Stochastic Methods + Lab

Session 6

September 20, 2016

Notes:

- The computational work is to be submitted up to one week after the session via `git`.
- For this task sheet, it is sufficient to submit the code so long as it runs and produces the requested output.
- Please do not submit ipython notebooks. If you prefer working in the ipython shell, you can convert the notebook to a standalone script writing

  ```
  ipython3 nbconvert --to python filename.ipynb
  ```

  or

  ```
  jupyter nbconvert --to python filename.ipynb
  ```

  (depending on your installation). Make sure the file runs; you may have to add print statements expressions resulting from output cells.
- Make sure that your submission runs the requested test case.
- The theoretical questions can be submitted handwritten on paper.

1. Consider a 10-year level coupon bond with an annual coupon rate of 8% compounded annually and a par value of EUR 1 000. Plot the price of the bond vs. the yield.
2. Plot price volatility vs. time to maturity for level coupon bonds with annual coupon rates of 2%, 6%, and 12% paid semiannually. Assume a yield of 6% and a par value of EUR 1 000. To see the different volatility behaviors, take a range from 0 up to 100 years to maturity.

3. Plot the bond value (forward value) of an 8% 15-year bond compounded semi-annually vs. years to maturity under three rate scenarios: (a) the interest rate decreases instantaneously to 6%, (b) the interest rate remains unchanged, and (c) the interest rate increases instantaneously to 10%.

   Note: The forward value of the bond required here is the value of the bond at time zero based on the new market interest rate times the compounded interest factors corresponding to different times to maturity.

4. Plot the future value of a 30-year bond at a coupon rate of 10% compounded annually after a 10-year horizon as a function of yield. Find the minimum of the horizon price numerically, e.g., using `scipy.optimize.brent`.

5. Visit the web site of the European Central Bank (ECB) and look for their yield curve data. What data is contained in the files?

   (This is to be submitted as a written answer!)

6. The ECB publishes spot rate data in a downloadable CSV file. Find online documentation on how to read CSV files, read in the ECB spot rates, and plot the yield curve.

7. Write out a consistent set of expressions for the Macaulay duration and the immunization of a level coupon bond assuming continuous compounding of interest.

8. Suppose you want to immunize a liability at time D with two bonds of Macaulay durations D_1 and D_2, respectively. Show that the fraction ω_1 and ω_2 of the two bonds in the initial portfolio need to satisfy

$$\omega_1 + \omega_2 = 1\,,$$
$$\omega_1\, D_1 + \omega_2\, D_2 = D\,.$$