Stochastic Methods + Lab

Session 16

October 18, 2016

It is known that the stochastic differential equation

$$\mathrm{d}S(t) = \mu\, S(t)\, \mathrm{d}t + \sigma\, S(t)\, \mathrm{d}W(t)\,, \qquad (*)$$
$$S(0) = S_0$$

is solved by geometric Brownian motion

$$S(t) = S_0\, \mathrm{e}^{(\mu - \sigma^2/2)\, t + \sigma\, W(t)}\,.$$

1. Use the Euler–Maruyama method to solve (*) with $\mu = 2$, $\sigma = 1$, and $S_0 = 1$ up to final time $T = 1$. Compare pathwise against the exact solution.

2. Find the *strong order of convergence*, i.e., an exponent p such that

   $$\mathbb{E}[|S_N - S(T)|] \le c\, \Delta t^p$$

   where $S(T)$ denotes true geometric Brownian motion and S_N its Euler–Maruyama approximation at the final time T.

3. Find the *weak order of convergence*, i.e., an exponent q such that

   $$|\mathbb{E}[S_N] - \mathbb{E}[S(T)]| \le c\, \Delta t^q\,.$$