

Exploring Quarto and Latex

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1 ANTIDIFFERENTIATION AND INDEFINITE INTEGRALS

1.1 Integration by Substitution

Theorem 4.1.11 (Substitution Rule). If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I , then

$$\int f(g(x)) \cdot g'(x) dx = \int f(u) du.$$

Example 4.1.12.

1. $\int (1 - 4x)^{\frac{1}{2}} dx$

If we let $u = 1 - 4x$, then $du = -4dx$. We multiply the integrand by $\frac{-4}{-4}$. Thus,

$$\int (1-4x)^{1/2} dx = \int (1-4x)^{1/2} \cdot \frac{-4}{-4} dx = \int (u)^{1/2} \left(-\frac{du}{u}\right) = -\frac{1}{4} \int u^{1/2} du = -\frac{1}{4} \cdot \frac{2u^{3/2}}{3} + C.$$

We put the final answer in terms of x by substituting $u = 1 - 4x$. Therefore,

$$\int (1 - 4x)^{1/2} dx = \frac{(1 - 4x)^{3/2}}{6} + C$$