Exercises week 3

Overview

Week 3: Single qubit gates

Keywords: Microwave control, DRAG, universal gate set, fidelity.

[1, Chapters: IVA-D] Philip Krantz, Morten Kjaergaard, Fei Yan, Terry P. Orlando, Simon Gustavsson, and William D. Oliver. A Quantum Engineer's Guide to Superconducting Qubits. *Applied Physics Reviews*, 6(2):021318, June 2019. arXiv: 1904.06560

[2] F. Motzoi, J. M. Gambetta, P. Rebentrost, and F. K. Wilhelm. Simple pulses for elimination of leakage in weakly nonlinear qubits. *Phys. Rev. Lett.*, 103:110501, Sep 2009

E1

Exercises concerning [1].

(a) Derive Eqs. (84), (89) - (94) starting from Eq. (79) as detailed in the text.

(b) Assume that you choose a phase offset $\phi = 0$ and a pulse envelope $s(t)$ giving $\Theta = \pi$, meaning that the resulting gate is a σ_x. Show that performing a Z_θ-gate, $e^{-i\theta\sigma_z/2}$, after the σ_x gives the same result as driving a pulse with the same pulse envelope $s(t) \rightarrow \Theta = \pi$ but with a phase offset ϕ.

E2

Let's make a little piece of code that computes the eigenvalues and eigenstates of the tunable transmon. This time we do it in the charge basis and you might like to consult Ref. [3]. The charge basis is often much more efficient to work with compared to the flux basis.

(a) Define sparse matrices for Eqs. (15) and (18). You can set the charge cutoff to $n_{cutoff} = 10$.

(b) Define a function that returns the Hamiltonian in Eq. (22) from Ref. [1] as a sparse matrix. Like last week, the function should take inputs such as the Josephson energies, charging energy, charge offset and external flux.

(c) Compute the eigenenergies and eigenfunctions. Plot the three lowest eigenfunctions. As a rule of thumb, we are typically only interested in the few lowest eigenfunctions and as long as the wavefunctions are contained in the charge window $[-n_{cutoff}; n_{cutoff}]$, the numerics are precise despite the relatively small cutoff. Do you think that $n_{cutoff} = 10$ is appropriate now? Adjust it accordingly and make sure that the eigenenergies converge.

(d) Compute the Fourier transform of the three lowest eigenstates using Eq. (21), plot them, and compare them to what you found last week in the flux basis.

E3

Let's make some gates!

(a) To simplify, we will only consider the three lowest eigenstates of the transmon (please use the charge basis). By extracting the eigenenergies, we can construct the qubit Hamiltonian,

$$H_Q = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \omega_1 - \omega_0 & 0 \\ 0 & 0 & \omega_2 - \omega_0 \end{pmatrix}.$$

Next is the coupling Hamilton with elements $H_C^{ij} = \langle\psi_i|\,\hat{n}\,|\psi_j\rangle$ (see e.g. Eq. (76) in Ref. [1]) where $\hat{n}$ is the charge operator (really, it is just the number operator which is related to the proper charge operater $\hat{q} = 2e\hat{n}$, but the name is used interchangably).

Compute H_Q and show that

$$H_C \approx \begin{pmatrix} 0 & g_1 & 0 \\ g_1 & 0 & g_2 \\ 0 & g_2 & 0 \end{pmatrix}.$$

(b) Write a small function that is the pulse envelope $s(t)$. It could be a Gaussian, a flat top pulse with a $\sin^2(t/T_{ramp})$ ramp up/down or something else.

(c) We can now put everything together and construct the full time dependent Hamiltonian $H(t) = H_Q + As(t)\cos((\omega_1 - \omega_0)t)H_C$ where A determines the rotation angle Θ. Note that for a π-pulse, $A = \pi/(2g_1 \int \mathrm{d}t\, s(t))$.

(d) Use an ODE-solver to solve the time dependent Schrödinger equation with $H(t)$, using the ground state as the initial state and attempt to make a σ_x gate. Plot $|\langle\psi_1|\psi(t)\rangle|^2$ as a function of time. You may intepret $|\langle\psi_1|\psi(T)\rangle|^2$ as the gate fidelity in this case.

(e) Try to implement a DRAG pulse and make a plot with the DRAG scaling parameter λ on the x-axis and the gate fidelity on the y-axis. What is the optimal λ?

References

[1] Philip Krantz, Morten Kjaergaard, Fei Yan, Terry P. Orlando, Simon Gustavsson, and William D. Oliver. A Quantum Engineer's Guide to Superconducting Qubits. *Applied Physics Reviews*, 6(2):021318, June 2019. arXiv: 1904.06560.

[2] F. Motzoi, J. M. Gambetta, P. Rebentrost, and F. K. Wilhelm. Simple pulses for elimination of leakage in weakly nonlinear qubits. *Phys. Rev. Lett.*, 103:110501, Sep 2009.

[3] Philipp Aumann, Tim Menke, William D. Oliver, and Wolfgang Lechner. CircuitQ: An open-source toolbox for superconducting circuits. *arXiv:2106.05342 [quant-ph]*, June 2021. arXiv: 2106.05342.