ASSIGNMENT 02

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- **Question:** ***Find the Inverse and QR Decomposition of the following.***

Exercise 2.93:

$$A = \begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix}$$

- **Solution:**

INVERSE OF A:

We are given with a matrix

$$A = \begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix}$$

Now,

DetA=

$$2 * 4 - 1 * 7$$

$$8 - 7 = 1$$

Also,

$$AdjA = \begin{pmatrix} 4 & -1 \\ -7 & 2 \end{pmatrix}$$

Now,

A^{-1}, $can \quad be \quad calculated \quad by \quad the \quad formula,$

A^{-1} $= AdjA/DetA$

Therefore,

$$A^{-1} = \begin{pmatrix} 4 & -1 \\ -7 & 2 \end{pmatrix}$$

QR DECOMPOSITION OF A:

The QR Decomposition of a matrix is a decomposition of the matrix into an orthogonal matrix and an upper triangular matrix. A QR decomposition of a real square matrix A is a decomposition of A as,

$$A = QR$$

where Q is the orthogonal matrix and R is the upper triangular matrix.

Given

$$A = \begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix}$$

Let a and b be the column vectors of the given matrix such that,

$$a = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$$

$$b = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

The above vectors can be expressed as:

$$a = t_1 u_1$$

$$\mathrm{b}{=}\mathrm{s}_1 u_1 + t_2 u_2$$

where,

$$t_1 = ||a||$$

$$\mathrm{u}_1 = a/t_1$$

$$\mathrm{s}_1 = u_1^T * b/||u_1||^2$$

$$\mathrm{u}_2 = b - s_1 * u_1/||b - s_1 u_1||$$

$$\mathrm{t}_2 = u_2^T b$$

The Values of a and b can be written as,

$$\begin{pmatrix} a & b \end{pmatrix} = \begin{pmatrix} u_1 & u_2 \end{pmatrix} * \begin{pmatrix} t_1 & s_1 \\ 0 & t_2 \end{pmatrix}$$

$$\begin{pmatrix} a & b \end{pmatrix} = QR$$

and,

$$Q^T * Q = I$$

$Now,\ using\ the\ equations\ of\ t_1, u_1, s_1, u_2\ and\ t_2\ we\ get\ the\ values\ of\ t_1, u_1, s_1, u_2\ and\ t_2$

$$t_1 = \sqrt{2^2 + 7^2} = \sqrt{51}$$

$$u_1 = 1/\sqrt{51} * \begin{pmatrix} 2 \\ 7 \end{pmatrix}$$

$$s_1 = \begin{pmatrix} 2/\sqrt{51} & 7/\sqrt{51} \end{pmatrix} * \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$s_1 = (2/\sqrt{51} * 1) + 7/\sqrt{51} * 4$$
$$\mathrm{s}_1 = 30/\sqrt{51}$$

$$u_2 = 1/\sqrt{51} * \begin{pmatrix} 7 \\ -2 \end{pmatrix}$$

$$t_2 = \begin{pmatrix} 7/\sqrt{51} & -2/\sqrt{51} \end{pmatrix} * \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$t_2 = 7/\sqrt{51} + (-8\sqrt{51})$$
$$\mathrm{t}_2 = -1/\sqrt{51}$$

substituting the values of t_1, u_1, s_1, u_2 *and* t_2 *in the matrix*

$$\begin{pmatrix} a & b \end{pmatrix} = QR$$

we get the required QR decomposition of A.

$$\begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix} = \begin{pmatrix} 2/\sqrt{51} & 7/\sqrt{51} \\ 7\sqrt{51} & -2/\sqrt{51} \end{pmatrix} * \begin{pmatrix} \sqrt{51} & 30\sqrt{51} \\ 0 & -1\sqrt{51} \end{pmatrix}$$

Exercise 2.94:

$$A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$$

- **Solution:**

INVERSE OF A:

We are given with a matrix

$$A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$$

Now,

DetA=

$$2 * 3 - 1 * 5$$

$$6 - 5 = 1$$

Also,

$$AdjA = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$$

Now,

A$^{-1}, can \quad be \quad calculated \quad by \quad the \quad formula,$

A$^{-1} = AdjA/DetA$

Therefore,

$$A^{-1} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$$

QR DECOMPOSITION OF A:

The QR Decomposition of a matrix is a decomposition of the matrix into an orthogonal matrix and an upper triangular matrix. A QR decomposition of a real square matrix A is a decomposition of A as,

$$A = QR$$

where Q is the orthogonal matrix and R is the upper triangular matrix.

Given

$$A = \begin{pmatrix} 2 & 3 \\ 1 & 5 \end{pmatrix}$$

Let a and b be the column vectors of the given matrix such that,

$$a = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$b = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

The above vectors can be expressed as:

$$a = t_1 u_1$$

$$\text{b=s}_1 u_1 + t_2 u_2$$

where,

$$t_1 = ||a||$$

$$\text{u}_1 = a/t_1$$

$$\text{s}_1 = u_1^T * b/||u_1||^2$$

$$\text{u}_2 = b - s_1 * u_1/||b - s_1 u_1||$$

$$\text{t}_2 = u_2^T b$$

The Values of a and b can be written as,

$$\begin{pmatrix} a & b \end{pmatrix} = \begin{pmatrix} u_1 & u_2 \end{pmatrix} * \begin{pmatrix} t_1 & s_1 \\ 0 & t_2 \end{pmatrix}$$

$$\begin{pmatrix} a & b \end{pmatrix} = QR$$

and,

$$Q^T * Q = I$$

$Now,\ using\ the\ equations\ of\ t_1, u_1, s_1, u_2\ and\ t_2\ we\ get\ the\ values\ of\ t_1, u_1, s_1, u_2\ and\ t_2$

$$t_1 = \sqrt{2^2 + 3^2} = \sqrt{13}$$

$$u_1 = 1/\sqrt{13} * \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$s_1 = \begin{pmatrix} 2/\sqrt{13} & 1/\sqrt{13} \end{pmatrix} * \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$$s_1 = (2/\sqrt{13} * 5) + 1/\sqrt{13} * 3$$
$$\mathrm{s}_1 = 13/\sqrt{13}$$

$$u_2 = 1/\sqrt{13} * \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$t_2 = \begin{pmatrix} 1/\sqrt{13} & -2/\sqrt{13} \end{pmatrix} * \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$$t_2 = 5/\sqrt{13} + (-6\sqrt{13})$$
$$\mathrm{t}_2 = -1/\sqrt{13}$$

$$substituting\ \ the\ \ values\ \ of\ \ t_1, u_1, s_1, u_2\ \ and\ \ t_2\ \ in\ \ the\ \ matrix$$
$$\begin{pmatrix} a & b \end{pmatrix} = QR$$
$$we\ \ get\ \ the\ \ required\ \ QR\ \ decomposition\ \ of\ \ A.$$

$$\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 2/\sqrt{13} & 1/\sqrt{13} \\ 1\sqrt{13} & -2/\sqrt{13} \end{pmatrix} * \begin{pmatrix} \sqrt{13} & 13\sqrt{13} \\ 0 & -1\sqrt{13} \end{pmatrix}$$