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A UKF Algorithm Based on the Singular Value Decomposition of State Covariance*

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***Abstract*—One of the existing problems of General UKF is the stability problem due to the strong nonlinearity and the complexity of the system. When the algorithm can't ensure the state covariance to be positive semidefinite, the precision will be decreased or even general UKF will be halted. Based on the singular value decomposition of state covariance, this paper proposes a modified UKF algorithm which can enhance the state covariance to be positive semidefinite. Comparing to the general UKF, the SVDUKF algorithm releases the condition of the positive semi-definite of state covariance. Numeric simulation experiments demonstrate the effectiveness of the algorithm.**

***Index Terms* - diagonal similar decomposition, symmetrical sample UKF, stability, positive semidefinite**

I. INTRODUCTION

Online estimation technique aims mainly to obtain high precise states or parameters of a dynamic system with a set of noisy measured signals. Linear Kalman filter (KF) provides acceptable solutions in many situations [24]-[26]. However, in a larger of real applications, such as robots, industrial process, and even economy systems, the evolution of systems is governed by nonlinear functions, and this make the validity of KF is dropped greatly [27]. Thus, nonlinear estimation now constitutes an interesting and growing research area.

In the past decades, many approximate optimal nonlinear estimation methods, including the extended Kalman filter (EKF) and the unscented Kalman filter (UKF), have been developed and applied to all kinds of practical nonlinear systems. Although EKF, which generalizes KF method to nonlinear systems by linearizing the nonlinear functions using Taylor series expansions, is most commonly used in real applications, it also exhibits several severe disadvantages, e.g. heavy computational burden due to the Jacobian Matrix and poor estimation precision for systems with high nonlinearities. In order to improve the performance of EKF method, researchers proposed a new KF based nonlinear estimation strategy – UKF[1]-[3]. UKF can deal with nonlinearities better on the basis of unscented transform (UT), a mechanism for propagating mean and covariance through a nonlinear transformation. It has been shown that UKF can approximate the posterior mean and covariance of the Gaussian random variable with a second order accuracy[15]. This makes UKF have higher precision than EKF algorithm.

In recent years, UKF has obtained extensive attentions, and many researches on the performance of UKF can be found. These research works can be divided into two parts: 1) algorithm improvement; 2) applications.

[8]-[12] present the applications of UKF algorithm in all kinds of fields. Such as, Chen et al. [8] studied UKF parameters estimation based on nonlinear observation and nonlinear kinetics model; Yamakita et al. [11] compared the recursive least squares, adaptive observer, EKF and UKF with a conclusion that the smaller error and higher precision algorithm is UKF; Ge Zhe-Xue et al. [12] brought forward a fault detect method based on UKF; UKF has been also introduced to the aircrafts modeling [9].

Besides the application researches, many researchers try to improve the performance of nominal UKF algorithm and many new version UKF algorithms are brought forward. For example, in 2001, square root UKF [13] was proved by Van der Merwe and Wan that the numeric stability and parameter estimation efficiency was higher (for special situation of parameter estimation is $O(L^2)$). K. Xiong[15] proposed a revised UKF which could reach Cramér–Rao lower bound (CRLB) with a certain conditions. Song Qi [4] could estimate the covariance of noise online with the proposed Adaptive UKF (AUKF).

Although UKF has been the focus of research of estimation online, there are still some to be improved with the algorithm. When the state covariance matrix is negative semidefinition, the estimation of states and outputs maybe complex, which is not consistent with the measurement. In this paper, a modified UKF, which is based on singular value decomposition, is proposed to ensure the positive semi definition of the state covariance matrix. Details of the modified UKF, proof of singular value decomposition of symmetrical matrix are presented in Section III.

A brief mathematical formulation of the UKF is given in Section II. In Section Ⅳ discuss the numeric simulation. Section Ⅴ is the conclusion.

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II. General UKF algorithm

A. Unscented transformation (UT)

Since UKF is based on the UT technique, a mechanism for propagating mean and covariance through a nonlinear transformation. We will first introduce the basic idea of UT.

The unscented transformation (UT) is to calculate the statistics of a random variable which undergoes a nonlinear transformation [23]. Consider a random variable $\boldsymbol{x}$ (dimension N), which has mean $\bar{x}$ and covariance P, propagating through a nonlinear function, $y = g(x)$. To calculate the statistics of y, matrix $\mathbf{x}$ of 2N+1 sigma points should be calculated as follow:

$$\begin{cases} \mathbf{x}_0 = \bar{x} \\ \mathbf{x}_i = \bar{x} + (\sqrt{(N+\lambda)P})_i \qquad i = 1,\dots,N \\ \mathbf{x}_i = \bar{x} + (\sqrt{(N+\lambda)P})_{i-N} \quad i = N,\dots,2N \end{cases} \tag{1}$$

$$\begin{cases} w_0^m = \dfrac{\lambda}{N+\lambda} \\ w_0^c = \dfrac{\lambda}{N+\lambda} + (1-\alpha^2+\beta) \\ w_i^m = w_i^c = \dfrac{1}{2(N+\lambda)} \qquad i = 1,\dots,2N \end{cases} \tag{2}$$

where $\lambda = (N+\kappa)\alpha^2 - N$, α is the constant to control the *Sigma point* distribution, κ is a scaling parameter which is usually set to 0, and β is nonnegative constant. These sigma vectors are propagated through the nonlinear function,

$$\mathbf{y}_i = g(\mathbf{x}_i) \quad i = 0,\dots,2N$$

Then the mean and covariance of y is approximated by a weighted sample mean and covariance of the posterior sigma points

$$\begin{aligned} \bar{y} &\approx \sum_{i=0}^{2N} w_i^c \mathbf{y}_i \\ P_y &\approx \sum_{i=0}^{2N} w_i^m (\mathbf{y}_i - \bar{y})(\mathbf{y}_i - \bar{y})^{\mathrm{T}} \end{aligned} \tag{3}$$

A simple sketch map of the UT method is shown in Fig 1.

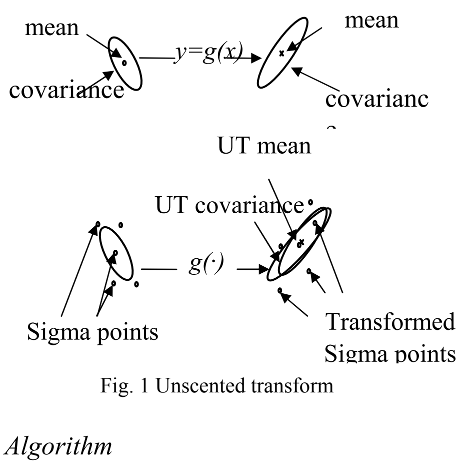

Fig. 1 Unscented transform

B. UKF Algorithm

Consider the following nonlinear system with addictive noises,

$$\begin{cases} x_k = f(x_{k-1}, u_k) + v_k \\ y_k = Hx_k + w_k \end{cases} \tag{3}$$

where, $x_k \in R^n$ is the states vector, $u_k \in R^r$ and $y_k \in R^p$ are the input and output vectors at time instant k, respectively. v_k and w_k are the process noise and measurement noise such that

$$E[v_k] = 0, E[w_k] = 0,$$
$$E[w_k w_j^T] = R_k \delta_{kj},$$
$$E[v_k v_j^T] = Q_k \delta_{kj}$$

Here, we suppose that v_k, w_k, x_k are not correlative to each other.

Nominal UKF algorithm for the associated noisy nonlinear system is described as follow,

1) Initialization

$$\begin{cases} \bar{x}_0 = E[x_0] \\ P_0 = E\left[(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T\right] \end{cases} \tag{5}$$

$$\begin{cases} w_0^m = \dfrac{\lambda}{n+\lambda} \\ w_0^c = \dfrac{\lambda}{n+\lambda} + (1-\alpha^2+\beta) \\ w_i^m = w_i^c = \dfrac{1}{2(n+\lambda)} \qquad i = 1,\dots,2n \end{cases} \tag{6}$$

where $\lambda = n(\alpha^2 - 1)$, α is the constant to control the *Sigma point* distribution, and β is nonnegative constant.

2) Computation of the Sigma points

$$\chi_{k-1} = \left[\bar{x}_{k-1}, \bar{x}_{k-1} + \sqrt{(n+\lambda)P_{k-1}}, \bar{x}_{k-1} - \sqrt{(n+\lambda)P_{k-1}}\right] \tag{7}$$

3) Time update

$$\begin{cases} \chi^*_{k|k-1} = f(\chi_{k-1}, u_k) \\ \hat{x}_{k|k-1} = \sum_{i=0}^{2n} w_i^m \chi^*_{i,k|k-1} \\ P_{k|k-1} = \sum_{i=0}^{2n} w_i^c (\chi^*_{i,k|k-1} - \hat{x}_{k|k-1})(\chi^*_{i,k|k-1} - \hat{x}_{k|k-1})^T + Q \\ \gamma_{k|k-1} = h(\chi^*_{k|k-1}) \\ \hat{y}_{k|k-1} = \sum_{i=0}^{2n} w_i^m \gamma_{i,k|k-1} \end{cases} \tag{8}$$

4）Measurement update

$$\begin{cases} P_{\bar{y}_k\bar{y}_k} = \sum_{i=0}^{2n} w_i^c (\gamma_{i,k|k-1} - \bar{y}_{k|k-1}) \cdot (\gamma_{i,k|k-1} - \bar{y}_{k|k-1})^T + Q^n \\ P_{\bar{x}_k\bar{y}_k} = \sum_{i=0}^{2n} w_i^c (\chi_{i,k|k-1} - \bar{x}_{k|k-1}) \cdot (\gamma_{i,k|k-1} - \bar{y}_{k|k-1})^T \\ K_k = P_{\bar{x}_k\bar{y}_k} P^{-1}_{\bar{y}_k\bar{y}_k} \\ P_k = P_{k|k-1} - K_k P_{\bar{y}_k\bar{y}_k} K_k^T \\ \bar{x}_k = \bar{x}_{k|k-1} + K_k (y_k - \bar{y}_{k|k-1}) \end{cases} \tag{9}$$

The computational cost of the UKF algorithm is similar as the EKF, but computational precision is much higher. Furthermore the Jacobian Matrix of the system function needs not to be computed any more.

To reduce the computation burden, the sequence UKF [16] is used, which is in the measurement stage equation (9) is replaced by following equation (10)

$$\begin{cases} P_k^0 = P_{k,k-1} \\ x_k^0 = x_{k,k-1} \\ x_k^i = x_k^{i-1} + K_k^i (y_k^i - H_k^i x_k^{i-1}) \\ K_k^i = P_k^{i-1} H_k^{iT} (H_k^i P_k^{i-1} H_k^{iT} + R_k^i)^{-1} \\ P_k^i = (I - K_k^i H_k^i) H_k^{i-1} \quad (i = 1, 2, \cdots m) \\ x_k = x_k^m \\ P_k = P_k^m \end{cases} \tag{10}$$

Where, H_k^i is the $i-th$ column of H_k, y_k^i is the $i-th$ element of the measurement y_k, K_k^i is the $i-th$ column of K_k, x_k^i and P_k^i are the $i-th$ circle of the state vector x_k and state covariance matrix P_k respectively.

Although UKF has been the focus of research of estimation online, it requires initial value of state covariance should be nonnegative definition [1]-[3], which limit the application of the method.

III. Singular Value Decomposition Based UKF (SVD-UKF)

A. Singular Value Decomposition of Symmetrical Positive definite Matrix

Given the real symmetrical matrix $M \in R^{n\times n}$, $MM^T \geq 0$, it must be a symmetrical matrix and its eigenvalues are all nonnegative.

Define

$$\sigma_i^2 = \sqrt{\lambda_i (M^{\mathrm{T}} M)}$$

Then σ_i is called singular value of M [22], and here must exist orthogonal matrixes U and V such that

$$M = U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} V^T \tag{11}$$

The equation (10) is called singular value decomposition [22], where

$$\Sigma = \mathrm{diag}(\sigma_1, \sigma_2, \sigma_3, \cdots \sigma_r),$$
$$\sigma_i (i = 1, 2, \cdots r)$$

are all the nonzero singular values of M.

Lemma: if $M \in R^{n\times n}$ is a real symmetrical positive semidefinite matrix, it can be decomposed as

$$\begin{aligned} M &= U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} U^{\mathrm{T}} \\ &= U \begin{bmatrix} \sqrt{\Sigma} & 0 \\ 0 & 0 \end{bmatrix} U^{\mathrm{T}} U \begin{bmatrix} \sqrt{\Sigma} & 0 \\ 0 & 0 \end{bmatrix} U^{\mathrm{T}} \end{aligned} \tag{12}$$

where U is the orthogonal matrix. Σ is a diagnal matrix with positive diagnal element.

□*Proof:*

Because M is symmetrical and nonnegative definite, the eigenvalues of M are all nonnegative and real. From the definition of the singular value we find that the singular values are its eigenvalues, that is $\sigma_i(M) = \lambda_i(M)$.

Thereby

$$\begin{aligned} \sigma_i^2 x &= M^{\mathrm{T}} M x = M^2 x \\ &= MMx = M\lambda_i(M)x = \lambda_i^2(M)x \end{aligned}$$

and the eigenvectors of $M^{\mathrm{T}}M$ are that of M.

Supposed that U is the orthogonal matrix composing of orthogonal eigenvectors of M, and $V = U$, we can get equation (11), which is singular value decomposition of M.

Supposed

$$S = U \begin{bmatrix} \sqrt{\Sigma} & 0 \\ 0 & 0 \end{bmatrix} U^{\mathrm{T}}$$

where $\Sigma = \mathrm{diag}(\sigma_1, \sigma_2, \sigma_3, \cdots \sigma_r)$

$\sigma_i (i = 1, 2, \cdots r.r \leq n)$ are all the nonzero singular values of M. Decomposing matrix M, we get

$$\begin{aligned} M &= U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} U^{\mathrm{T}} \\ &= SS \end{aligned}$$

Then we get the equation (12). □

B. Algorithm based on singular value decomposition of state covariance matrix

From theorem above, the state covariance matrix P is decomposed to chose the sigma points, which approximate the posterior mean and covariance of the Gaussian random variable, propagating mean and covariance through a nonlinear transformation.

Replacing the equation (7) of the UKF algorithm, the new measurement update in algorithm is

$$\begin{cases} [U, D, V] = f_{\mathrm{svd}}(P_{k-1}) \\ C = U\left(\sqrt{(n+\lambda)D}\right)U^{\mathrm{T}} \\ \chi_{k-1} = [\hat{x}_{k-1}, \hat{x}_{k-1} - C, \hat{x}_{k-1} + C] \end{cases} \tag{13}$$

where, $D = \mathrm{diag}({\sigma^2}_1, {\sigma^2}_2, {\sigma^2}_3, \cdots {\sigma^2}_n)$, $\sigma_i (i = 1, 2, \cdots n)$ are the singular values of P (including 0). $F_{\mathrm{svd}}(\bullet)$ is the function to get the eigenvectors and singular values of the matrix (•).

C. The qualitative analysis of the stability

Only the nonpositive semidefinition situation of the state covariance matrix is analyzed qualitative.

When the state covariance matrix P is positive semidefinition, the new algorithm is consistent with the general UKF algorithm. While the state covariance matrix has at least one eigenvalue with negative real part, the new algorithm will work automatically to change the one-step predicted state covariance matrix to

$$\begin{aligned} \overline{P}_{k-1} &= U \begin{bmatrix} \sqrt{\Sigma} & 0 \\ 0 & 0 \end{bmatrix} U^{\mathrm{T}} U \begin{bmatrix} \sqrt{\Sigma} & 0 \\ 0 & 0 \end{bmatrix} U^{\mathrm{T}} \\ &= U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} U^{\mathrm{T}} \end{aligned}$$

where, $\Sigma = \mathrm{diag}(\sigma_1, \sigma_2, \sigma_3, \cdots \sigma_n)$, $\sigma_i (i = 1, 2, \cdots n)$ are the singular values of P (including 0).

Although $\overline{P} \neq P$, $\lambda_i(\overline{P}) = |\lambda_i(P)|$. This will ensure the sigma point belong to the real field and enhance the positive semidefinition of the state covariance matrix P, so it releases the requirement of the state covariance matrix and ensure the precision of the estimation.

IV. NUMERIC SIMULATION

Used a fighter as the background, a nonlinear system model is built to validate the new algorithm.

In this paper, the fixed-wing UAV model is used to verify the feasibility and validity of the proposed algorithm.

The nonlinear function is as follows,

$$\begin{aligned}
m\dot{v} =& -D\cos\beta + Y\sin\beta + XT\cos\alpha\cos\beta + YT\sin\beta \\
& + ZT\sin\alpha\cos\beta - mg(\cos\alpha\cos\beta\sin\theta - \sin\beta\sin\phi\cos\theta \\
& - \sin\alpha\cos\beta\cos\phi\cos\theta)
\end{aligned}$$

$$\dot{\alpha} = q - \tan\beta(p\cos\alpha + r\sin\alpha) + \frac{-L + ZT\cos\alpha - XT\sin\alpha + mg(\cos\alpha\cos\phi\cos\theta + \sin\alpha\sin\theta)}{mv\cos\beta}$$

$$\begin{aligned}
\dot{q} =& (LI_2 + MI_4 + NI_5 - p^2(I_{xz}I_4 - I_{xy}I_5) + \\
& pq(I_{xz}I_2 - I_{xz}I_4 - D_zI_5) - pr(I_{xy}I_2 + D_yI_4 - I_{yz}I_5) \\
& + q^2(I_{yz}I_2 - I_{xy}I_5) - qr(D_xI_2 - I_{xy}I_4 + I_{xz}I_5) \\
& - r^2(I_{yz}I_2 - I_{xz}I_4)) / \Delta I
\end{aligned}$$

$$\begin{aligned}
\dot{p} =& (LI_1 + MI_2 + NI_3 - p^2(I_{xz}I_2 - I_{xy}I_3) + \\
& pq(I_{xz}I_1 - I_{yz}I_2 - D_zI_3) - pr(I_{xy}I_1 + D_yI_2 - I_{yz}I_3) \\
& + q^2(I_{yz}I_1 - I_{xy}I_3) - qr(D_xI_1 - I_{xy}I_2 + I_{xz}I_3) \\
& - r^2(I_{yz}I_1 - I_{xz}I_2)) / \Delta I
\end{aligned}$$

$$\begin{aligned}
\dot{r} =& (LI_3 + MI_5 + NI_6 - p^2(I_{xz}I_5 - I_{xy}I_6) + \\
& pq(I_{xz}I_3 - I_{yz}I_5 - D_zI_6) - pr(I_{xy}I_3 + D_yI_5 - I_{yz}I_6) \\
& - r^2(I_{yz}I_3 - I_{xz}I_5)) / \Delta I
\end{aligned}$$

$$\dot{\theta} = q\cos\phi - r\sin\phi$$

$$\dot{\phi} = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta$$

$$\dot{\psi} = r\cos\phi\sec\theta + q\sin\phi\sec\theta$$

$$\begin{aligned}
\dot{\beta} =& p\sin\alpha - r\cos\alpha + \\
& (D\sin\beta + Y\cos\beta - X_T\cos\alpha\sin\beta + Y_T\cos\beta \\
& - Z_T\sin\alpha\sin\beta + mg(\cos\alpha\sin\beta\sin\theta + \\
& \cos\beta\sin\phi\cos\theta - \sin\alpha\sin\beta\cos\phi\cos\theta)) / (mv)
\end{aligned}$$

$$\begin{aligned}
\dot{h} =& v(\cos\alpha\cos\beta\sin\theta - \sin\beta\sin\phi\cos\theta \\
& - \sin\alpha\cos\beta\cos\phi\cos\theta)
\end{aligned}$$

where XT,YT and so on are the corresponding parameters of the fighter.

Compact above equations as:

$$\begin{aligned} \dot{x} &= f(x, u, t) + v(t) \\ y &= x + w(t) \end{aligned} \tag{14}$$

where, $x = [V, \alpha, q, \theta, p, \phi, r, \psi, \beta, h]^{\mathrm{T}}$ is the state vector, u and y are input and output vectors respectively, $u = [\delta_H \delta_{lf} \delta_{rf} \delta_A \delta_F \delta_R]^{\mathrm{T}}$. f is the nonlinear differential function. $v(t)$ *and* $w(t)$ are the independent white noises. Euler method is used to discretize the system

$$\begin{aligned} x_{k+1} &= x_k + Tf(x, u, t) + Tv(t) \\ y_k &= x_k + w(t) \end{aligned} \tag{15}$$

where, T is the step time of the simulation. The simulator is built on the matlab7.5, the two UKF algorithms are both realized, and only the real part of equation (7) of general UKF algorithm is used to ensure program running (This will deteriorate the performance of the filter greatly, as shown in our simulation results).

The initial values of the simulator：

$$T = 0.1$$

$$x_0 = [550,\ 0.0891, 0.0891,\ 0,0,0,0,0,0,9800]^T$$

To prove the superiority of SVDUKF, the initial values of the algorithms are initialized

$$\hat{x}_0 = 0.8x_0$$

$$\hat{P}_0 = -diag([35;1;1;1;1;1;1;1;1;300]^2)$$

$$Q = 10^{-8} I$$

$$R = diag([1.5;0.0015;0.0015;0.0015;0.0015;0.0015;0.0015;0.0015;0.0015;60]^2)$$

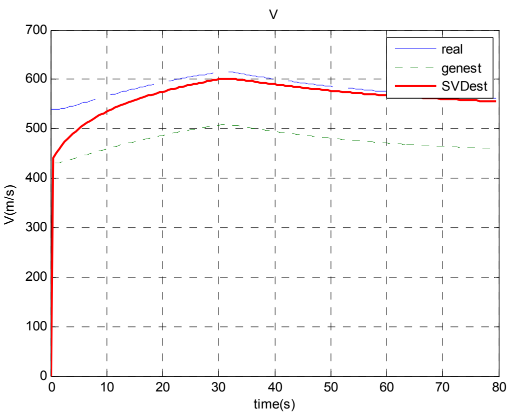

Fig.2 Estimation of velocity

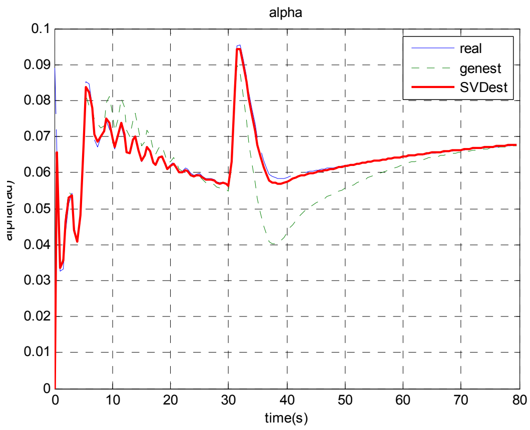

Fig.3 Estimation of angle of attack

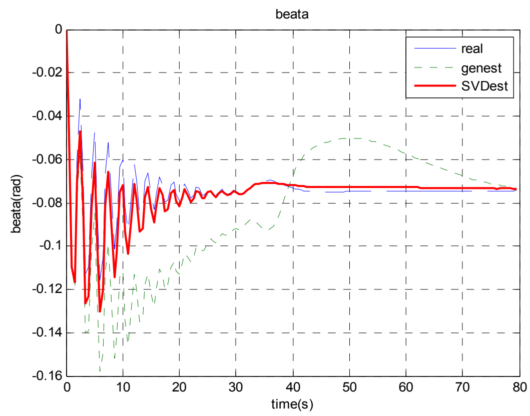

Fig.4 Estimation of angle of side-slip

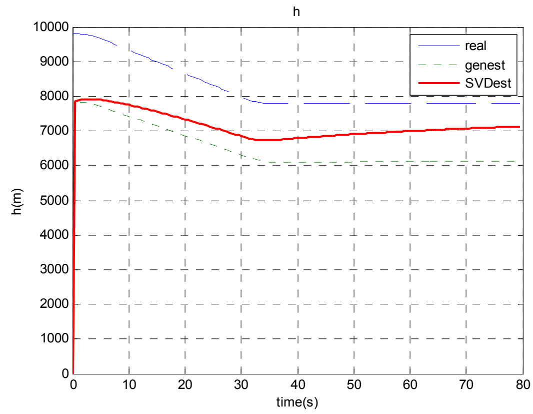

Fig.5 Estimation of height

Figure 2~5 indicate that when errors exist in initial estimation values and the state covariance matrix is the situation of nopositive semidefinition, the two algorithm work much differently. Tab.1 lists the MSE of some state estimation error. All the results indicate that SVDUKF improve the stability and converge of the algorithm, and release the requirement to the positive semidefinition of state covariance matrix. In the table GENUKF is represented the general UKF algorithm and SVDUKF is represented the SVDUKF algorithm.

Tab.1 comparison of MSE

Method	V(ft/s)	α (°)	β (°)	h(ft)
GENUKF	——	1.032×10^{-4}	8.66×10^{-4}	——
SVDUKF	337.47	5.78×10^{-7}	1.995×10^{-5}	9.74×10^{5}

V. Conclusion

A modified UKF algorithm is proposed, which is based on the singular value decomposition of state covariance (SVDUKF). The proposed algorithm is proved to be superiority to the general UKF algorithm by the numeric simulation. The positive semidefinition can be ensured in SVDUKF algorithm, so it releases the requirement of the state covariance matrix and ensure the precision of the estimation. There is deduction of the algorithm as well as qualitative analysis in the paper.

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