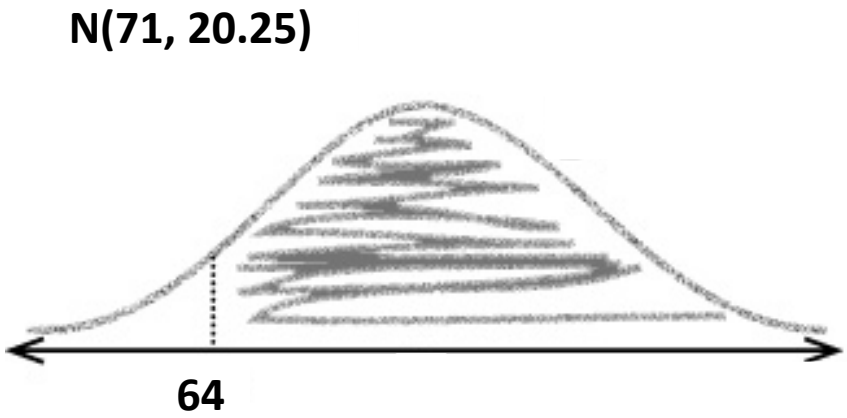
   This gives us $\frac{X-71}{4.5} \sim N(0,1)$

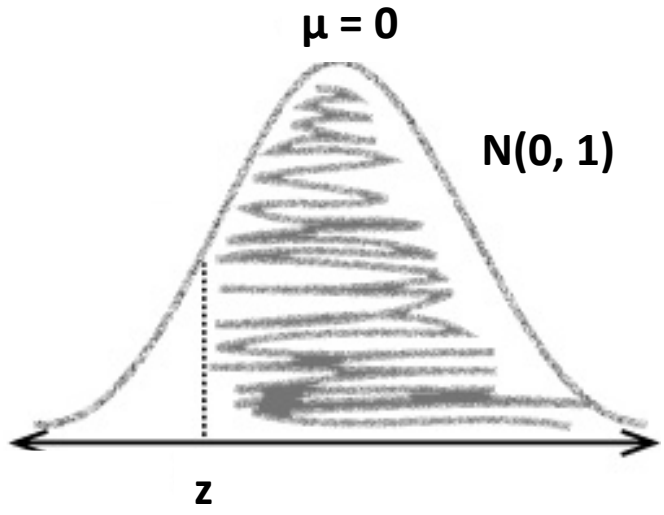


$Z = \frac{X-\mu}{\sigma}$ is called the Standard Score or the z-score.

Calculating Normal Probabilities

Step 2: Standardize to Z ~ N(0,1)





Note: R does this step internally but you must understand the concept of z-score as this is fundamental to most statistical thinking

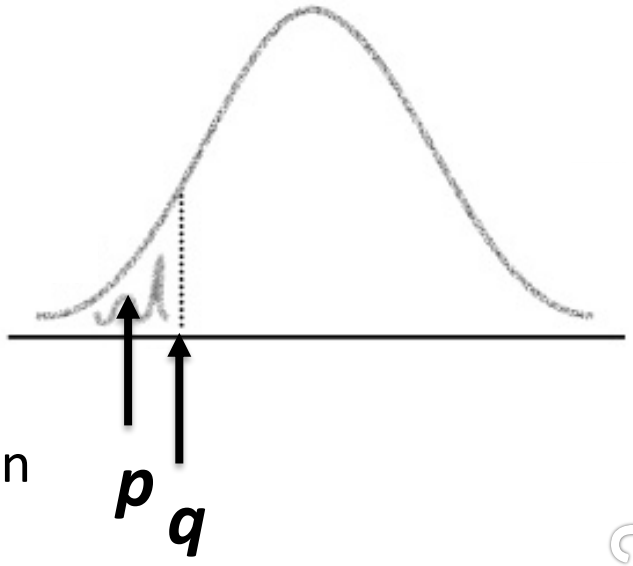
$Z = \frac{64-71}{4.5} = -1.56$ in the case of our problem.

Calculating Normal Probabilities

Step 3: Look up the probability in the tables

Note the tables give P(Z<z).

In R functions, the distribution is abbreviated and prefixed with an alphabet.



*p*norm: Probability (Cumulative Distribution Function, CDF) in a *Normal Distribution*

*q*norm: Quantile (Inverse CDF) in a *Normal Distribution* – The value corresponding to the desired probability.

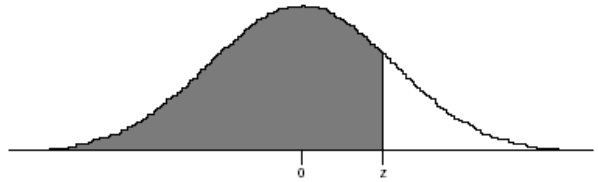
Calculating Normal Probabilities

Step 3: Look up the probability in the tables

Note the tables give P(Z<z).

$z = \frac{64-71}{4.5} = -1.56$ in the case of our problem.

P(Z>-1.56) = 1- P(Z<-1.56) = 1- 0.0594 = 0.9406



Normal Deviate z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-4.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.9	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.7	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379

Calculating Normal Probabilities

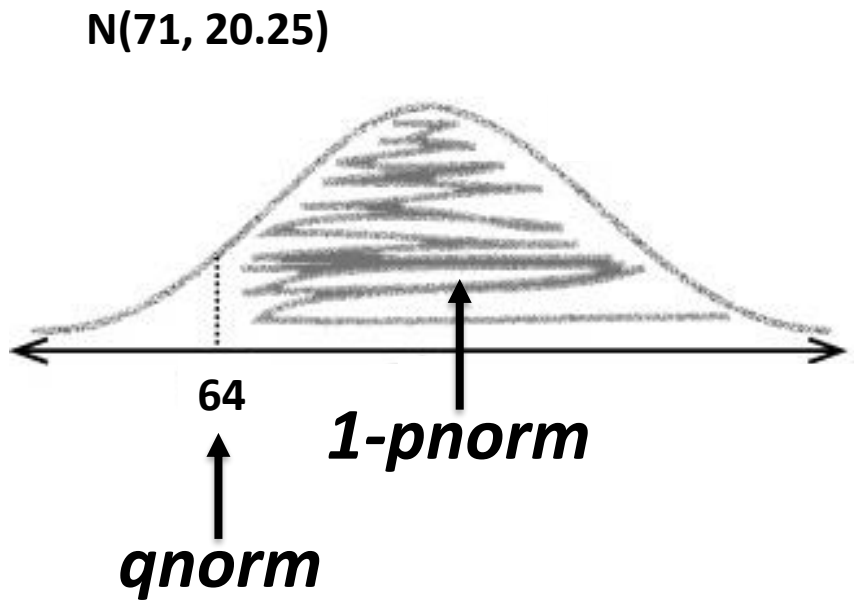
Step 3: Get the probability from R

1-pnorm(64, mean=71, sd=sqrt(20.25))
or
1-pnorm(64, 71, 4.5)

Answer: 1-0.0599 = 94.01%

qnorm(0.0599, 71, 4.5)

Answer: 64



Attention Check

Q. What is the standard score for N(10,4), value 6?

A. $z = \frac{6-10}{2} = -2$

Q. The standard score of value 20 is 2. If the variance is 16, what is the mean?

A. $2 = \frac{20-\mu}{4}$. $\therefore \ \mu = 20 - 8 = 12$

Attention Check

Q. Julie just realized that she wants her date to be taller when she is wearing her heels, which are 5” high. Find the new probability that her date will be taller.

A. $z = \frac{69-71}{4.5} = -0.44;$

P(Z<-0.44) = 0.33,

∴ P(Z>-0.44) = 0.67 or 67%

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

A. 1-pnorm(69, 71, 4.5). This gives P(X>69) = 67%

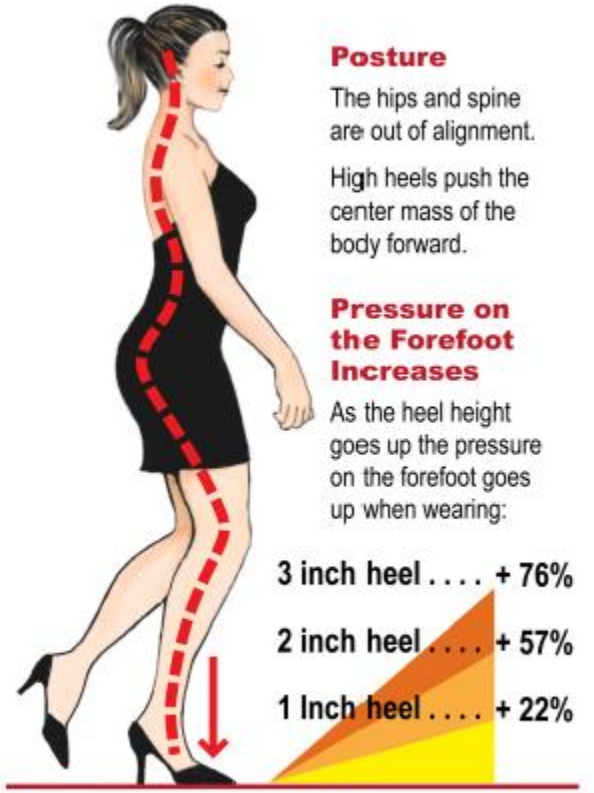
Attention Check

Q. Julie wants to have at least 80% probability of finding the right guy. What is the maximum size of heels she can wear?

A. qnorm(0.20, 71, 4.5). This gives a value of 67.2". As Julie is 64" tall, the maximum heel size she should wear is about 3".

Attention Check

Q. Julie is convinced of the dangers of high heels and decides to stick with only 1” heels. What is the probability of finding the right guy now?



A. 1-pnorm(65, 71, 4.5). This gives a P(X>65)=90.9%.

FLAT REFUSAL

It's not just high heels that can be a pain, flat footwear is equally damaging

PRIYANKA PRAVEEN
DECCAN CHRONICLE

Almost everyone's favourite pair of 'killer' high heels have been notorious for bad posture and foot aches amongst other issue. Now reports say that its simple cousin — the flats — aren't really goody two shoes either.

Even celebrities like Victoria Beckham, who swear by their stilettos, have on quite a few occasions traded them for a pair of flats, but doctors feel that this really might not be the best thing for our feet. From agonising pain, spinal damage and even disorders — flats, are responsible for a host of problems.

Flats can cause spinal problems and inflammation of the thick band of tissues that connects the heel and the toes

"Our foot consists of the toes, the arch and the heel, this mechanism works so well that when we walk our entire weight is distributed equally," explains Dr Mithin Aachi, Senior Orthopedician. "The arch is what helps with the equal distribution of weight and so when we wear flat footwear unequal distribution of weight takes place and undue stress is put on the heel. This leads to several problems including plantar fasciitis and an inflammation of the thick band of tissues that connects the heel and the toes," he adds. In such cases, the pain is, several times, unbearable. Dr Praveen Rao, Orthopedic Surgeon, says, "When this happens, people find it difficult to walk after sitting for a long time."

Apart from pain, the lack of a cushioning and an arch in these footwear can eventually lead to spine troubles. "Since the pressure is on the heel, the gait of the person changes over the years and that leads to spinal problems and causes severe pain," explains Dr Rao.

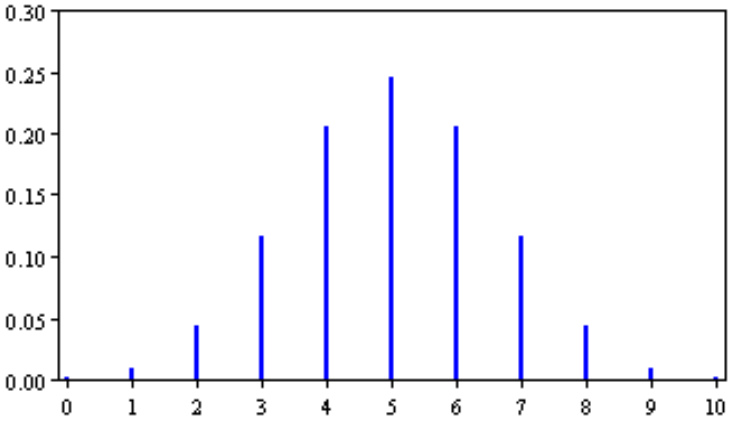
Doctors believe that we need to find a middle ground. "It's okay to wear high heels once in a while and since flats are more convenient, you can wear them occasionally, but you will need to find a balance. It helps to take a 'foot holiday' once a week by giving flats and heels a break and opting for an arched and cushioned footwear," explains Dr Aachi.

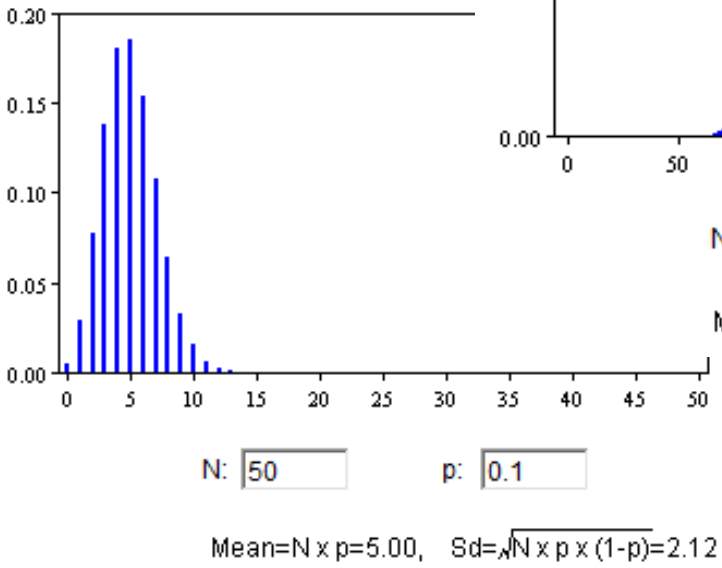
So, is there an ideal heel height that one needs to follow? "There isn't a number as such, but heels above one inch should be avoided regularly. Also wearing cushioned footwear with a small block-heel sometimes is fine," adds Dr. Rao.

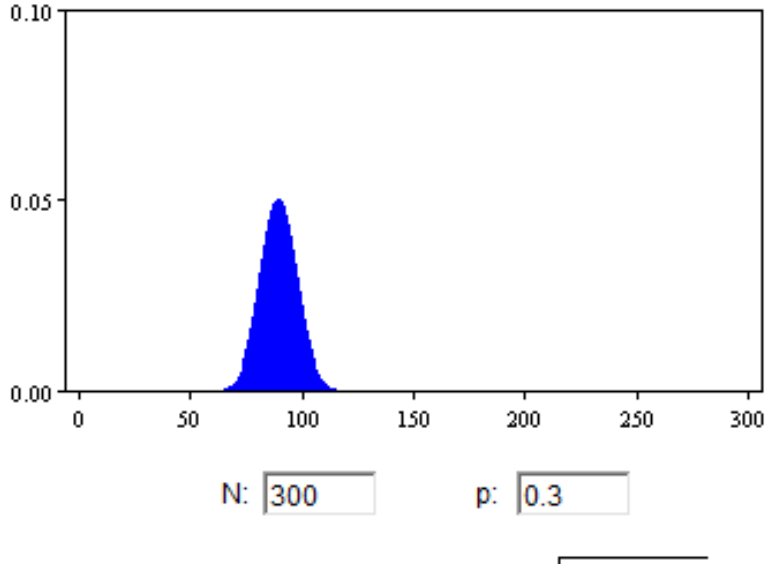
ALL TOO FLAT: Wearing flats regularly can be bad for your feet

Normal Distribution

Binomial distribution can be approximated to a Normal distribution if np>5 and nq>5 (Continuity Correction required).

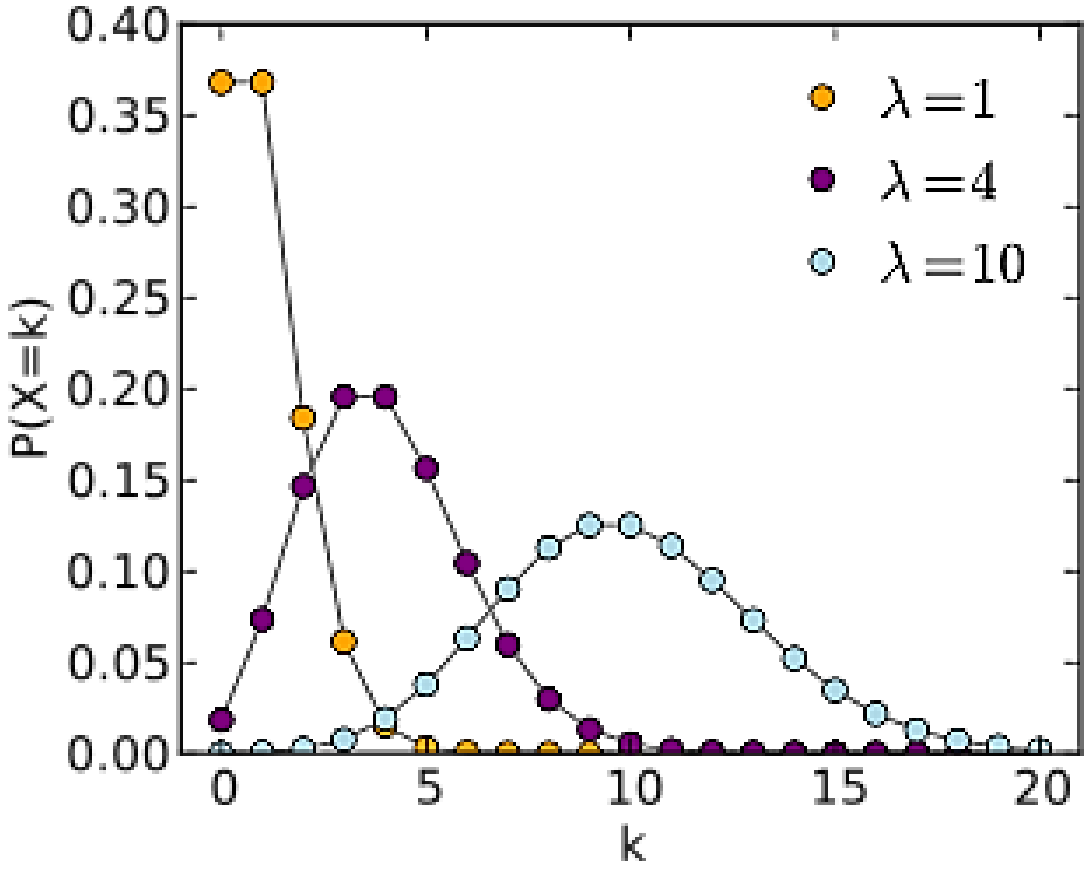






Normal Distribution

Poisson distribution can be approximated to a Normal distribution when $\lambda > 15$ (Continuity Correction required).



Continuity Correction?

You are playing Who Wants to Win a 

What is the probability you will get 5 or fewer correct out of 12, given that each question has only 2 possible choices? You have no lifelines.

X ~ B(12, 0.5) and we need to find P(X < 6).

$$P(X = 0) = {}^{12}C_0(0.5)^0(0.5)^{12-0} = 0.5^{12}$$
$$P(X = 1) = {}^{12}C_1(0.5)^1(0.5)^{12-1} = 12 * 0.5^{12}$$
$$P(X = 2) = {}^{12}C_2(0.5)^2(0.5)^{12-2} = 66 * 0.5^{12}$$
$$P(X = 3) = {}^{12}C_3(0.5)^3(0.5)^{12-3} = 220 * 0.5^{12}$$
$$P(X = 4) = {}^{12}C_4(0.5)^4(0.5)^{12-4} = 495 * 0.5^{12}$$
$$P(X = 5) = {}^{12}C_5(0.5)^5(0.5)^{12-5} = 792 * 0.5^{12}$$
$$\therefore P(X < 6) = (1 + 12 + 66 + 220 + 495 + 792) * 0.5^{12} \cong 0.387$$

Continuity Correction?

X ~ B(12, 0.5) can be approximated to X ~ N(6, 3). How/Why?

n = 12, p = 0.5 and q = 0.5. Since np and nq are both > 5, the Binomial distribution can be approximated to a Normal distribution, i.e., X ~ B(n, p) can be approximated to X ~ N(np, npq).

If we want to get P(X < 6), what is the next step to do in the Normal distribution?

Calculate the z-score (or the standard-score).

$$z = \frac{x - \mu}{\sigma} = \frac{6 - 6}{\sqrt{3}} = 0$$

What do we do with the z-score?

Look it up in the probability tables or R.

What is the probability corresponding to the z-score of 0?

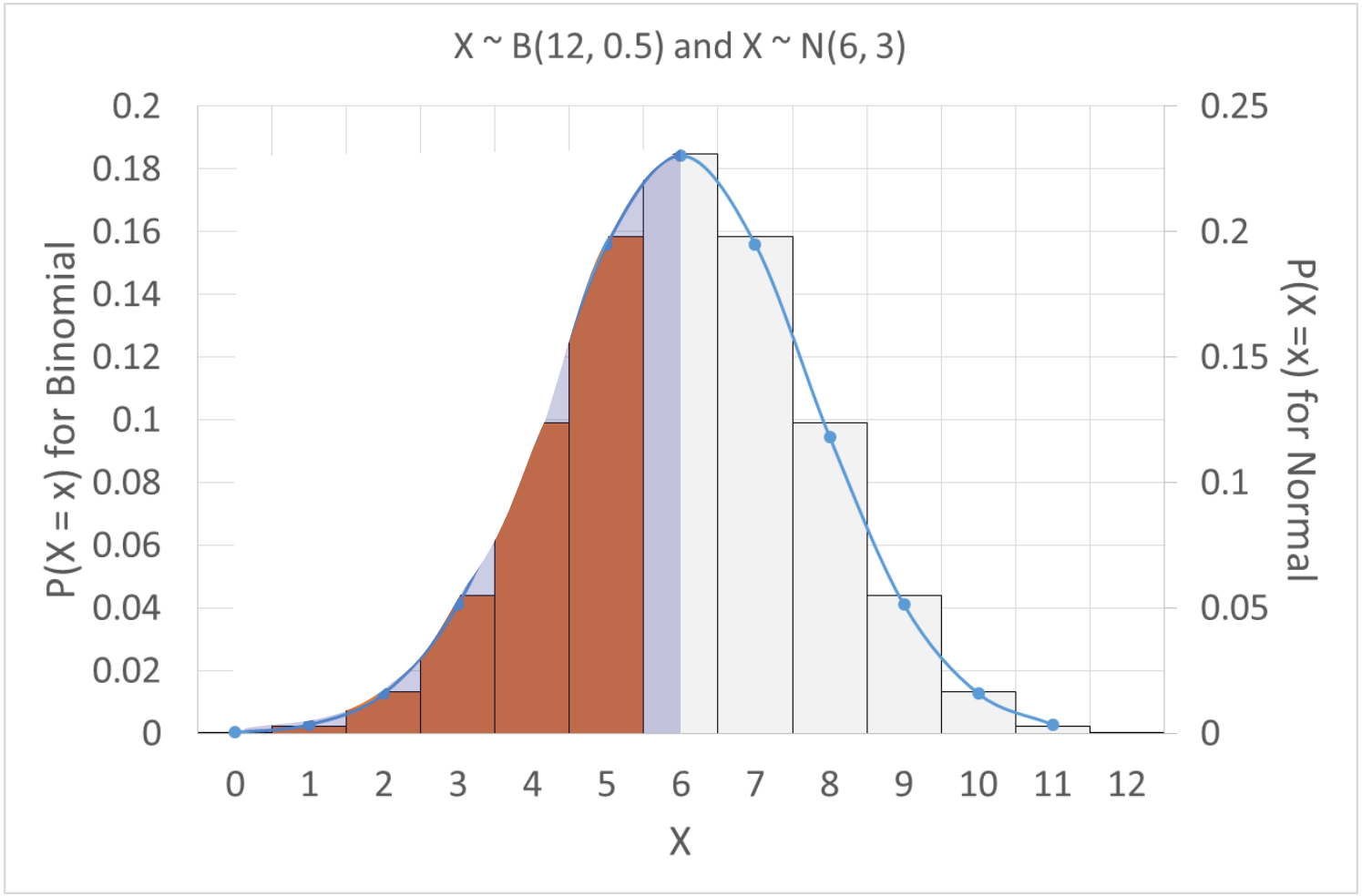
P(X < 6) = 0.5

Z	0.00	0.01	0.02	0.03	0.04	0.05
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647

Continuity Correction?

So, P(X < 6) = 0.387 for X ~ B(12, 0.5)

and P(X < 6) = 0.5 for X ~ N(6, 3). Is this a good approximation?



Continuity Correction?

So, P(X < 6) = 0.387 for X ~ B(12, 0.5)

and P(X < 6) = 0.5 for X ~ N(6, 3).

$$z = \frac{5.5 - 6}{\sqrt{3}} = -0.29$$

P(X < 5.5) = 0.3859 for X ~ N(6, 3)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Continuity Correction

Identify the right continuity correction for each discrete probability distribution.

Discrete	Continuous
$X < 3$	$X < 2.5$
$X > 3$	$X > 3.5$
$X \leq 3$	$X < 3.5$
$X \geq 3$	$X > 2.5$
$3 \leq X < 10$	$2.5 < X < 9.5$
$X = 0$	$-0.5 < X < 0.5$
$3 \leq X \leq 10$	$2.5 < X < 10.5$
$3 < X \leq 10$	$3.5 < X < 10.5$
$X > 0$	$X > 0.5$
$3 < X < 10$	$3.5 < X < 9.5$

Continuity Correction

You are playing Who Wants to Win a 

What is the probability of getting at least 30 out of 40 questions correct, where each question has 2 possible choices?

$P(X \geq 30)$ where $X \sim B(40, 0.5)$

Can we approximate it to the normal distribution?

$P(X > 29.5)$ where $X \sim N(20, 10)$

Normal Distribution

You have designed a new game, Angry Buds. The key to success is that it should not be so difficult that people get frustrated, nor should it be so easy that they don't get challenged. Before building the new level, you want to know what the mean and standard deviation are of the number of minutes people take to complete level 1. You know the following:

1. The # of minutes follows a normal distribution.
2. The probability of a player playing for less than 5 minutes is 0.0045.
3. The probability of a player playing for less than 15 minutes is 0.9641.

Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

$P(X < 5) = 0.0045$

$z_1 = -2.61$

Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

$P(X < 15) = 0.9641$

$z_2 = 1.8$

Normal Distribution

$-2.61 = \frac{5-\mu}{\sigma}$ and $1.8 = \frac{15-\mu}{\sigma}$

Solving for the above 2 equations, we get

$\mu = 5 + 2.61\sigma$

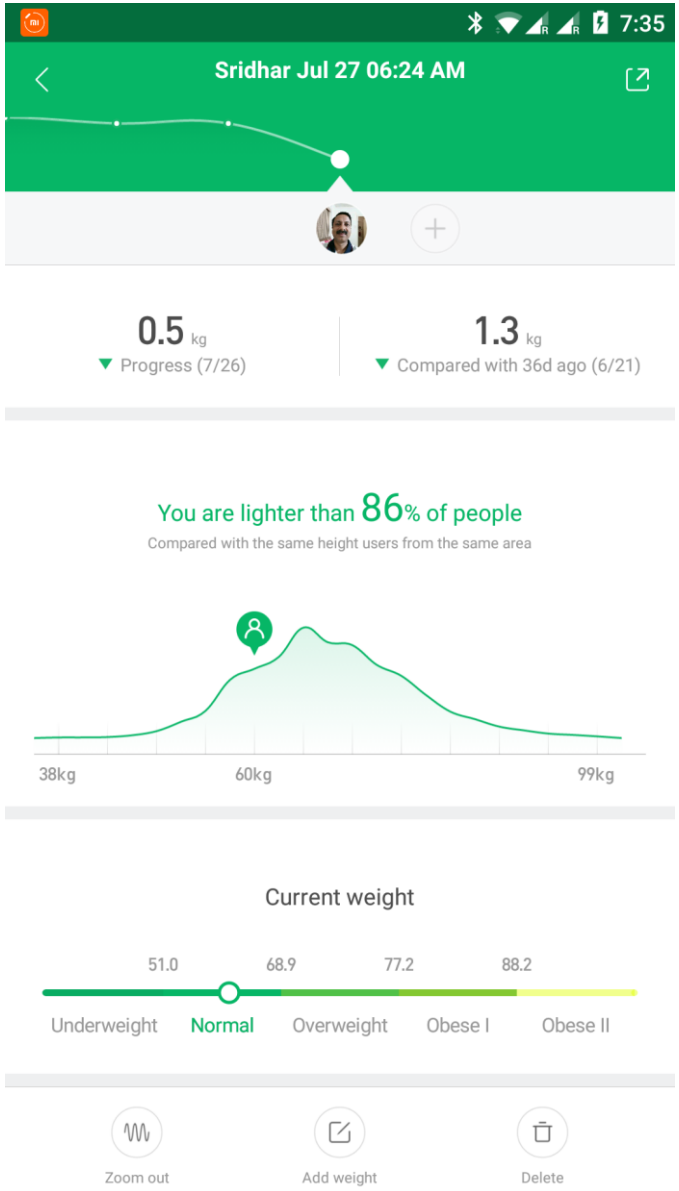
$\mu = 15 - 1.8\sigma$

Subtracting the two, we get

$0 = -10 + 4.41\sigma \Rightarrow \ \sigma = 10 \div 4.41 = 2.27$

Substituting this value of σ in either of the above 2 equations, we get $\mu = 5 + 2.61 * 2.27 = 10.925$

Normal Distribution



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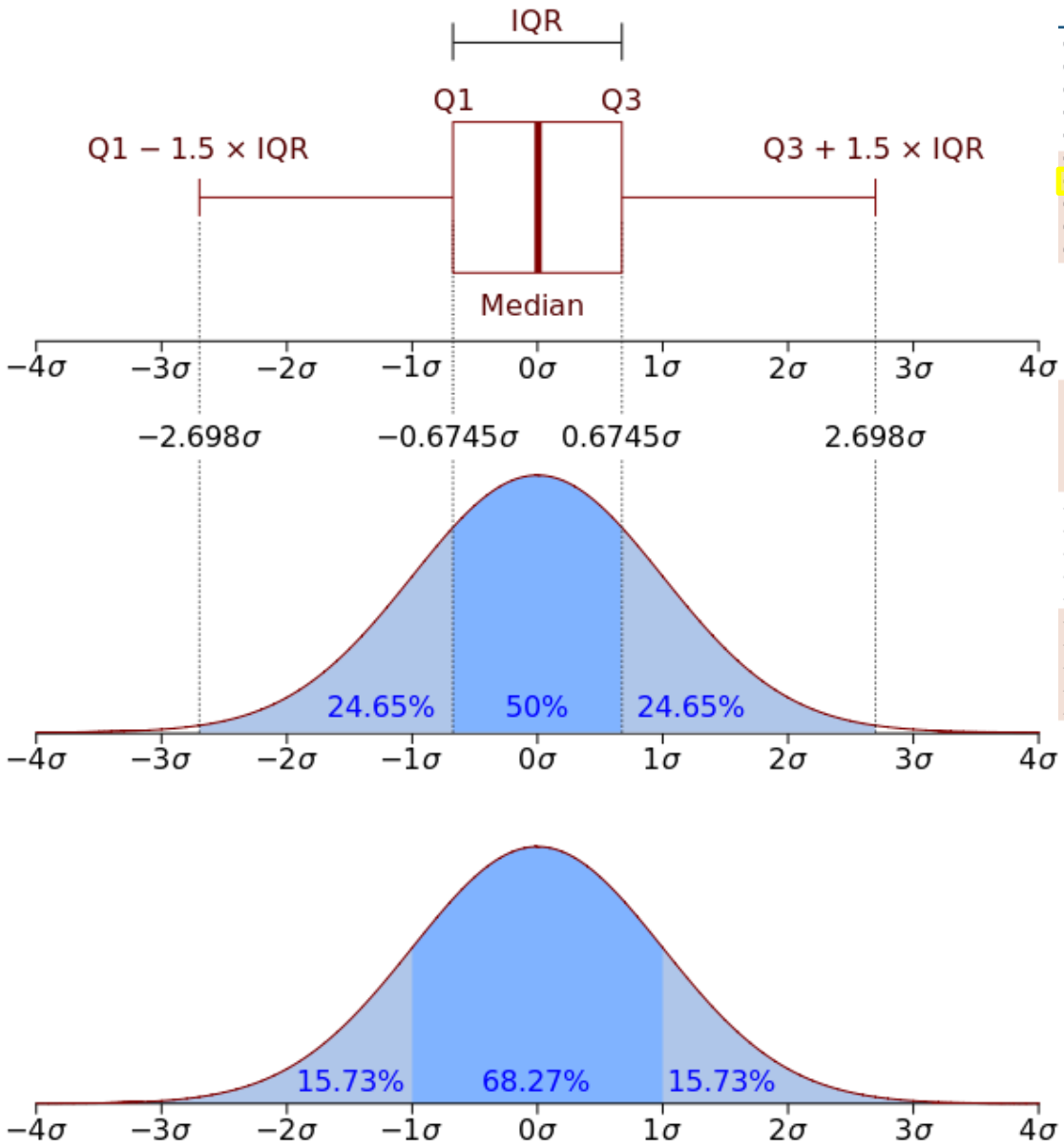

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0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
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0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
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1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
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2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Academic Exercise

0.75 is not in the z-table. The closest values are 0.7486 corresponding to a z-score of 0.67 and 0.7517 corresponding to a z-score of 0.68.

Method 1: Take the closest value, 0.7486 in this case.

Method 2: Interpolate.

z-value	P(0<Z<z)	Proportional distance from top
0.67	0.7486	0.0000
c (to be found)	0.75 (desired)	0.4516
0.68	0.7517	1.0000

$$\frac{0.75 - 0.7486}{0.7517 - 0.7486} = \frac{0.0014}{0.0031} = 0.4516$$

$$\Rightarrow \frac{c - 0.67}{0.68 - 0.67} = 0.4516$$

$$\therefore c = 0.67 + 0.4516 * 0.01 = 0.6745$$

Central Limit Theorem (CLT)

SAMPLING DISTRIBUTION OF MEANS

Sampling Distribution of the Means

- The sampling distribution of means is what you get if you consider all possible samples of size *n* taken from the same population and form a distribution of their means.
- Each randomly selected sample is an independent observation.

Central Limit Theorem

- http://onlinestatbook.com/2/sampling_distributions/clt_demo.html
- As sample size goes large and number of buckets are high, the means will follow a normal distribution with same mean (μ) and $\frac{1}{n}$ of variance (σ^2).

Expectation and Variance for $\bar{X}$

$$E(\bar{X}) = \mu$$

Mean of all sample means of size *n* is the mean of the population.

$$Var(\bar{X}) = \frac{\sigma^2}{n}$$

Standard deviation of $\bar{X}$ tells how far away from the population mean the sample mean is likely to be. It is called the **Standard Error of the Mean** and is given by

$$Standard\ Error\ of\ the\ Mean = \frac{\sigma}{\sqrt{n}}$$

$\bar{X}$ ~ N(μ, σ^2/n)

When an Attribute is Not Normal

- Let us assume it is a sample from infinite data
- So, if we take many such samples of large sample size (>30 as a thumb rule), the mean values, $\bar{x}$, will be hovering close to the population mean, μ, with a standard deviation, $s = \frac{\sigma}{\sqrt{n}}$, where σ is the population standard deviation and *n* is the sample size.

Using the Central Limit Theorem

Let us say the mean number of Gems per packet is 10, and the variance is 1. If you take a sample of 30 packets, what is the probability that the sample mean is 8.5 Gems per packet or fewer?

Using the Central Limit Theorem

We know that $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$, μ = 10, σ^2 = 1 and n = 30.

We need the value of P($\bar{X}$ < 8.5) when $\bar{X}$ ~ N(10, 0.0333).

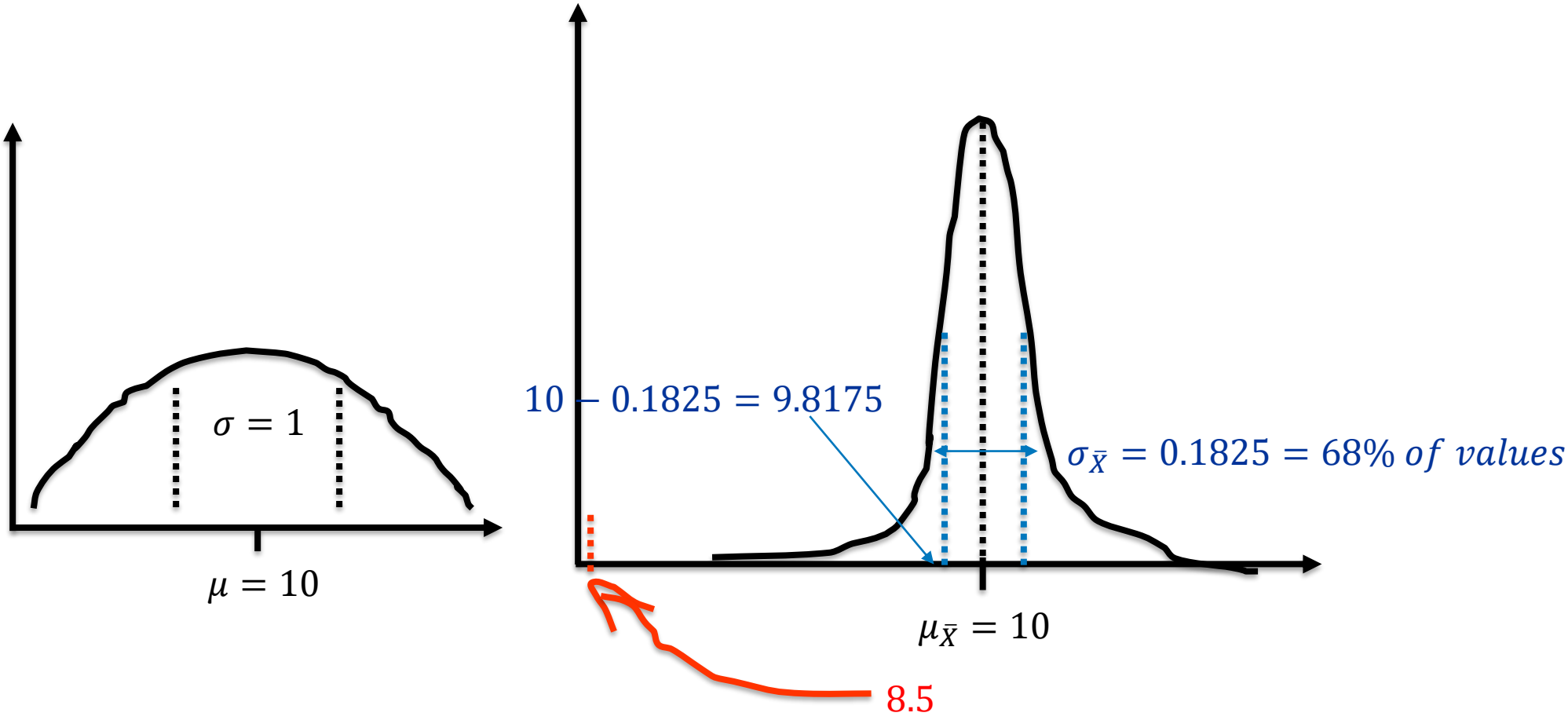
$$z = \frac{8.5 - 10}{\sqrt{0.0333}} = -8.22$$

P(Z < z) = P(Z < -8.22)

This doesn't exist in probability tables. What does it mean?

Using the Central Limit Theorem

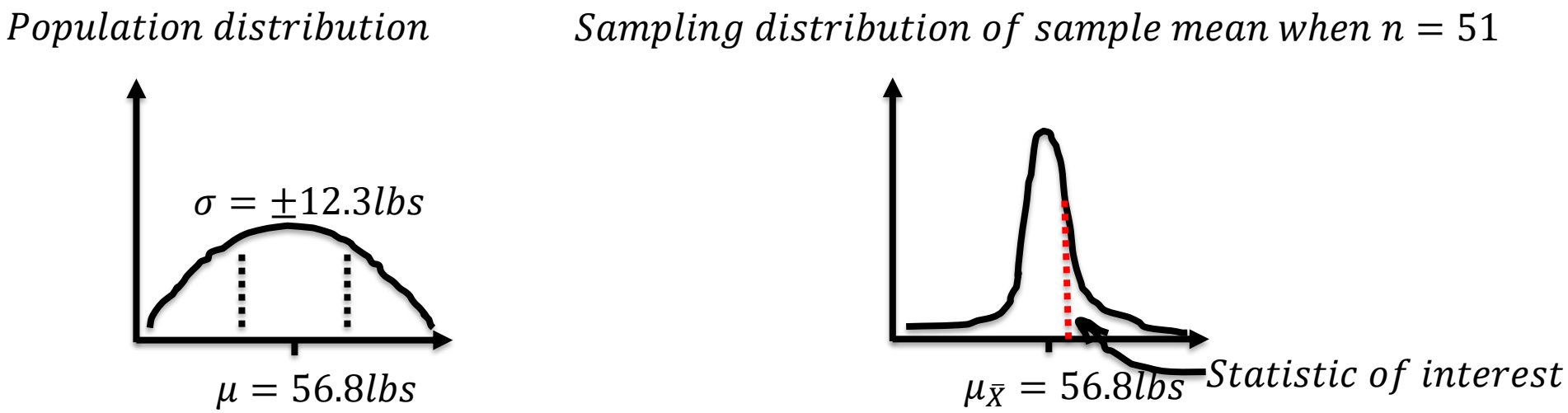
How do we visualize it?



Using the Central Limit Theorem

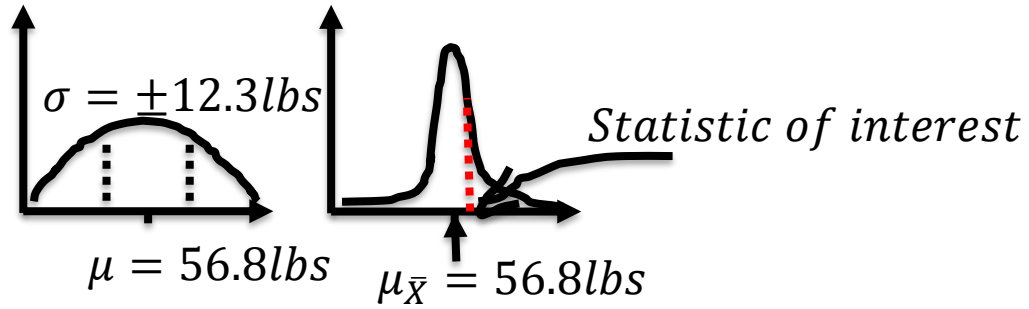
The Aluminum Association of America reports that the average American household uses 56.8 lbs of aluminium in a year. A random sample of 51 households is monitored for one year to determine aluminium usage. If the population standard deviation of annual usage is 12.3 lbs, what is the probability that the sample mean will be $> 60\ lbs$?

Sampling Distribution



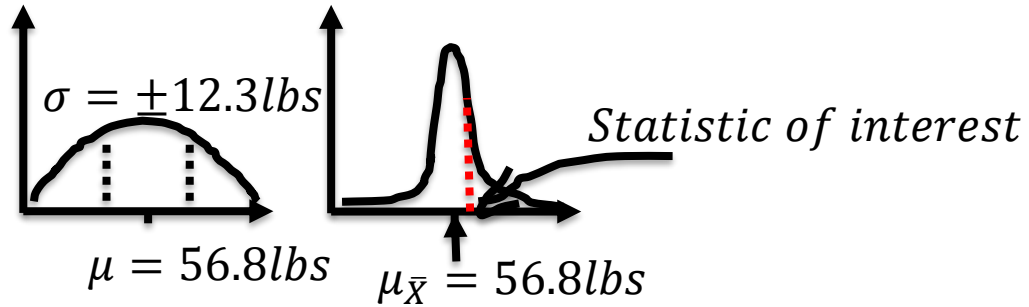
- Step 1: List all known parameters and values
- Step 2: Calculate others, or estimate if cannot be calculated
- Step 3: Find probabilities using tables, Excel or R

Sampling Distribution



- Step 1: List all known parameters and values
  - Population mean, $\mu = 56.8\ lbs$
  - Population standard deviation, $\sigma = 12.3\ lbs$
  - Sample size, $n = 51$
  - Sample mean, $\bar{x} > 60\ lbs$
  - Mean of sample means, $\mu_{\bar{X}} = \mu = 56.8\ lbs$

Sampling Distribution



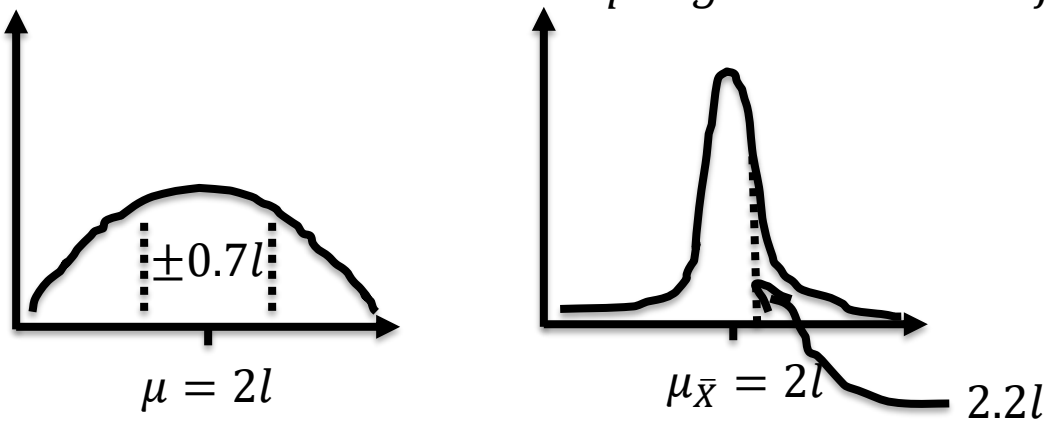
- Step 2: Calculate others or estimate, if cannot be calculated
  - Standard deviation of sample means, $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{12.3}{\sqrt{51}} = 1.72$
  - $\therefore z = \frac{60-56.8}{1.72} = 1.86$
- Step 3: Find probabilities using tables, Excel or R
  - Excel: 1-NORM.S.DIST(z,TRUE) = 0.0316
  - Please calculate these for:
    - $> 58\ lbs$
    - $> 56\ lbs < 57\ lbs$
    - $< 50\ lbs$

Sampling Distribution

The average male drinks $2l$ of water when active outdoors with a standard deviation of $0.7l$. You are planning a trip for 50 men and bring $110l$ of water. What is the probability that you will run out of water?

$\mu = 2, \sigma = 0.7$

$P(run\ out) \Rightarrow P(use > 110l) \Rightarrow P(average\ water\ use\ per\ male > 2.2l)$



$\mu_{\bar{X}} = \mu = 2l, {\sigma_{\bar{X}}}^2 = \frac{\sigma^2}{n} = \frac{0.49}{50}$

$\Rightarrow \sigma_{\bar{X}} = 0.099$

$z = \frac{2.2 - 2}{0.099} = 2.02$

$P(\bar{X} < 2.02) = 0.9783$

The probability of running out is

1-0.9783 = 0.0217 or 2.17%

Source: https://www.khanacademy.org/math/probability/statistics-inferential/sampling_distribution/v/sampling-distribution-example-problem

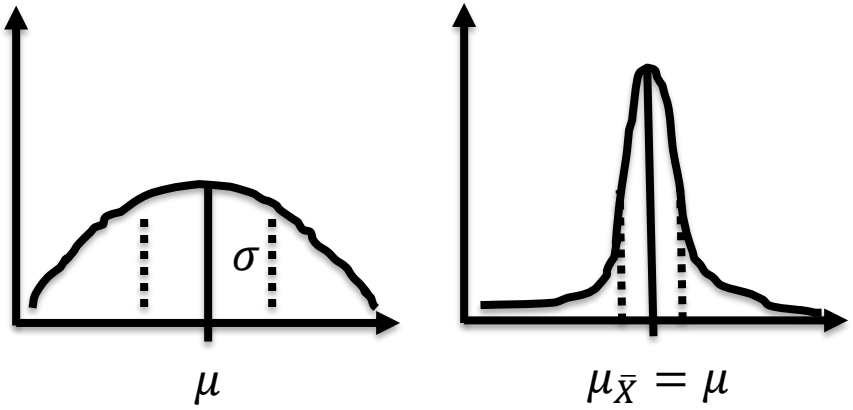
Last accessed: May 9, 2014

Using the Central Limit Theorem

You sample 36 apples from your farm's harvest of 200,000 apples. The mean weight of the sample is 112g with a 40g sample standard deviation. What is the probability that the mean weight of all 200,000 apples is between 100 and 124g?

Sampling Distribution

$Population\ distribution$ $Sampling\ distribution\ of\ sample\ mean\ when\ n = 36$



$$\sigma_{\bar{X}}{}^2 = \frac{\sigma^2}{n} = \frac{\sigma^2}{36} \Rightarrow \sigma_{\bar{X}} = \frac{\sigma}{6}$$

What are we trying to find out?

We need to know if population mean, μ, is within $\pm$12g of the sample mean, $\bar{X}$.

This is the same as saying that we need to know if sample mean, $\bar{X}$, is within $\pm$12g of the population mean, μ. Since $\mu = \mu_{\bar{X}}$, we can now use the sampling distribution of the means.

Source: https://www.khanacademy.org/math/probability/statistics-inferential/confidence-intervals/v/confidence-interval-1
Last accessed: May 9, 2014

Sampling Distribution

$Population\ distribution$ $Sampling\ distribution\ of\ sample\ mean\ when\ n = 36$

We need to find out how many standard deviations away from $\mu_{\bar{X}}$ is 12g. But, we don't know $\sigma_{\bar{X}}$ because we don't know σ. We use the sample standard deviation, s (40g), as the best estimate of population standard deviation. $\sigma \approx s = \pm 40g.\ \therefore \sigma_{\bar{X}} = \frac{\sigma}{6} = \frac{40}{6} = 6.67.$ So 12g is 12/6.67 = 1.8 standard deviations.

The z-table gives the probability as 0.9641 but that is the entire region below +1.8 z.

Find the region between -1.8 and +1.8 z.

0.9282. How would you get this answer if you did not have the negative z table?

Source: https://www.khanacademy.org/math/probability/statistics-inferential/confidence-intervals/v/confidence-interval-1
Last accessed: May 9, 2014

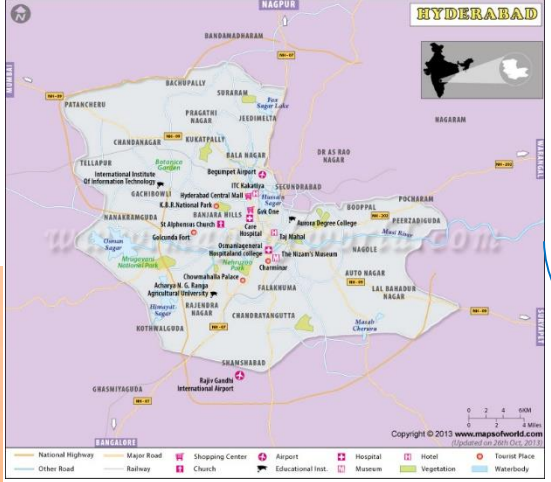
Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

(0.9641-0.5000)*2

= 0.4641*2

= 0.9282



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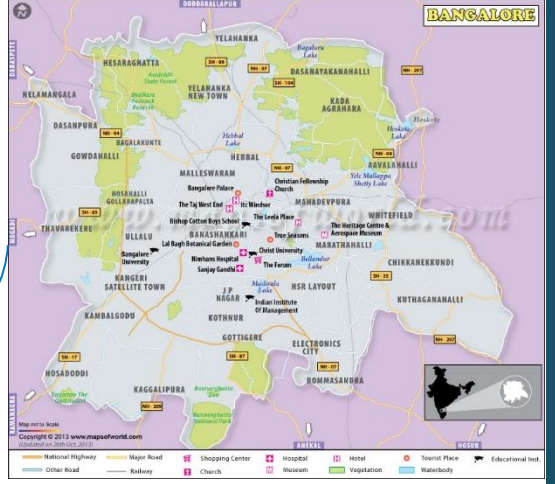

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