Exercises week 5

Overview

Week 5: Two-qubit gates (transmon)

Keywords: iSWAP, CPHASE, tunable coupler.

[1, Chapters: IIC, IVE-F, Fig. 18] Philip Krantz, Morten Kjaergaard, Fei Yan, Terry P. Orlando, Simon Gustavsson, and William D. Oliver. A Quantum Engineer's Guide to Superconducting Qubits. *Applied Physics Reviews*, 6(2):021318, June 2019. arXiv: 1904.06560

[2] Fei Yan, Philip Krantz, Youngkyu Sung, Morten Kjaergaard, Daniel L. Campbell, Terry P. Orlando, Simon Gustavsson, and William D. Oliver. Tunable coupling scheme for implementing high-fidelity two-qubit gates. *Phys. Rev. Applied*, 10:054062, Nov 2018

[3, Abstract, Introduction, Figures and Conclusion] Youngkyu Sung, Leon Ding, Jochen Braumüller, Antti Vepsäläinen, Bharath Kannan, Morten Kjaergaard, Ami Greene, Gabriel O. Samach, Chris McNally, David Kim, Alexander Melville, Bethany M. Niedzielski, Mollie E. Schwartz, Jonilyn L. Yoder, Terry P. Orlando, Simon Gustavsson, and William D. Oliver. Realization of High-Fidelity CZ and Z Z -Free iSWAP Gates with a Tunable Coupler. *Physical Review X*, 11(2):021058, June 2021

E1

Exercises concerning Ref. [1]:

(a) Draw a diagram similar to Fig. 3(a) and label the nodes as we learned in week 1.

(b) Derive Eq. (27) starting from the Lagrangian and taking the limit $C_g \ll C_1, C_2$.

(c) Continue to derive Eq. (31) and (32). Remember the coefficients left out in $n \propto i(a - a^\dagger)$ and $\phi \propto (a + a^\dagger)$.

(d) Do you recover Eq. (105)?

E2

Exercises concerning Ref. [1]:

(a) Consider the product state

$$|a\rangle \otimes |b\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 b_1 \\ a_1 b_2 \\ a_2 b_1 \\ a_2 b_2 \end{pmatrix}.$$

Compare it to another state $|c\rangle = (c_1, c_2, c_3, c_4)^T$ which you do not know whether is a product state or an entangled state. Show that if $c_1 c_4 = c_2 c_3$, then $|c\rangle$ is a product state.

(b) The iSWAP-gate in Eq. (110) is maximally entangling and the $\sqrt{\text{iSWAP}}$ is only "half" entangling, in the sense that you need two $\sqrt{\text{iSWAP}}$-gates in order to fully entangle two qubits. Using a trial product state $|PS\rangle = \frac{1}{\sqrt{2}}(1,1)^T \otimes \frac{1}{\sqrt{2}}(1,1)^T = \frac{1}{2}(1,1,1,1)^T$, show using the rule in (a) that iSWAP $|PS\rangle$ and $\sqrt{\text{iSWAP}}\,|PS\rangle$ are not product states.

(c) Using the results from (a) and (b), can you conjecture a way to calculate a entanglement measure that works for the iSWAP-gate family in Eq. (109)?

(d) Test you conjecture, or alternatively, your approach in (b), to check if the CPHASE and CNOT-gates in Eqs. (63) and (64) are entangling. If they are not entangling, can you find another trial product state that becomes entangled by the gate?

E3

Exercises concerning Ref. [2]. In this exercise, we will make the simplest numerical implementation of a two-qubit gate using the tunable coupler.

(a) Reproduce Fig. 2(b) using the parameters in the figure text and Eq. (5).

(b) Using Eq. (5) as the effective qubit-qubit coupling in Eq. (2), make a numerical integration of the Schrödinger equations that simulate an iSWAP-gate. How do you like to visually represent what your gate does? Make a figure that you would use in a paper and write an appropriate caption.

You can consult the text for inspiration to the simulation. For example, how do they tune the effective coupling/the coupler frequency?

References

[1] Philip Krantz, Morten Kjaergaard, Fei Yan, Terry P. Orlando, Simon Gustavsson, and William D. Oliver. A Quantum Engineer's Guide to Superconducting Qubits. *Applied Physics Reviews*, 6(2):021318, June 2019. arXiv: 1904.06560.

[2] Fei Yan, Philip Krantz, Youngkyu Sung, Morten Kjaergaard, Daniel L. Campbell, Terry P. Orlando, Simon Gustavsson, and William D. Oliver. Tunable coupling scheme for implementing high-fidelity two-qubit gates. *Phys. Rev. Applied*, 10:054062, Nov 2018.

[3] Youngkyu Sung, Leon Ding, Jochen Braumüller, Antti Vepsäläinen, Bharath Kannan, Morten Kjaergaard, Ami Greene, Gabriel O. Samach, Chris McNally, David Kim, Alexander Melville, Bethany M. Niedzielski, Mollie E. Schwartz, Jonilyn L. Yoder, Terry P. Orlando, Simon Gustavsson, and William D. Oliver. Realization of High-Fidelity CZ and Z Z -Free iSWAP Gates with a Tunable Coupler. *Physical Review X*, 11(2):021058, June 2021.