

Report Title: Magnetically coupled circuits; Concept, analysis and applications

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To: AUM-Shell team's instructors

Date: Aug 29,2023

1.Abstract

Electric power can be transmitted by conductive connection circuits or magnetically coupled circuits. Magnetic transmission of power allows it to go through air or vacuum or any other material which solves a lot of problems caused by wiring connections.

This report provides some information about magnetically coupled circuits, the theory of operation, how to analyze them and what is their applications in the real life.

2.Introduction

When two circuits affects each other without wiring connection between them, this called magnetically circuits. The relation between magnetism and electricity was proposed firstly by a Danish physicist called H.C. Oersted. He showed that an electric current a magnetic field. Then the French scientist Andre Marie Ampere expressed this relationship in a formulation called Amperes Law. After that , the English scientist Faraday showed that the electricity can be generated by an electric field.

The magnetic field can be produced by electric current going through a conductor. Also if a changing magnetic field cuts a conductor, a voltage will be induced according Faraday's law.

$$e = - \frac{d\Phi}{dt}$$

, Where e is the induced voltage, $d\Phi/dt$ is the rate of change of magnetic flux with respect to time.

Magnetic fields allow energy to be transferred form one circuit to another, the quantities used to express the strength of the magnetic field are the magnetic flux (WB), the magnetic flux density **B** (Tesla) and the magnetic field intensity **H** (A/m).

The flux of a magnetic circuit should go in a closed path starting from the north pole and ends at the south pole.

The driving force of the Magnetic flux is called the magnetomotive force F and it depends on the number of turns of the coil and the electric current passing through it.

3. Laws and concepts of Magnetism

3.1 Faraday's Laws

Faraday's laws of electromagnetism consists of two laws. The first one describes the induced emf in an electric conductor and the second one determines the quantity of that induced emf. The first Law states that:

Whenever a conductor is placed in a varying magnetic field, an electromotive force is induced. If the conductor circuit is closed, a current is induced, which is called induced current. (BYJu's, n.d.).

Faraday's second law of electromagnetic induction states that

The induced emf in a coil is equal to the rate of change of flux linkage. (BYJu's, n.d.)

$$e = -\frac{d\Phi}{dt}$$

3.2 Ampere's Law

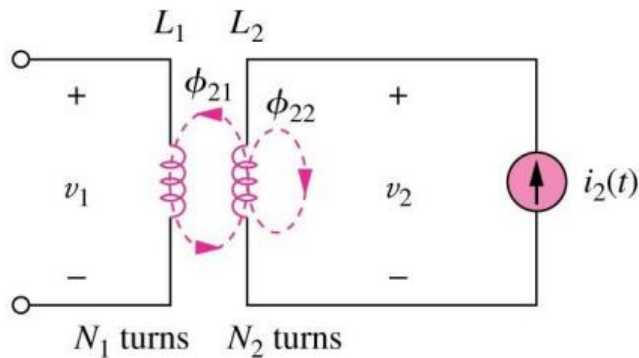
Ampere's Law states that the magnetic field intensity H in the vicinity of a conductor is related to the current carried by the conductor; thus Ampere's law establishes the relationship with Faraday's law.

Mathematically, it states that the integral of the vector magnetic field intensity H around a closed path is equal to the total current linked by the closed path i :

$$\oint \mathbf{H} \cdot d\mathbf{L} = I_{enc}$$

(Rizzoni)

3.3 Self- and Mutual Inductance



The figure shows two coils, one of

Figure 3.3.1 Mutual inductance

Which, \$L_2\$, is excited by a current \$i_2\$ therefore a magnetic field is developed and a voltage \$v_2\$ is induced. The second coil, \$L_1\$, links some of the flux generated by current \$i_2\$ around \$L_2\$ because of its close proximity to the first coil. This magnetic coupling between the two coils is called mutual inductance and defined by the symbol \$M\$.

$$V_1 = M \frac{di_2}{dt}$$

\$M\$ is defined as the ability of one inductor to induce a voltage across a neighboring inductor, measured in henrys (H)

Coupling coefficients

The coupling coefficient is used to measure the coupling between two coils. The coupling between two coils is mathematically given using the following equation,

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

where \$k\$ is the coupling coefficient, \$M\$ is the mutual inductance, \$L_1\$ is the self-inductance of the first coil, and \$L_2\$ is the self-inductance of the second coil. The values of \$k\$ vary between 0 to 1. The values of the coupling coefficient and their representations are as follows -

- \$k = 0\$ means that the coils are not coupled.
- \$k = 1\$ means that the coils are perfectly coupled.
- \$k < 0.5\$ means that the coils are loosely coupled.
- \$k > 0.5\$ means that the coils are tightly coupled.

(bartleby, n.d.)

4. Magnetically coupled circuits' analysis

Inductance is the parameter that relates a voltage to a time-varying current in the same circuit; thus, inductance is more precisely referred to as self-inductance. (Nilsson)

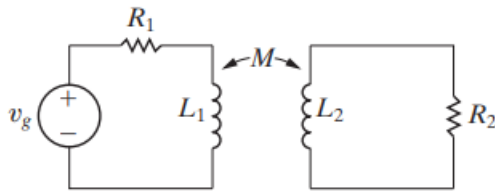


Figure 4.1

The easiest way to analyze circuits containing mutual inductance is to use mesh currents. The problem is to write the circuit equations that describe the circuit in terms of the coil currents. First, choose the reference direction for each coil current.

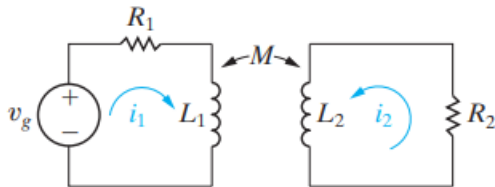


Figure 4.2

Figure 4.2 shows arbitrarily selected reference currents. After choosing the reference directions for and sum the voltages around each closed path. Because of the mutual inductance M , there will be two voltages across each coil, namely, a self-induced voltage and a mutually induced voltage. The self induced voltage is the product of the self-inductance of the coil and the first derivative of the current in that coil. The mutually induced voltage is the product of the mutual inductance of the coils and the first derivative of the current in the other coil. Consider the coil on the left in Fig. 6.20 whose self-inductance has the value L_1 . The self-induced voltage across this coil is $L_1 \frac{di_1}{dt}$ and the mutually induced voltage across this coil is $M \frac{di_2}{dt}$. But what about the polarities of these two voltages? Using the passive sign convention, the self-induced voltage is a voltage drop in the direction of the current producing the voltage. But the polarity of the mutually induced voltage depends on the way the coils are wound in relation to the reference direction of coil currents. In general, showing the details of mutually coupled windings is very cumbersome. Instead, we keep track of the polarities by a method known as the dot convention, in which a dot is placed on one terminal of each winding, as shown in Fig. 4.3. These dots carry the sign information and allow us to draw the coils schematically rather than showing how they wrap around a core structure. The rule for using the dot convention to determine the polarity of mutually induced voltage can be summarized as follows: When the reference direction for a current enters the dotted

terminal of a coil, the reference polarity of the voltage that it induces in the other coil is positive at its dotted terminal. Or, stated alternatively, When the reference direction for a current leaves the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is negative at its dotted terminal.

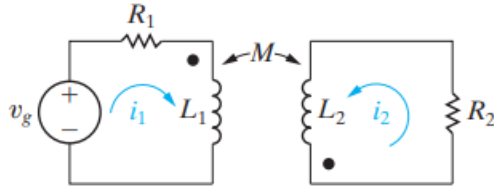


Figure 4.3

(Nilsson)

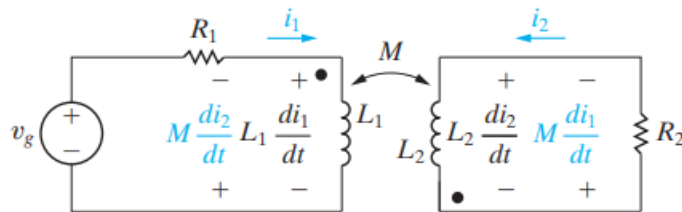


Figure 4.4

the sum of the voltages around each closed loop. In Eqs. 4. 1 and 4.2, voltage rises in the reference direction of a current are negative:

Equation 4.1 & 4.2

$$-v_g + i_1 R_1 + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} = 0,$$

$$i_2 R_2 + L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} = 0.$$

Energy Calculations

For linear magnetic coupling,

(1) $M_{12} = M_{21} = M$, and (2) $M = k\sqrt{L_1 L_2}$, where $0 \leq k \leq 1$.

We use the circuit shown in Fig. 6.30 to derive the expression for the total energy stored in the magnetic fields associated with a pair of linearly coupled coils. We begin by assuming that the currents and are zero and that this zero-current state corresponds to zero energy stored in the coils. Then we let increase from zero to some arbitrary value

and compute the energy stored when Because the total power input into the pair of coils is and the energy stored is

Equation 4.3

$$\int_0^{W_1} dw = L_1 \int_0^{I_1} i_1 di_1,$$

$$W_1 = \frac{1}{2} L_1 I_1^2.$$

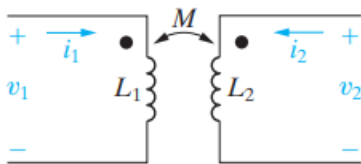


Figure 4.5

Now we hold constant at and increase from zero to some arbitrary value During this time interval, the voltage induced in coil 2 by is zero because is constant. The voltage induced in coil 1 by is Therefore, the power input to the pair of coils is

$$p = I_1 M_{12} \frac{di_2}{dt} + i_2 v_2.$$

The total energy stored in the pair of coils when is

Equation 4.4

$$\int_{W_1}^W dw = \int_0^{I_2} I_1 M_{12} di_2 + \int_0^{I_2} L_2 i_2 di_2,$$

or

$$\begin{aligned} W &= W_1 + I_1 I_2 M_{12} + \frac{1}{2} L_2 I_2^2, \\ &= \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + I_1 I_2 M_{12}. \end{aligned}$$

If we reverse the procedure—that is, if we first increase from zero to I_2 and then increase i_1 from zero to I_1 , the total energy stored is

Equation 4.5

$$W = \frac{1}{2}L_1I_1^2 + \frac{1}{2}L_2I_2^2 + I_1I_2M_{21}.$$

5. Applications of Magnetically coupled circuits

The Magnetically coupled circuits is widely used in so many applications. One of the most common structures in everyday application is the transformer figure5.1. A **transoformer** is a device which can step an AC voltage up and down by a fixed ratio, with a corresponding decrease or increase in current (Rizzoni)

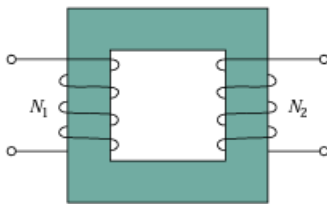


Figure 5.1

A simple transformer consists of two coils of wire wound around an iron core. The input side/coil of the transformer is called the primary winding, and has N_p number of turns. The output side/coil of the transformer is called the secondary winding, and has N_s number of turns. (electronic reference , n.d.)

Another electromechanical device that finds common applications in industrial practice is the **relay**. A relay is an electromechanical switch that allows to open and close the electrical contact by means of an electromagnetic structure. (Rizzoni)

Another important class of electromechanical transducers is that of moving-coil transducers. This class of transducers includes a number of common devices such as microphones, loudspeakers, doorbell figure 5.2 and all electric motors and generators. (Rizzoni)

The basic principle of operation of electromechanical transducers is that a magnetic field exerts a force on a charge moving through it.

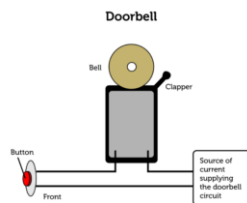


Figure 5.2

Summary

- Magnetically coupled circuits are those which depends on energy transferring in the magnetic field between two different circuits or a magnetic source and an electric circuit.
- The analysis and principles of electromagnetism was introduced by some famous scientists like Faraday and Ampere.
- Because of the properties of the magnetic circuits, they provide so many important applications like transformers, relays and electric machines.

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