① Encuentre mediante el teorema de ito, la representación integral de:

$$S_t = W_t^2 \cdot t^3 = F(t, W_t)$$

$$\begin{cases} S_0 = 0 \\ F'_t = W_t^2 \cdot 3t^2 \\ F'_{W_t} = 2W_t \cdot t^3 \\ F''_{W_t;W_t} = 2 \cdot t^3 \end{cases}$$

$$\sum_0^T \Delta S_t \cong \sum_0^T W_t^2 \cdot 3t^2 \cdot \Delta t + \frac{1}{2} \cdot 2t^3 \, \Delta t$$

$$+ \sum_0^T 2W_t \cdot t^3 \, \Delta W_t$$

$$S_t = \overbrace{S_0}^{=0} + \int_0^T \Big[\underbrace{3W_t^2 \cdot t^2 + t^3}_{\alpha_t}\Big] dt + \int_0^T \underbrace{2 \cdot W_t \cdot t^3}_{\beta_t} dW_t$$

② Encuentre mediante teorema generalizado de ito la representación integral de:

$$V_t = \ln(S_t) - t$$

$$\begin{cases} F'_t = -1 \\ F'_{S_t} = \dfrac{1}{S_t} \\ F''_{S_t} = -\dfrac{1}{S_t^2} \end{cases}$$

$$\sum_0^T \Delta V_t \cong \sum_0^T \Big[-1 + \frac{1}{S_t} \alpha_t - \frac{1}{2} \cdot \frac{1}{S_t^2} \cdot \beta_t^2\Big] \Delta t$$

$$+ \sum_0^T \frac{1}{S_t} \cdot \beta_t \, \Delta S_t$$

$$V_T = V_0 + \int_0^T \Big[-1 + \frac{\alpha_t}{S_t} - \frac{1}{2} \cdot \frac{\beta_t^2}{S_t^2}\Big] dt + \int_0^T \frac{\beta_t}{S_t} dW_t$$

reemplazo α_t, β_t y S_t

$$\underbrace{\alpha_t = 3 \cdot W_t^2 \cdot t^2 + t^3} \qquad \underbrace{\beta_t = 2 \cdot W_t \cdot t^3} \qquad \underbrace{S_t = W_t^2 \cdot t^3}$$

$$V_T = \int_0^T \left[-1 + \frac{(3W_t^2 \cdot t^2 + t^3)}{W_t^2 \cdot t^3} - \frac{1}{2} \cdot \underbrace{\frac{(2W_t \cdot t^3)^2}{(W_t^2 \cdot t^3)^2}}_{\frac{4W_t^2 \cdot t^5}{W_t^4 \cdot t^5}} \right] dt$$

$$+ \int_0^T \frac{2 \cdot W_t \cdot t^3}{W_t^2 \cdot t^3} \, dW_t$$

$$\boxed{V_T = \int_0^T \left[-1 + \frac{(3W_t^2 \cdot t^2 + t^3)}{W_t^2 \cdot t^3} - \frac{1}{2} \frac{(2W_t \cdot t^3)^2}{(W_t^2 \cdot t^3)^2} \right] dt + \int_0^T \frac{2}{W_t} \, dW_t}$$

③ Dado la VA $X = (2+T)(2+W_T)$

Calcular $H_t = E[X / F_t]$

$H_t = E\left[(2+T)(2+W_T) / F_t\right]$

$H_t = E\left[4 + 2W_T + 2T + T \cdot W_T / F_t\right] \rightarrow$ separo lo que me interesa que es W_T

$H_t = E\left[(4+2T) / F_t\right] + E\left[(2+T) W_T / F_t\right]$

$H_t = (4+2T) + (2+T)\, E[W_T / F_t]$

$H_t = (4+2T) + (2+T) \cdot E[\underbrace{W_T - W_t}_{\Delta W_{T-t}} + W_t / F_t]$

$H_t = (4+2T) + (2+T) \cdot \left[\underbrace{E[\Delta W_{T-t} / F_t]}_{= 0} + \underbrace{E[W_t / F_t]}_{= W_t}\right]$

$$\boxed{H_t = (4+2T) + (2+T) \cdot W_t} \rightarrow F(T; W_t)$$

④ Calcule ΔH_t

$F'_T = 2 + W_t$

$F'_{W_t} = 2 + T$

$F''_{W_t} = 0$

$\Delta H_t = \sum_0^T (2 + W_t)\, \Delta T + \frac{1}{2} \cdot 0 + \sum_0^T (2 + \cancel{T})\, \Delta W_t$

$= \sum (2 + W_t) \underbrace{\Delta T}_{= 0} + \sum_0^T (2+T)\, \Delta W_t$

$= \sum (2+T)\, \Delta W_t$

② $X = W_T^2$

① Calcular el proceso $H_t = E[X/F_t]$

$H_t = E[X/F_t]$

$H_t = E[W_T^2/F_t]$

$= E[(W_T - W_t + W_t)^2/F_t]$

$= E[(\Delta W_{T-t}^2 + W_t^2 + 2 \cdot \Delta W_{T-t} \cdot W_t)/F_t]$

Aplico esperanza

$= E(\Delta W_{T-t}^2/F_t) + E(W_t^2/F_t) + 2 \cdot W_t \cdot E(\Delta W_{T-t}/F_t)$

Por prop

$= T - t + W_t^2 + 2 \cdot W_t \cdot 0 = T - t + W_t^2$

$\boxed{H_t = T - t + W_t^2}$

② Compruebe que cumple con la tower property

$H_s = E[E[X/F_t]/F_s]$

$= E[T-t+W_t^2/F_s]$

Aplico esperanza

$= E[T/F_s] - E[t/F_s] + E[W_t^2/F_s]$

$= T-t + E[(W_t - W_s + W_s)^2/F_s]$

$= T-t + E[\Delta W_{t-s}^2 + W_s^2 + 2 \cdot W_s \cdot \Delta W_{t-s} / F_s]$

$= T-t + E[\Delta W_{t-s}^2/F_s] + E[W_s^2/F_s] + 2 \cdot W_s\, E[\Delta/F_s]$

$= T-t + t-s + W_s^2 + 2 \cdot W_s \cdot 0$

$= T-s+W_s^2$

③ representar H_t por Ito $\quad H_t = T-t+W_t^2$

$H_0 = T-0+W_0^2 = T$

$F'_t = -1$

$F'_{W_t} = 2W_t$

$F''_{W_t;W_t} = 2$

$\sum \Delta H_t = \sum_0^T -1 \cdot \Delta t + \frac{1}{2} \cdot 2\,\Delta t + \sum_0^T 2W_t \cdot \Delta W_t$

$H_t = \underbrace{T}_{H_0} + \int_0^T (-1+1)\,dt + \int_0^T 2 \cdot W_t\, dW_t$

$H_t = T + \int_0^T 2W_t\, dW_t$