

ASSIGNMENT 02

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- **Question:** *Find the Inverse and QR Decomposition of the following.*

Exercise 2.93:

$$A = \begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix}$$

• **Solution:**

INVERSE OF A:

We are given with a matrix

$$A = \begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix}$$

Now,

DetA=

$$2 * 4 - 1 * 7$$

$$8 - 7 = 1$$

Also,

$$AdjA = \begin{pmatrix} 4 & -1 \\ -7 & 2 \end{pmatrix}$$

Now,

A^{-1} , can be calculated by the formula,

$$A^{-1} = AdjA / DetA$$

Therefore,

$$A^{-1} = \begin{pmatrix} 4 & -1 \\ -7 & 2 \end{pmatrix}$$

QR DECOMPOSITION OF A:

The QR Decomposition of a matrix is a decomposition of the matrix into an orthogonal matrix and an upper triangular matrix. A QR decomposition of a real square matrix A is a decomposition of A as,

$$A = QR$$

where Q is the orthogonal matrix and R is the upper triangular matrix.

Given

$$A = \begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix}$$

Let a and b be the column vectors of the given matrix such that,

$$a = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$$

$$b = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

The above vectors can be expressed as:

$$a = t_1 u_1$$

$$b = s_1 u_1 + t_2 u_2$$

where,

$$t_1 = \|a\|$$

$$u_1 = a/t_1$$

$$s_1 = u_1^T * b / \|u_1\|^2$$

$$u_2 = (b - s_1 * u_1) / \|b - s_1 u_1\|$$

$$t_2 = u_2^T b$$

The Values of a and b can be written as,

$$\begin{pmatrix} a & b \end{pmatrix} = \begin{pmatrix} u_1 & u_2 \end{pmatrix} * \begin{pmatrix} t_1 & s_1 \\ 0 & t_2 \end{pmatrix}$$

$$\begin{pmatrix} a & b \end{pmatrix} = QR$$

and,

$$Q^T * Q = I$$

Now, using the equations of t_1, u_1, s_1, u_2 and t_2 we get the values of t_1, u_1, s_1, u_2 and t_2

$$t_1 = \sqrt{2^2 + 7^2} = \sqrt{51}$$

$$u_1 = 1/\sqrt{51} * \begin{pmatrix} 2 \\ 7 \end{pmatrix}$$

$$s_1 = (2/\sqrt{51} \quad 7/\sqrt{51}) * \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$s_1 = (2/\sqrt{51} * 1) + 7/\sqrt{51} * 4$$

$$s_1 = 30/\sqrt{51}$$

$$u_2 = 1/\sqrt{51} * \begin{pmatrix} 7 \\ -2 \end{pmatrix}$$

$$t_2 = (7/\sqrt{51} \quad -2/\sqrt{51}) * \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$t_2 = 7/\sqrt{51} + (-8/\sqrt{51})$$

$$t_2 = -1/\sqrt{51}$$

substituting the values of t_1, u_1, s_1, u_2 and t_2 in the matrix

$$\begin{pmatrix} a & b \end{pmatrix} = QR$$

we get the required QR decomposition of A .

$$\begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix} = \begin{pmatrix} 2/\sqrt{51} & 7/\sqrt{51} \\ 7\sqrt{51} & -2/\sqrt{51} \end{pmatrix} * \begin{pmatrix} \sqrt{51} & 30/\sqrt{51} \\ 0 & -1/\sqrt{51} \end{pmatrix}$$

Exercise 2.94:

$$A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$$

• **Solution:**

INVERSE OF A:

We are given with a matrix

$$A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$$

Now,

DetA=

$$2 * 3 - 1 * 5$$

$$6 - 5 = 1$$

Also,

$$Adj A = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$$

Now,

A^{-1} , can be calculated by the formula,

$$A^{-1} = Adj A / Det A$$

Therefore,

$$A^{-1} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$$

QR DECOMPOSITION OF A:

The QR Decomposition of a matrix is a decomposition of the matrix into an orthogonal matrix and an upper triangular matrix. A QR decomposition of a real square matrix A is a decomposition of A as,

$$A = QR$$

where Q is the orthogonal matrix and R is the upper triangular matrix.

Given

$$A = \begin{pmatrix} 2 & 3 \\ 1 & 5 \end{pmatrix}$$

Let a and b be the column vectors of the given matrix such that,

$$a = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$b = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

The above vectors can be expressed as:

$$a = t_1 u_1$$

$$b = s_1 u_1 + t_2 u_2$$

where,

$$t_1 = \|a\|$$

$$u_1 = a/t_1$$

$$s_1 = u_1^T * b / \|u_1\|^2$$

$$u_2 = (b - s_1 * u_1) / \|b - s_1 u_1\|$$

$$t_2 = u_2^T b$$

The Values of a and b can be written as,

$$\begin{pmatrix} a & b \end{pmatrix} = \begin{pmatrix} u_1 & u_2 \end{pmatrix} * \begin{pmatrix} t_1 & s_1 \\ 0 & t_2 \end{pmatrix}$$

$$\begin{pmatrix} a & b \end{pmatrix} = QR$$

and,

$$Q^T * Q = I$$

Now, using the equations of t_1, u_1, s_1, u_2 and t_2 we get the values of t_1, u_1, s_1, u_2 and t_2

$$t_1 = \sqrt{2^2 + 3^2} = \sqrt{13}$$

$$u_1 = 1/\sqrt{13} * \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$s_1 = (2/\sqrt{13} \quad 1/\sqrt{13}) * \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$$s_1 = (2/\sqrt{13} * 5) + 1/\sqrt{13} * 3$$

$$s_1 = 13/\sqrt{13}$$

$$u_2 = 1/\sqrt{13} * \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$t_2 = (1/\sqrt{13} \quad -2/\sqrt{13}) * \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$$t_2 = 5/\sqrt{13} + (-6/\sqrt{13})$$

$$t_2 = -1/\sqrt{13}$$

substituting the values of t_1, u_1, s_1, u_2 and t_2 in the matrix

$$\begin{pmatrix} a & b \end{pmatrix} = QR$$

we get the required QR decomposition of A .

$$\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 2/\sqrt{13} & 1/\sqrt{13} \\ 1/\sqrt{13} & -2/\sqrt{13} \end{pmatrix} * \begin{pmatrix} \sqrt{13} & 13/\sqrt{13} \\ 0 & -1/\sqrt{13} \end{pmatrix}$$