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Extended Kalman Filter (EKF)

X_0, P_0 $\longrightarrow$ $X_{kp} = F \cdot X_{k-1} + u_{k-1}$ Predict state

Initialize state

$$P_{kp} = F \cdot P_{k-1} \cdot F^T + Q_{k-1}$$

$\downarrow$

$$y_k = \underset{(\text{sensor})}{Z_k} - h(x')_k$$

$$K_k = \frac{(P_{kp} \cdot H_j^T)}{H_j \cdot P_{kp} \cdot H_j^T + R}$$

update state

$\downarrow$

$$X_k = X_{kp} - K_k \cdot y_k$$

$$P_k = (I - K_k \cdot H_j) \cdot P_{kp}$$

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Unscented Kalman Filter (UKF)

$$\mathcal{X}_{k|k} = x_{k|k} \cdot x_{k|k} + \sqrt{(\lambda + n_x) P_{k|k}} \cdot$$

$$x_{k|k} - \sqrt{(\lambda + n_x) P_{k|k}}$$

Generate sigma points

$$\downarrow$$

$$\mathcal{X}_{k+1|k} = f(x_k, v_k)$$

process model

predict sigma points

$$\downarrow$$

a) $x_{k+1|k} = \sum_{i=0}^{2n_a+1} W_i \cdot \mathcal{X}_{k+1|k,i}$

Predicted state mean.

b) $P_{k+1|k} = \sum_{i=0}^{2n_a+1} W_i (\mathcal{X}_{k+1|k,i} - x_{k+1|k}) \cdot (\mathcal{X}_{k+1|k,i} - x_{k+1|k})^T$

Predicted state covariance.

Continental

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$\downarrow$

Predict Measurement:

a) $Z_{k+1|k} = h(x_{k+1})$

Measurement model.

b) $z_{k+1|k} = \sum_{i=0}^{2n_a} W_i \cdot Z_{k+1|k,i}$

predict measurement mean

c) $S_{k+1|k} = \sum_{i=0}^{2n_a} W_i \left(Z_{k+1|k,i} - z_{k+1|k}\right) \cdot \left(Z_{k+1|k,i} - z_{k+1|k}\right)^T + R$

predicted measurement covariance.

$\downarrow$

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Update state

a) $T_{k+1|k} = \sum_{i=0}^{2n_a} W_i \left(X_{k+1|k,i} - x_{k+1|k}\right) \cdot \left(Z_{k+1|k,i} - z_{k+1|k}\right)^T$

cross-correlation betn sigma points in state space & measurement space.

b) $K_{k+1|k} = T_{k+1|k} \cdot S_{k+1|k}^{-1}$

Kalman Gain

c) $x_{k+1|k+1} = x_{k+1|k} + K_{k+1|k} \cdot \left(z_{k+1\,(\text{sensor})} - z_{k+1|k}\right)$

state update

d) $P_{k+1|k+1} = P_{k+1|k} - K_{k+1|k} \cdot S_{k+1|k} \cdot K_{k+1|k}^T$.

State Covariance update.

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EKF steps.

1) Initialization
2) Prediction
3) Update.

UKF steps

1) Generate sigma points in state space (Variables of axis are state vector elements).

2) Predict sigma points in state space + assign weights to each sigma points.

3) Predict mean & Covariance of sigma points in state space.

4) Predict measurement in measurement space.
(Variables of axis are sensor

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measurement elements.
Hence we need to convert from state space to measurement space in order to equate measurements of same unit).

5) Update State.
By taking cross correlation between matrix of sigma points in state space & Measurement space.

* Variations in the values of 'P' matrix.

✓ Ideally, the values of 'P' matrix goes on reducing at each iteration, when the road is straight & the vehicle is moving at constant velocity.

✓ But in real-time roads won't be straight always, it may be curvy, more curvy. Also vehicle not always moves with constant velocity depending on traffic & road conditions may accelerate or decelerate, in such conditions the values of 'P' matrix may increase Gradually & then decreases Gradually.

✓ This happens because filter sometimes at specific conditions of roads (mostly non-linear) breaks or we can say ~~this~~ fails to handle (couldn't better linearize the non-linearity).

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* Variations in the values of 'Q' matrix

- ✓ Once the Variable (σ_{ax}^2, σ_{ay}^2, Δt) in Q matrix are evaluated, they remain same and does not vary at any conditions.

- ✓ Different road scenarios & vehicle speed doesn't make any difference for 'Q' matrix.

- ✓ 'Q' matrix represents the errors that are expected in designing the state transition equations

- ✓ σ_{ax}^2, σ_{ay}^2 → are co-variance (error) in the acceleration, these values we are going to assign initially i.e while designing the filter.

  Δt ⇒ It is the time difference at which the filter receives sensor measurement, this value also fixed while designing the filter.

- ✓ Hence values of 'Q' matrix never changes, It's objective is, at each iteration values of 'Q' matrix summed up with values of 'P_{k-1}' matrix (previous state covariance matrix) to give P_{kp} (predicted state covariance matrix).

IMU

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Acceleration

↓

Velocity

↓

Displacement

↓

Coordinates
(lat & long)

$$\boxed{s = R \cdot \theta}$$

R = Rad. of earth
s = Dist. travelled (Dist. bet^n any 2 points on a earth).
θ = ?

$$lat = \frac{s \cdot \sin(\theta)}{R}$$

$$long = \frac{s \cdot \cos(\theta)}{R}$$

θ = Central angle between any 2 points on the earth.