Stochastic Methods + Lab

Session 17

November 1, 2016

Let $X = X(t)$ be an Itō process, i.e., a solution of the stochastic differential equation

$$dX = f(X,t)\, dt + g(X,t)\, dW \,,$$

interpreted in the sense of the Itō stochastic integral. Let $F(X,t)$ be twice continuously differentiable. Then the stochastic chain rule, also known as the Itō formula, reads

$$dF(X,t) = \left(\frac{\partial F(X,t)}{\partial t} + f(X,t)\, \frac{\partial F(X,t)}{\partial X} + \frac{1}{2}\, g(X,t)^2\, \frac{\partial^2 F(X,t)}{\partial X^2} \right) dt + g(X,t)\, \frac{\partial F(X,t)}{\partial X}\, dW \,.$$

Verify the Itō formula numerically for the example when $X(t)$ is geometric Brownian motion with $\mu = 0.2$ and $\sigma = 2.0$, and where

$$F(X,t) = (1+t)\,\sqrt{X}\,.$$

Hint: You have to compare direct evaluation of this expression with a numerical solution of the stochastic differential equation which you obtain from the Itō formula.