Quasi-Oracle Estimation of Heterogeneous Treatment Effects

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Abstract

We propose ***R-learning*** for heterogeneous treatment effect estimation in observational studies.

Estimate: Treatment Propensity, Marginal Effect → form → *R-Objective Function* → *Treatment Effect Estimate*

4-th Root Consistency → *Oracle Regret Bound*

Existing Literature

There are promising methods for this problem, based on *BART* (Chipman et al., 2010; Hill 2011); *Boosting* (Powers et al., 2017); *Deep nets* (Hartford et al., 2017; Shalit et al., 2017); *Lasso* (Imai and Ratkovic, 2013), *Forests* (Wager and Athey, 2018), *Trees* (Athey and Imbens, 2016; Su et al., 2009).

Q: Can we get something general that works with blackbox learners *without* per instance twiddling? **YES!**

Existing black-box model based approaches

◆ S-learner

$\mu(x,w) := E[Y_i^{obs} \mid X_i = x, W_i = w]$

◆ T-learner

$\hat\tau(x) = \hat\mu(x,1) - \hat\mu(x,0)$

$\mu_w(x) := E[Y^w \mid X_i = x]$

$\hat\tau(x) = \hat\mu_1(x) - \hat\mu_0(x)$

◆ X-learner

[Künzel, Sekhon, Bickel, Yu, 2017]

$\hat\tau_1(x) := M_1[Y^1 - \hat\mu_0(x) \sim X^1]$

$\hat\tau_0(x) := M_0[\hat\mu_1(x) - Y^0 \sim X^0]$

$e^*(x) = E(W_i \mid X_i = x)$

$\hat\tau(x) = \hat e(x)\hat\tau_0(x) + (1-\hat e(x))\hat\tau_1(x)$

Our inspiration: *Robinson's Transformation* (1988)

Assume we have a partially linear treatment effect model,

$$Y_i = f^*(X_i) + \tau^* W_i + \varepsilon_i, \qquad \boxed{\begin{aligned} e^*(x) &:= E(W_i \mid X_i = x) \\ m^*(x) &:= E(Y_i \mid X_i = x)\end{aligned}}$$

Rearrange, $Y_i - m^*(X_i) = \tau^*(W_i - e^*(X_i)) + \varepsilon_i$.

The induced estimator is **efficient** (Robinson, 1988),

$$\hat\tau = OLS\{Y_i - \hat m(X_i) \sim (W_i - \hat e(X_i))\}.$$

We extend this idea to **non-parametric** settings.

* See also Chernozhukov et al. (2017), Zhao, Ertefaie, and Small (2017), Athey, Tibshirani, and Wager (2016)

Our proposal: *R-learning*

We assume a non-parametric treatment effect model:

$$Y_i = f^*(X_i) + \tau^*(X_i) W_i + \varepsilon_i, \qquad \boxed{\begin{aligned} e^*(x) &:= E(W_i \mid X_i = x) \\ m^*(x) &:= E(Y_i \mid X_i = x)\end{aligned}}$$

Recall from Robinson's Transformation:

$$Y_i - m^*(X_i) = \tau^*(X_i)\cdot(W_i - e^*(X_i)) + \varepsilon_i.$$

This suggests a natural **oracle learner**:

$$\tilde\tau(\cdot) = \arg\min_\tau \frac{1}{n}\sum_{i=1}^n (Y_i - m^*(X_i) - (W_i - e^*(X_i))\tau(X_i))^2 + \underbrace{\Lambda_n(\tau(\cdot))}_{\text{regularizer}}.$$

Q: What about the plug-in version with $\hat m(\cdot)$ and $\hat e(\cdot)$? Overfitting!

The *R-learning* framework:

① Fit $\hat m(\cdot)$ and $\hat e(\cdot)$ via any black-box supervised learning for high predictive accuracy

② Estimate *treatment effects* via a cross-fit estimator:

$$\hat\tau(\cdot) = \operatorname{argmin}_\tau \frac{1}{n}\sum_{i=1}^n (Y_i - \hat m^{(-i)}(X_i) - (W_i - \hat e^{(-i)}(X_i))\tau(X_i))^2 + \Lambda_n(\tau(\cdot)).$$

The Quasi-Oracle Property (in the case of regularized regression in RKHS)

Assumptions:

- H is an RKHS with kernel K and norm $\|\cdot\|_H$
- For some $0 < p < 1$, the eigenvalues σ_j of K satisfy $\sup_{j\ge1} j^{1/p}\sigma_j < \infty$
- For some $0 < \alpha < 0.5$, the true τ^* satisfies $\| T_K^\alpha(\tau^*)\|_H < \infty$

The Oracle Regret Bound (Mendelson and Neeman, 2010)

Assuming overlap and using an oracle estimator:

$$\tilde\tau(\cdot) = \operatorname{argmin}_\tau \frac{1}{n}\sum_{i=1}^n (Y_i - m^*(X_i) - (W_i - e^*(X_i))\tau(X_i))^2 + \Lambda_n(\|\tau\|_H),$$

the best available MSE bounds scale as $\tilde O_P(n^{(1-2\alpha)/(p+1-2\alpha)})$.

The Quasi-Oracle Regret Bound

Theorem. (Nie and Wager, 2018) Suppose that

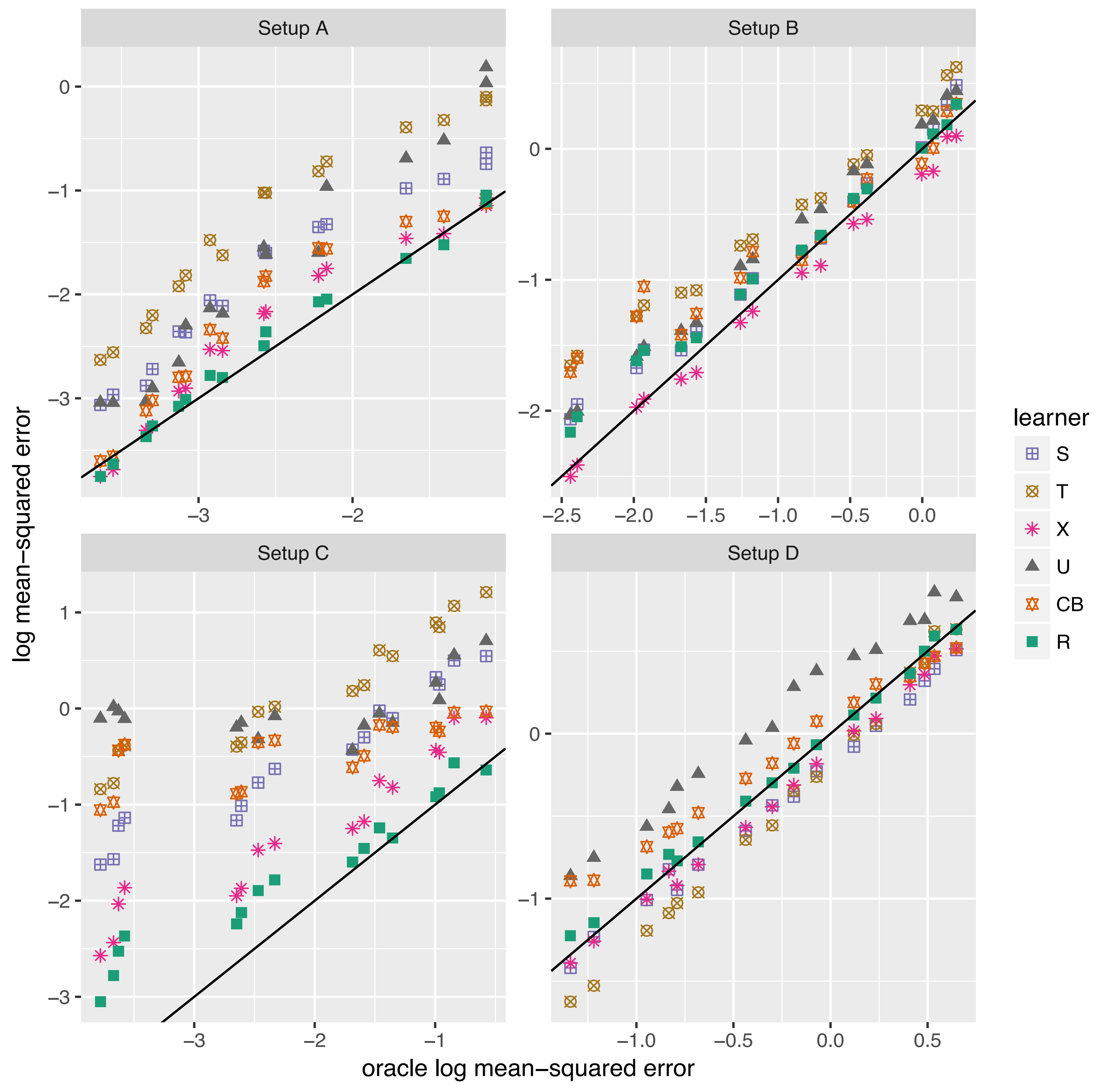
- Nuisance components $\hat m(\cdot)$ and $\hat e(\cdot)$ are $o_p(n^{-1/4})$-consistent
- The smoothness parameter is bounded by $2\alpha \le 1 - p$

Then, the minimizer of the regularized plug-in loss satisfies the same regret bound.

* Connections with semiparametric efficiency (Bickel et al. 1998, Newey 1994, Robins and Rotnitzky 1995, Robinson 1998, Tsiatis 2007), TMLE (Scharfstein et al. 1999, ver der Laan and Rubin 2006), policy learning (Athey and Wager, 2017, Dudik et al. 2011, Luedtke and van der Laan 2016, Zhang et al. 2012).

Simulation (with boosting)

$$X_i \sim P_d, W_i \mid X_i \sim Bernoulli(e^*(X_i)), \varepsilon_i \mid X_i \sim N(0,1),$$
$$Y_i = b^*(X_i) + (W_i - 0.5)\tau^*(X_i) + \sigma\varepsilon_i$$



- U is proposed by Künzel et al. (2017), similar to R in spirit, but suffers from instability
- CB is Causal Boosting (Power et al, 2017).

Setup A

Difficult nuisance component $m^*(\cdot)$ and $e^*(\cdot)$, easy HTE function $\tau^*(\cdot)$

$X_i \sim Unif(0,1)^d, e^*(X_i) = \sin(\pi X_{i1} X_{i2})$

$b^*(X_i) = \sin(\pi X_{i1} X_{i2}) + 2(X_{i3} - 0.5)^2 + X_{i4} + 0.5 X_{i5}$

$\tau^*(X_i) = (X_{i1} + X_{i2})/2$

Setup B

Randomized Trial

$X_i \sim N(0, I_{d\times d}), e^*(X_i) = 0.5$

$b^*(X_i) = \max\{X_{i1} + X_{i2}, X_{i3}, 0\} + \max\{X_{i4} + X_{i5}, 0\}$

$\tau^*(X_i) = X_{i1} + \log(1 + e^{X_{i2}})$

Setup C

Easy propensity score $e^*(\cdot)$, difficult baseline $b^*(\cdot)$.

$X_i \sim N(0, I_{d\times d}), e^*(X_i) = 1/(1 + e^{X_{i2} + X_{i3}})$

$b^*(X_i) = 2\log(1 + e^{X_{i1} + X_{i2} + X_{i3}}), \tau^*(X_i) = 1$

Setup D

Unrelated treatment and control arms.

$X_i \sim N(0, I_{d\times d}), e^*(X_i) = 1/(1 + e^{-X_{i1}} + e^{-X_{i2}})$

$b^*(X_i) = (\max\{X_{i1} + X_{i2} + X_{i3}, 0\} + \max\{X_{i4} + X_{i5}, 0\})/2$

$\tau^*(X_i) = \max\{X_{i1} + X_{i2} + X_{i3}, 0\} - \max\{X_{i4} + X_{i5}, 0\}$