Stochastic Methods + Lab

Session 12

October 11, 2016

1. Plot the binomial distribution

$$b(j, n; p) = p^j \, q^{n-j} \binom{n}{j}$$

   where $q = 1 - p$ into a coordinate system where the values on the x-axis correspond to j according to

$$x_j = \frac{j - np}{\sqrt{npq}}$$

   and the y-values are given by $\sqrt{npq}\, b(j, n; p)$.

   Compare the graphs for $n = 10$, $n = 100$, and the graph of the standard Gaussian

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \, \mathrm{e}^{-x^2/2}$$

   in the same plot. Comment *briefly* on what you see.

2. Generate $N = 10\,000$ samples of the binomial distribution (number of successes in n independent trials). Rescale the samples via

$$X = \frac{J - \mathbb{E}[J]}{\sqrt{\mathrm{Var}[J]}}$$

   where you use the sample mean to approximate $\mathbb{E}[J]$ and the sample variance to approximate $\sqrt{\mathrm{Var}[J]}$. (These can be computed via the `numpy`-functions `mean()` and `std()`.)

   Generate $N = 10\,000$ samples of the standard normal distribution.

   Plot the sorted samples for X vs. the sorted samples for the standard normal distribution. Comment briefly on what you see.

   Note: This is called a QQ-plot and is more generally used to empirically compare two probability distributions.

3. Compute an ensemble of standard Brownian paths $W(t)$ over the interval $[0,1]$ partitioned into $N = 500$ time steps. Plot the empirically determied mean and standard deviation of the ensemble as a function of time.

4. Similarly, compute an ensemble of geometric Brownian paths

$$S(t) = \exp((\mu - \tfrac{1}{2}\,\sigma^2)\,t + \sigma\,W(t))$$

with $\mu = 0.05$ and $\sigma = 0.3$ and plot mean and standard deviation as a function of time on the interval $[0,1]$.