

Extended Kalman Filter (EKF)

Initialize state

$$\begin{bmatrix} X_0 \\ P_0 \end{bmatrix} \rightarrow \begin{bmatrix} X_{kp} = F \cdot X_{k-1} + U_{k-1} \\ P_{kp} = F \cdot P_{k-1} \cdot F^T + Q_{k-1} \end{bmatrix} \quad \begin{matrix} \text{predict} \\ \text{state} \end{matrix}$$

update state

$$\begin{bmatrix} y_k = z_k - \cancel{\text{XXXXXXXXXX}} h(x')_k \\ K_k = \frac{(P_{kp} \cdot H_j^T)}{H_j \cdot P_{kp} \cdot H_j^T + R} \end{bmatrix}$$

$$\begin{bmatrix} X_k = X_{kp} - K_k \cdot y_k \\ P_k = (I - K_k \cdot H_j) \cdot P_{kp} \end{bmatrix}$$

Unscented Kalman Filter (UKF)

$$\bar{X}_{k|k} = X_{k|k} \cdot \lambda_{k|k} + \sqrt{(\lambda + \eta \pi) P_{k|k}}$$

$$\lambda_{k|k} = \sqrt{(\lambda + \eta \pi) P_{k|k}}$$

Generate
sigma
points



$$\bar{X}_{k+1|k} = f(\bar{x}_k, V_k)$$

Process
model

predict
sigma points



$$a) \bar{x}_{k+1|k} = \sum_{i=0}^{2m+1} W_i \cdot \bar{X}_{k+1|k,i}$$

Predicted state mean.

$$b) P_{k+1|k} = \sum_{i=0}^{2m+1} K_i (\bar{X}_{k+1|k,i} - \bar{x}_{k+1|k}) (\bar{X}_{k+1|k,i} - \bar{x}_{k+1|k})^T$$

Predicted state
Covariance



Predict Measurement:

$$a) \quad Z_{k+1|k} = h(x_{k+1})$$

measurement model

$$b) \quad \bar{z}_{k+1|k} = \sum_{i=0}^{2na} w_i \cdot z_{k+1|k,i}$$

predict measurement mean

$$c) \quad S_{k+1|k} = \sum_{i=0}^{2na} k_i \left(Z_{k+1|k,i} - \bar{z}_{k+1|k} \right) \cdot \left(Z_{k+1|k,i} - \bar{z}_{k+1|k} \right)^T + R$$

predicted measurement
covariance



✓

Update state

$$a) T_{k+1|k} = \sum_{i=0}^{2n_a} W_i (X_{k+1|k,i} - \bar{X}_{k+1|k}) (Z_{k+1|k,i} - \bar{Z}_{k+1|k})^T$$

cross-correlation betⁿ sigma points
in state space & measurement space.

$$b) K_{k+1|k} = T_{k+1|k} \cdot P_{k+1|k}^{-1}$$

Kalman Gain

$$c) X_{k+1|k+1} = \bar{X}_{k+1|k} + K_{k+1|k} \cdot$$

$$(Z_{k+1}(\text{sensor}) - \bar{Z}_{k+1|k})$$

state update

$$d) P_{k+1|k+1} = P_{k+1|k} - K_{k+1|k} \cdot S_{k+1|k} \cdot K_{k+1|k}^T$$

State Covariance
update.

EKF steps

- 1) Initialization
- 2) Prediction
- 3) Update

UKF steps

- 1) Generate sigma points in state space (Variables of axis are state vector elements).
- 2) Predict sigma points in state space + assign weights to each sigma point.
- 3) Predict mean & Covariance of sigma points in state space.
- 4) Predict measurement in measurement space.
(Variables of axis are sensor

measurement elements.

Hence we need to convert from state space to measurement space in order to equate measurements of same unit).

5) Update State.

By taking cross correlation ~~betw~~ matrix of sigma points in state space & measurement space.

* Variations in the values of 'P' matrix.

- ✓ Ideally, the values of 'P' matrix goes on reducing at each iteration, when the road is straight & the vehicle is moving at constant velocity.
- ✓ But in real-time roads won't be straight always, it may be curvy, more curvy. Also vehicle not always moves with constant velocity depending on traffic & road conditions may accelerate or decelerate, in such conditions the values of 'P' matrix may increase gradually & then decreases gradually.
- ✓ This happens because filter sometimes at specific conditions of roads (mostly non-linear) breaks or we can say ~~it~~ fails to handle (couldn't better linearize the non-linearity).

* Variations in the values of 'Q' matrix

- ✓ Once the Variable (σ_{ax}^2 , σ_{ay}^2 , Δt) in Q matrix are evaluated, they remain same and does not vary at any conditions,
- ✓ Different road scenarios & vehicle speed doesn't make any difference for 'Q' matrix.
- ✓ 'Q' matrix represents the errors that are expected in designing the state transition equations
- ✓ σ_{ax}^2 , $\sigma_{ay}^2 \rightarrow$ are Co-variance (error) in the acceleration, these values are going to assign initially i.e. while designing the filter.

$\Delta t \rightarrow T$ is the time difference at which the filter receives sensor measurement, this value also fixed while designing the filter.

- ✓ Hence values of 'Q' matrix never changes, It's objective is, at each iteration values of 'Q' matrix summed up with values of ' P_{k-1} ' matrix (previous state covariance matrix) to give ' P_k ' (predicted state covariance matrix).

IMU

Date: ___/___/___

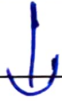
Acceleration



Velocity



Displacement



Coordinates
(lat & long)

$$S = R \cdot \theta$$

R = Rad. of earth

S = Dist. travelled (Dist. betⁿ any 2 points on earth).

θ = ?

$$\text{lat} = \frac{S \cdot \sin(\theta)}{R}$$

$$\text{long} = \frac{S \cdot \cos(\theta)}{R}$$

θ = Central angle between any 2 points on the earth.