Stochastic Methods + Lab

Session 10

October 4, 2016

1. Rewrite your code so that it stores the option value at each node of the tree. Then visualize the tree using `imshow`. Think about an appropriate color map, how to mask the missing values (Hint: use Numpy's masked arrays), and how to best map the computed values to pixel coordinates.

2. The price of a European Call option with current stock price S, strike price K, annualized volatility σ, annual risk-free interest rate r, and maturity time T can be computed explicitly from the Black–Scholes formula

   $$C = S\,\Phi(x) - K\,\mathrm{e}^{-rT}\,\Phi(x - \sigma\sqrt{T})\,,$$

   where

   $$x = \frac{\ln(S/K) + (r + \sigma^2/2)\,T}{\sigma\sqrt{T}}$$

   and Φ denotes the cumulative distribution function of the standard normal distribution with mean zero and variance one.

   Compare your call option prices from the binomial tree model against those computed by the Black–Scholes formula. Does the error scale like a power of N? (Plot the logarithm of the error vs. N. Do you obtain a straight line?)

3. Show that an American call option should be exercised at expiration for maximal profit.

4. Modify your binomial tree algorithm to price an American put option (the holder may exercise the option at any time before expiration). Is the price of an American put higher or lower than that that of a European put with otherwise identical parameters?