Exercises week 1

Overview

Week 1: Introduction to circuit quantization and the transmon

Keywords: Qubit basics, circuit quantization, transmon, anharmonicity.

[1, Chapters: I, IIa-b, Fig. 14(a-c)] Philip Krantz, Morten Kjaergaard, Fei Yan, Terry P. Orlando, Simon Gustavsson, and William D. Oliver. A Quantum Engineer's Guide to Superconducting Qubits. *Applied Physics Reviews*, 6(2):021318, June 2019. arXiv: 1904.06560

[2, Chapters: 1-2.1, 3.1.1-3.1.2] Uri Vool and Michel Devoret. Introduction to quantum electromagnetic circuits. *International Journal of Circuit Theory and Applications*, 45(7):897–934, jun 2017

[3, Chapters: I-II, VI] Jens Koch, Terri M. Yu, Jay Gambetta, A. A. Houck, D. I. Schuster, J. Majer, Alexandre Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf. Charge-insensitive qubit design derived from the Cooper pair box. *Physical Review A*, 76(4):042319, October 2007

E1

Exercises concerning [1]:

(a) Derive equations (5) - (9) and (13) as detailed in the text.

(b) Derive ω_r as found in Eq. (14) and the text above. A trick is to compare Eq. (13) to the standard harmonic oscillator,

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega_r^2 x^2,$$

and isolate ω_r.

E2

Exercises concerning [1] and Fig. 1:

(a) Derive Eq. (24) from the diagram in Fig. 1.

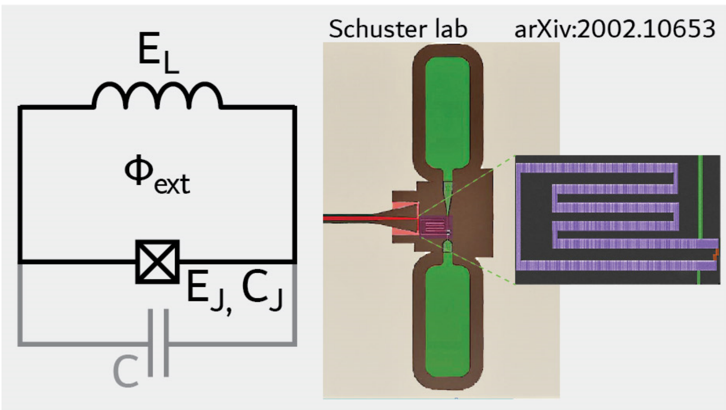

Figure 1: Diagram and image of a fluxonium qubit.

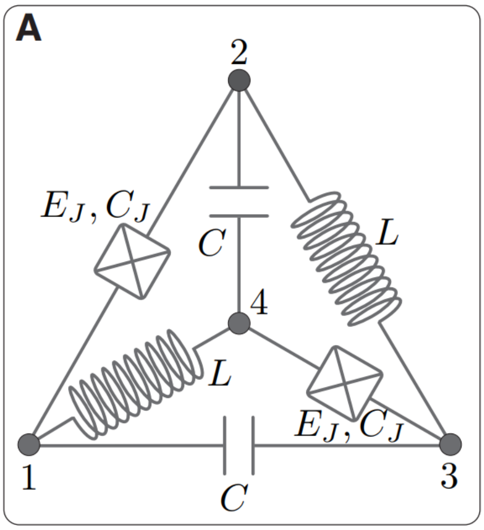

Figure 2: An advanced circuit adapted from Gyenis et al.

(b) The inductive element typically consists of an array of large Josephson junctions as shown in purple in Fig. 1. Draw a diagram of N identical Josephson junctions, each with Josephson energy γE_J, connected in series with a total phase drop ϕ across the entire array of junctions.

Assume the "distributed phase approximation" which says that the phase drop across each junction in the array is ϕ/N and derive $E_L = E_J\gamma/N$ as in the text surrounding Eq. (24).

(c) Expand the cosine in Eq. (24) at half flux bias ($\phi_e = \pi$) to fourth order. What should γ/N be for the quadratic term to vanish?

(d) Plot the potential for $\gamma/N < 1$, $\gamma/N = 1$ and $\gamma/N > 1$. Describe qualitatively how the qubit states are different in the three cases and how the qubit frequency and anharmonicity might change.

(e) The energies of any potential of the form $\propto \phi^\nu$ can be found in the WKB approximation,

$$\omega_n \propto (n + 1/2)^{\frac{2\nu}{\nu+2}}.$$

Compute the relative anharmonicity α_r in the case of a quartic potential and for the infinite square well (remember that the energies here scale as $(n+1)^2$). The relative anharmonicity is the anharmonicity divided by the qubit frequency $\alpha_r = \alpha/\omega_r$.

E3

Exercises concerning Fig. 2.

(a) How many loops do you find in Fig. 2 and how many independent modes do we have?

(b) Using the coordinates

$$\begin{aligned}
\theta &= \phi_2 + \phi_3 - (\phi_1 + \phi_4),\\
\phi &= \phi_1 + \phi_3 - (\phi_2 + \phi_4),\\
\zeta &= \phi_1 + \phi_2 - (\phi_3 + \phi_4),
\end{aligned}$$

show that the Hamiltonian becomes

$$H = 4E_C^\theta(n_\theta - n_g^\theta)^2 + 4E_C^\phi(n_\phi - n_g^\phi)^2 - E_J\cos(\theta)\cos(\phi + \phi_e) + \frac{1}{2}E_L\phi^2 + H_\zeta.$$

What are $E_C^{\theta/\phi}$, E_L and H_ζ?

*Hint: It can be useful to express the branch variables in terms of the modes given above, for example $\phi_1 - \phi_2 = (\phi - \theta)/2$.

(c) Given that C_J is small and C is large, which modes are "heavy" and which are "light"?

(d) Plot the potential in ϕ and θ for small E_L for zero and half flux bias. Do you think that the qubit states will be qualitatively different in the two cases?

References

[1] Philip Krantz, Morten Kjaergaard, Fei Yan, Terry P. Orlando, Simon Gustavsson, and William D. Oliver. A Quantum Engineer's Guide to Superconducting Qubits. *Applied Physics Reviews*, 6(2):021318, June 2019. arXiv: 1904.06560.

[2] Uri Vool and Michel Devoret. Introduction to quantum electromagnetic circuits. *International Journal of Circuit Theory and Applications*, 45(7):897–934, jun 2017.

[3] Jens Koch, Terri M. Yu, Jay Gambetta, A. A. Houck, D. I. Schuster, J. Majer, Alexandre Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf. Charge-insensitive qubit design derived from the Cooper pair box. *Physical Review A*, 76(4):042319, October 2007.