Stochastic Methods + Lab

Session 14

October 18, 2016

1. On the previous task sheet, you computed an ensemble of geometric Brownian paths
$$S(t) = \exp((\mu - \tfrac{1}{2}\,\sigma^2)\,t + \sigma\,W(t))$$
with $\mu = 0.05$ and $\sigma = 0.3$ and plot mean and standard deviation as a function of time on the interval $[0, 1]$.

   Now add, into the same coordinate system, mean and standard deviation of the stock price paths which underlie the binomial tree model with $N = 500$ time steps calibrated with the same set of parameters $r = \mu$ and annualized volatility σ. What do you see?

2. Use the paths so obtained in a Monte–Carlo valuation of a European call option with $K = 0.9$, time of maturity $T = 1.0$ and risk free rate $r = \mu$. Compare your result against the Black–Scholes price by plotting the deviation from the Black–Scholes price against the number of samples in a doubly logarithmic plot.

   What is the order of the Monte–Carlo method as a function of the number of samples?

3. Look up stock option quotes for European call options on the stock of a major corporation. Plot the implied volatility vs. the strike price, while the time to maturity is fixed. (The applicable interest rate is the spot rate for zero coupon bonds of the same maturity.)

4. Approximate the Itō integral
$$I = \int_0^T X(t-)\,\mathrm{d}W(t) = \lim_{N\to\infty} \sum_{i=0}^{N-1} X(t_i)\,(W(t_{i+1}) - W(t_i))$$
and the Stratonovich integral
$$W = \int_0^T X(t)\,\mathrm{d}W(t) = \lim_{N\to\infty} \sum_{i=0}^{N-1} X\Big(\frac{t_{i+1}+t_i}{2}\Big)\,(W(t_{i+1}) - W(t_i))\,,$$
where $W(t)$ denotes standard Brownian motion, $t_i = i\,\Delta t$ with $\Delta t = T/N$, and we choose, as an example, $X(t) = W(t)$. Do they converge to the same value as $N \to \infty$?