ASSIGNMENT 02

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January 14, 2021

- **Question:** ***Find the Inverse and QR Decomposition of the following.***

Exercise 2.93:

$$A = \begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix}$$

- **Solution:**

INVERSE OF A:

We are given with a matrix

$$A = \begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix}$$

Now,

DetA=

$$2 * 4 - 1 * 7$$

$$8 - 7 = 1$$

Also,

$$AdjA = \begin{pmatrix} 4 & -1 \\ -7 & 2 \end{pmatrix}$$

Now,

A^{-1}, *can be calculated by the formula,*

A$^{-1} = AdjA/DetA$

Therefore,

$$A^{-1} = \begin{pmatrix} 4 & -1 \\ -7 & 2 \end{pmatrix}$$

QR DECOMPOSITION OF A:

The QR Decomposition of a matrix is a decomposition of the matrix into an orthogonal matrix and an upper triangular matrix. A QR decomposition of a real square matrix A is a decomposition of A as,

$$A = QR$$

where Q is the orthogonal matrix and R is the upper triangular matrix.

Given

$$A = \begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix}$$

Let a and b be the column vectors of the given matrix such that,

$$a = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$$

$$b = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

The above vectors can be expressed as:

$$a = t_1 u_1$$

$$\mathrm{b}{=}\mathrm{s}_1 u_1 + t_2 u_2$$

where,

$$t_1 = ||a||$$

$$\mathrm{u}_1 = a/t_1$$

$$\mathrm{s}_1 = u_1^T * b/||u_1||^2$$

$$\mathrm{u}_2 = b - s_1 * u_1/||b - s_1 u_1||$$

$$\mathrm{t}_2 = u_2^T b$$

The Values of a and b can be written as,

$$\begin{pmatrix} a & b \end{pmatrix} = \begin{pmatrix} u_1 & u_2 \end{pmatrix} * \begin{pmatrix} t_1 & s_1 \\ 0 & t_2 \end{pmatrix}$$

$$\begin{pmatrix} a & b \end{pmatrix} = QR$$

and,

$$Q^T * Q = I$$

$Now,\ using\ the\ equations\ of\ t_1, u_1, s_1, u_2\ and\ t_2\ we\ get\ the\ values\ of\ t_1, u_1, s_1, u_2\ and\ t_2$

$$t_1 = \sqrt{2^2 + 7^2} = \sqrt{53}$$

$$u_1 = 1/\sqrt{53} * \begin{pmatrix} 2 \\ 7 \end{pmatrix}$$

$$s_1 = \begin{pmatrix} 2/\sqrt{53} & 7/\sqrt{53} \end{pmatrix} * \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$s_1 = (2/\sqrt{53} * 1) + 7/\sqrt{53} * 4$$

$$\mathrm{s}_1 = 30/\sqrt{53}$$

$$u_2 = 1/\sqrt{53} * \begin{pmatrix} -7 \\ 2 \end{pmatrix}$$

$$t_2 = \begin{pmatrix} -7/\sqrt{53} & 2/\sqrt{53} \end{pmatrix} * \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$t_2 = -7/\sqrt{53} + (8/\sqrt{53})$$

$$\mathrm{t}_2 = 1/\sqrt{53}$$

$$substituting\ \ the\ \ values\ \ of\ \ t_1, u_1, s_1, u_2\ \ and\ \ t_2\ \ in\ \ the\ \ matrix$$

$$\begin{pmatrix} a & b \end{pmatrix} = QR$$

$$we\ \ get\ \ the\ \ required\ \ QR\ \ decomposition\ \ of\ \ A.$$

$$\begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix} = \begin{pmatrix} 2/\sqrt{53} & -7/\sqrt{53} \\ 7/\sqrt{53} & 2/\sqrt{53} \end{pmatrix} * \begin{pmatrix} \sqrt{53} & 30/\sqrt{53} \\ 0 & 1/\sqrt{53} \end{pmatrix}$$

Exercise 2.94:

$$A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$$

- **Solution:**

INVERSE OF A:

We are given with a matrix

$$A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$$

Now,

DetA=

$$2 * 3 - 1 * 5$$

$$6 - 5 = 1$$

Also,

$$AdjA = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$$

Now,

A^{-1}, *can be calculated by the formula,*

A$^{-1} = AdjA/DetA$

Therefore,

$$A^{-1} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$$

QR DECOMPOSITION OF A:

The QR Decomposition of a matrix is a decomposition of the matrix into an orthogonal matrix and an upper triangular matrix. A QR decomposition of a real square matrix A is a decomposition of A as,

$$A = QR$$

where Q is the orthogonal matrix and R is the upper triangular matrix.

Given

$$A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$$

Let a and b be the column vectors of the given matrix such that,

$$a = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$b = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

The above vectors can be expressed as:

$$a = t_1 u_1$$

b=s$_1 u_1 + t_2 u_2$

where,

$$t_1 = ||a||$$

u$_1 = a/t_1$

s$_1 = u_1^T * b/||u_1||^2$

u$_2 = b - s_1 * u_1/||b - s_1 u_1||$

t$_2 = u_2^T b$

The Values of a and b can be written as,

$$\begin{pmatrix} a & b \end{pmatrix} = \begin{pmatrix} u_1 & u_2 \end{pmatrix} * \begin{pmatrix} t_1 & s_1 \\ 0 & t_2 \end{pmatrix}$$

$$\begin{pmatrix} a & b \end{pmatrix} = QR$$

and,

$$Q^T * Q = I$$

$Now,\ using\ the\ equations\ of\ t_1, u_1, s_1, u_2\ and\ t_2\ we\ get\ the\ values\ of\ t_1, u_1, s_1, u_2\ and\ t_2$

$$t_1 = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$u_1 = 1/\sqrt{5} * \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$s_1 = \begin{pmatrix} 2/\sqrt{5} & 1/\sqrt{5} \end{pmatrix} * \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$$s_1 = (2/\sqrt{5} * 5) + 1/\sqrt{5} * 3$$

$$\mathrm{s}_1 = 13/\sqrt{5}$$

$$u_2 = 1/\sqrt{5} * \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$t_2 = \begin{pmatrix} -1/\sqrt{5} & 2/\sqrt{5} \end{pmatrix} * \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$$t_2 = -5/\sqrt{5} + 6/\sqrt{5}$$

$$\mathrm{t}_2 = 1/\sqrt{5}$$

$$substituting\ \ the\ \ values\ \ of\ \ t_1, u_1, s_1, u_2\ \ and\ \ t_2\ \ in\ \ the\ \ matrix$$
$$\begin{pmatrix} a & b \end{pmatrix} = QR$$
$$we\ \ get\ \ the\ \ required\ \ QR\ \ decomposition\ \ of\ \ A.$$

$$\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 2/\sqrt{5} & -1/\sqrt{5} \\ 1\sqrt{5} & 2/\sqrt{5} \end{pmatrix} * \begin{pmatrix} \sqrt{5} & 13/\sqrt{5} \\ 0 & 1/\sqrt{5} \end{pmatrix}$$