Q2:

a) The output for question 2a) is shown below:

Results for np.float64

n	rel error	cond(H)
2	6.661e-16	2.700e+01
3	9.992e-15	7.480e+02
4	6.114e-13	2.837e+04
5	6.494e-12	9.437e+05
6	7.989e-10	2.907e+07
7	6.708e-09	9.852e+08
8	5.217e-07	3.387e+10
9	3.558e-05	1.1e+12
10	5.06e-04	3.535e+13
11	1.416e-04	1.230e+15
12	9.672e-01	3.798e+16
13	4.755	4.276e+17

b) The output from question 2a shows that as the matrix grows, the condition number also goes up and the relative error also grows up. The data type I use in my code is numpy's float64 with a machine epsilon of 1e-16. The number of correct digits follows the theoretical bound of $\log_{10} cond(H)$, the for n = 13, the number of digit we loose is $\log_{10} cond(4.276e17) \approx 17$, which agree with the relative error we calculated (A relative error more than 1 means that no digits are correct). And the relative error for the computed solution is strictly with in the bound of $cond(H)\varepsilon_m$ as the Heath textbook suggested.

Q3:

a) The process of Guassian elimination is shown below:

$$\left[\begin{matrix}\varepsilon & 1\\ 1 & 1\end{matrix}\middle|\begin{matrix}1+\varepsilon\\ 2\end{matrix}\right]$$

subtract row1/ε from row2:

$$\left[\begin{matrix}\varepsilon & 1\\ 0 & \dfrac{\varepsilon-1}{\varepsilon}\end{matrix}\middle|\begin{matrix}1+\varepsilon\\ \dfrac{\varepsilon-1}{\varepsilon}\end{matrix}\right]$$

Then we have a matrix in echelon form.

b) To solve the equation, we use back substitution:

Subtract $\frac{\varepsilon}{\varepsilon-1}$row2 from row 1:

$$\left[\begin{matrix}\varepsilon & 0\\ 0 & \dfrac{\varepsilon-1}{\varepsilon}\end{matrix}\middle|\begin{matrix}\varepsilon\\ \dfrac{\varepsilon-1}{\varepsilon}\end{matrix}\right]$$

Divide row1 by ε and row2 by $\frac{\varepsilon}{\varepsilon-1}$:

$$\left[\begin{matrix}1 & 0\\ 0 & 1\end{matrix}\middle|\begin{matrix}1\\ 1\end{matrix}\right]$$

We have x1 = 1 and x2 = 1.

c) The code for this question is in HW3.py

d) The output from the code is shown below:

```
K | relative error
------------------
1 | 8.882e-16
2 | 1.101e-13
3 | 2.876e-11
4 | 6.077e-09
5 | 8.274e-08
6 | 1.331e-04
7 | 7.993e-04
8 | 1.220e+00
9 | 1.e+00
10 | 1.e+00
```

From the output of the code, we can see that as the value of epsilon decreases, the relative error produces by the code also increases. These results are still within the bound by the Heath textbook. The high relative error is likely due to the rounding error during the Gaussian elimination and back substitution step since some step in the procedure required to divide a number by ε and add it to other numbers, which may cause catastrophic cancellation during the process.