Homework 1

Chapter-4

Ans to the Question [4.2]

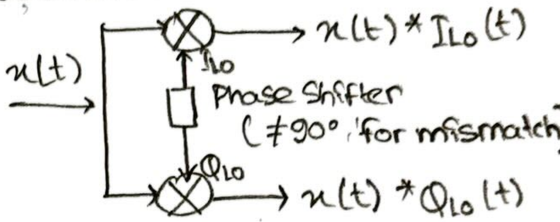
Here,

$x(t) = A\cos(2\pi f_c t) + B\sin(2\pi f_c t)$

$Q_{LO}(t) = 2(1-\frac{\epsilon}{2})\sin(2\pi f_c t - \theta/2)$

$I_{LO}(t) = 2(1+\frac{\epsilon}{2})\cos(2\pi f_c t + \theta/2)$

and $\omega = 2\pi f$



Now when there's a mix in signals;

then $r(t) = \underbrace{x(t) * I_{LO}(t)}_{\text{In-phase Component}} + \underbrace{x(t) * Q_{LO}(t)}_{\text{Quadrature component}}$

Inphase components; i.e. for $x(t) * I_{LO}(t)$,

$$x(t) * I_{LO}(t) = (A\cos\omega_c t + B\sin\omega_c t) * 2(1+\tfrac{\epsilon}{2})\cos(\omega_c t + \theta/2)$$

$$= (2A(1+\tfrac{\epsilon}{2})\cos\omega_c t + 2B(1+\tfrac{\epsilon}{2})\sin\omega_c t) * \cos(\omega_c t + \theta/2)$$

$$= (1+\tfrac{\epsilon}{2})\left[A\, 2\cos\omega_c t \cos(\omega_c t + \theta/2) + B\sin\omega_c t \cos(\omega_c t + \theta/2)\right]$$

Supposing, $p = \omega_c t$ and $q = \omega_c t + \theta/2$

$p+q = (2\omega_c t + \theta/2)$

$p-q = -\theta/2$

So, $$x(t) * I_{LO}(t) = A(1+\tfrac{\epsilon}{2})(\cos(2\omega_c t + \theta/2) + \cos(-\theta/2)) + B(1+\tfrac{\epsilon}{2})(\sin(2\omega_c t + \theta/2) + \sin(-\theta/2))$$

With filtering, the In-phase received baseband signal;

$$r(t)_{IBB} = A(1+\tfrac{\epsilon}{2})\cos(-\theta/2) + B(1+\tfrac{\epsilon}{2})\sin(-\theta/2) \quad \text{(i)}$$

Again, the Quadrature componet; for $x(t) * Q_{LO}(t)$,

$$x(t) * Q_{LO}(t) = (A\cos\omega_c t + B\sin\omega_c t) * (2(1-\tfrac{\epsilon}{2})\sin(\omega_c t - \theta/2))$$

PTO →

$$= \left[2A(1-\varepsilon/2)\cos\omega_c t + 2B(1-\varepsilon/2)\sin\omega_c t\right]\sin(\omega_c t - \theta/2)$$

$$= A(1-\varepsilon/2)\, 2\cos\omega_c t \sin(\omega_c t - \theta/2) + B(1-\varepsilon/2)\, 2\sin\omega_c t \sin(\omega_c t - \theta/2)$$

Similarly, $p = \omega_c t$ and $s = (\omega_c t - \theta/2)$ | $p+s = 2\omega_c t - \theta/2$; $p-s = \theta/2$

So,
$$2\cos(p)\sin(r) = \sin(p+s) - \sin(p-s)$$
$$2\sin(p)\sin(r) = \cos(p-s) - \cos(p+s)$$

Therefore,
$$x(t) * \varphi_{LO}(t) = A(1-\varepsilon/2)\left(\sin(2\omega_c t - \theta/2) - \sin\theta/2\right) + B(1-\varepsilon/2)\left(\cos\theta/2 - \cos(2\omega_c t - \theta/2)\right)$$

With filtering, the Quadrature received baseband signal;

$$r(t)_{Q_{BB}} = -A(1-\varepsilon/2)\sin\theta/2 + B(1-\varepsilon/2)\cos\theta/2 \quad \text{—— (ii)}$$

Now, with the equations from (i) & (ii), while drafting the four constellation points in a table;

A	B	A′ (I)	B′ (Q)
1	1	1.041	0.942
1	−1	1.059	−0.958
−1	1	−1.059	0.958
−1	−1	−1.041	−0.942

Table: 1 – Constellation points for the mismatch

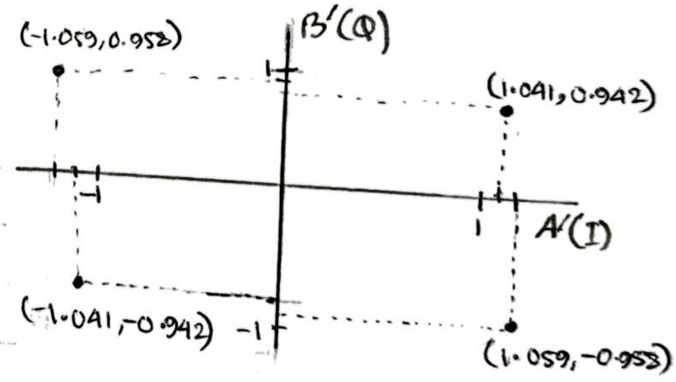


Fig: 1 – Constellation diagram for the mismatch.

Ans to the Question [4.3]

Here,

Error Vector Magnitude (EVM) can be illustrated as;

$$20 \log_{10} (|\text{error vector}| / |\text{ref. vector}|)$$

ref. vector = $\sqrt{2}$

error vector = EV1, EV2, EV3, EV4

Now, for the constellation diagram;

[• Constellation points (mismatch)
× Constellation points (QPSK)]

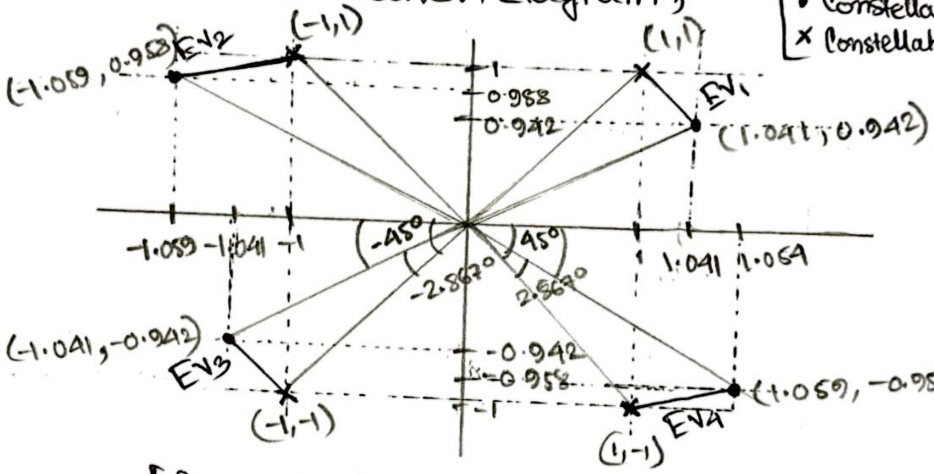


Figure 2: Constellation points diagram with both mismatch & QPSK

Based on the cosine rule, for EV1

$$EV_1^2 = 1.404^2 + 1.414^2 - 2(1.404)(1.414)\cos 2.86°$$
$$= 0.0050455$$

$\therefore EV_1 = 0.0707 = EV_3$

And the EVM $= 20 \log (0.0707/\sqrt{2})$

$= -26.022$ [That is the Magnitude]

Again, $EV_2 = EV_4$

$$EV_2^2 = 1.428^2 + 1.414^2 - 2(1.428)(1.414)\cos(-2.867)$$

PTO →

$= 0.005226$

$\therefore EV_2 = 0.0723 = EV_4$

And the $EVM = 20 \log(|\text{error vector}| / |\text{ref vector}|)$

$= 20 \log(0.0723 / 1.414)$

$= -25.826$

As we are determining magnitude, so the $EVM = 25.826$

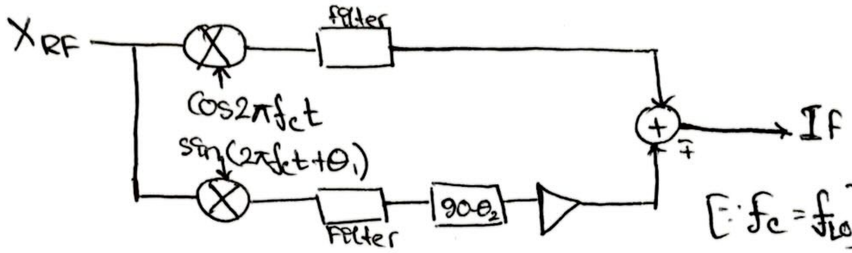
At the end, we can say;

$EV_1 = EV_2 = EV_3 = EV_4 \approx 26$

Ans to the Question [4.5]

Here,

$$X_{RF} = m_1(t)\cos(2\pi f_{RF_1} t) + m_2(t)\cos(2\pi f_{RF_2} t)$$



$[\because f_c = f_{LO}]$

The in-phase equation while considering the output of mixer 1,

$$X_{RF} * \cos(2\pi f_{LO} t) = \left(m_1(t)\cos(2\pi f_{RF_1} t) + m_2(t)\cos(2\pi f_{RF_2} t)\right) * \cos(2\pi f_{LO} t)$$

$$= m_1(t)\left[\cos(2\pi f_{RF_1} t)\cos(2\pi f_{LO} t)\right] + m_2(t)\left[\cos(2\pi f_{RF_2} t)\cos(2\pi f_{LO} t)\right]$$

Then, $\cos a \cdot \cos b = \frac{1}{2}\left[\cos(a+b) + \cos(a-b)\right]$ [∵ we know the formulae]

and $F_{LO} = F_{RF_1} + F_{IF} = F_{RF_2} - F_{IF}$

At the in-phase side (of mixer),

$$m_1(t)\left[\cos(2\pi f_{RF_1} t)\cos(2\pi f_{LO} t)\right] + m_2(t)\left[\cos(2\pi f_{RF_2} t)\cos(2\pi f_{LO} t)\right]$$

$$= m_1(t)/2\left[\cos(4\pi f_{RF_1} t + 2\pi f_{IF} t) + \cos(-2\pi f_{IF} t)\right.$$

$$\left. + m_2(t)/2\left[\cos(4\pi f_{RF_2} t - 2\pi f_{IF} t) + \cos(2\pi f_{IF} t)\right]\right]$$

Moving forward, the IF information with RF filtered out is got after the filtering stage;

$$\therefore m_1(t)/2 \cos(-2\pi f_{IF} t) + \frac{m_2(t)}{2}\cos(2\pi f_{IF} t)$$

$$= \frac{m_1(t)}{2}\cos(2\pi f_{IF} t) + \frac{m_2(t)}{2}\cos(2\pi f_{IF} t) \quad \text{——— (i)}$$

Again, at the quadrature side (of mixer),

PTO →

$$X_{RF} * \sin(2\pi f_{LO} t + \theta) = [m_1(t)\cos(2\pi f_{RF_1} t) + m_2(t)\cos(2\pi f_{RF_2} t)] * \sin(2\pi f_{LO} t + \theta_1)$$

$$= m_1(t)[\cos(2\pi f_{RF_1} t)\sin(2\pi f_{LO} t + \theta_1)] + m_2(t)[\cos(2\pi f_{RF_2} t)\sin(2\pi f_{LO} t + \theta_1)]$$

Then, $\cos a \sin b = \sin(a+b) - \sin(a-b)$ [$\because$ we know the formulae]

$$\therefore m_1(t)/2\,[\sin(4\pi f_{RF_1} t + 2\pi f_{IF} t + \theta_1) - \sin(-(2\pi f_{IF} t + \theta_1))$$
$$+ m_2(t)/2\,[\sin(4\pi f_{RF_2} t - 2\pi f_{IF} t + \theta_1) - \sin(2\pi f_{IF} t - \theta_1)]$$

All RF is filtered out after the stage of low-pass filter,

$$m_1(t)/2\,\sin(2\pi f_{IF} t + \theta_1) - m_2(t)/2\,\sin(2\pi f_{IF} t - \theta_1) \text{ ——— (ii)}$$

From (ii) which passing $(90-\theta_2)$ stage;

$$m_1(t)/2\,\sin(2\pi f_{IF} t + \theta_1 - (90-\theta_2)) - m_2(t)/2\,\sin(2\pi f_{IF} t - \theta_1 - (90-\theta_2))$$
$$= m_1(t)/2\,\sin(-(90 - (2\pi f_{IF} t + \theta_1 + \theta_2)) - m_2(t)/2\,\sin(-(90 - (2\pi f_{IF} t - \theta_1 + \theta_2))$$

[$\because \sin(90-\theta) = \cos\theta$]

After $(90-\theta_2)$ the Quadrature equation,

$$\text{So, } -m_1(t)/2\,\cos(2\pi f_{IF} t + \theta_1 + \theta_2) + m_2(t)/2\,\cos(2\pi f_{IF} t + \theta_2 - \theta_1)$$
$$= m_2(t)/2\,\cos(2\pi f_{IF} t - \theta_1 + \theta_2) - m_1(t)/2\,\cos(2\pi f_{IF} t + \theta_1 + \theta_2)$$

——— (iii)

[$\because$ And IF = Inphase + Quadrature] ,

$$= [m_1(t)/2\,\cos 2\pi(f_{IF} t) + m_2(t)/2\,\cos 2\pi(f_{IF} t)]$$
$$+ [(1+\varepsilon)\,m_2(t)/2\,\cos(2\pi f_{IF} t - \theta_1 + \theta_2) - (1+\varepsilon)\,m_1(t)/2\,\cos(2\pi f_{IF} t + \theta_1 + \theta_2)]$$

Again, getting the amplitudes from each stages;

$$m_1(t)/2 + m_2(t)/2 + (1+\varepsilon)(m_2(t)/2) - (1+\varepsilon)(m_1(t)/2)$$

PTO →

$$= m_1(t)/2\ [1-1-\varepsilon] + m_2(t)/2\ [1+1+\varepsilon]$$

$$= -m_1(t)/2\ [\varepsilon] + m_2(t)/2\ (2+\varepsilon]$$

Now, the image rejection Ratio; $10 \log \frac{P_{desired}}{P_{image}}$

With desired as $m_2(t)$, the image component is to be $m_1(t)$.

Hence, IRR (Image Rejection Ratio)

$$= 10 \log_{10} \left[\frac{\left(m_2(t)/2\ (2+\varepsilon)\right)^2}{\left(-m_1(t)/2\ \varepsilon\right)^2} \right]$$

$$= 20 \log_{10} \left[\frac{m_2(t)\,(2+\varepsilon)}{m_1(t)\,(\varepsilon)} \right].$$

Lastly, with $m_1(t) = m_2(t) = A$;

$$IRR = 20\log\left[\frac{\cancel{A}(2+\varepsilon)}{\cancel{A}(\varepsilon)}\right] = 20\log\left(\frac{2+\varepsilon}{\varepsilon}\right).$$