# Exercises week 2

## Overview

#### Week 2: Numerical experiments and decoherence

Keywords: Phase/charge basis, flux/charge decoherence,  $T_1$ ,  $T_2$ , noise power spectral density.

- [1, Chapters: I-IIIA] Philipp Aumann, Tim Menke, William D. Oliver, and Wolfgang Lechner. CircuitQ: An open-source toolbox for superconducting circuits. arXiv:2106.05342 [quant-ph], June 2021. arXiv: 2106.05342
- [2, Chapter: III] Philip Krantz, Morten Kjaergaard, Fei Yan, Terry P. Orlando, Simon Gustavsson, and William D. Oliver. A Quantum Engineer's Guide to Superconducting Qubits. *Applied Physics Reviews*, 6(2):021318, June 2019. arXiv: 1904.06560
- [3, Chapters: 4, App. A] Peter Groszkowski, A Di Paolo, A L Grimsmo, A Blais, D I Schuster, A A Houck, and Jens Koch. Coherence properties of the 0-  $\pi$  qubit. New Journal of Physics, 20(4):043053, April 2018

#### E1

Exercises concerning [1]. We start by working in the flux basis, which is the most intuitive. Our object is to find and plot the energies and eigenstates of the tunable transmon, i.e. Eq. (22) in Ref. [2]. You can use whatever programming language you are comfortable with (Python, Julia, etc.).

- (a) Define a sparse matrix that is the elementwise cosine of Eq. (9). Since the coordinate is periodic we define  $\phi_{max} = \pi$ . Sparse matrices can yield significant speed-ups for these kinds of problems.
- (b) Similarly, define sparse matrices for Eq. (10) and (11). Naturally, we take  $\hbar = 1$ . Note that due to the periodicity of the potential, we should include additional elements in the upper right and lower left corners.
- (c) Write a function that builds the Hamiltonian in Eq. (22) from Ref. [2]. It should take inputs such as the Josephson energies, charging energy, charge offset and external flux and return the Hamiltonian in the flux basis as a sparse matrix.
- (d) Compute the eigenvalues and eigenstates, and plot the potential, energies and eigenstates to reproduce something like Fig. 1a from [1].

#### E2

Exercises concerning [2]. We learn that there are many different noise sources. Let's focus on relaxation due to flux noise and charge noise. Specifically, let's focus on 1/f flux noise, 1/f charge noise, and ohmic charge noise for the tunable transmon.

(a) Write a function (or three functions - as you wish) which computes the above-mentioned relaxation times. You may use the following: The spectral functions are

$$S_{\lambda}^{\frac{1}{f}}(\omega) = \frac{2\pi A_{\lambda}^{2} \text{Hz}}{|\omega|}, \quad S_{\lambda}^{\Omega}(\omega) = \frac{B_{\lambda}^{2} \omega}{2\pi \times 1 \text{GHz}}, \tag{0.1}$$

- where  $A_{\lambda}$  and  $B_{\lambda}$  are noise amplitudes for 1/f and ohmic noise respectively. We use typical noise amplitudes  $A_{\Phi} = 10^{-6} \Phi_0 / \sqrt{\text{Hz}}$  [3],  $A_{n_g} = 10^{-4} e / \sqrt{\text{Hz}}$  [3] and  $B_{n_g} = 5.2 \times 10^{-9} e / \sqrt{\text{Hz}}$  [4].
- (b) Plot the 1/f flux relaxation time as a function of external flux. Do you see the sweet-spots? Should you?
- (c) Plot the 1/f and ohmic charge relaxation times as a function of  $E_J/E_C$ . Do you see the exponential dependence on  $E_J/E_C$ ? Should you?

### E3

Exercises concerning [3]. We now turn to 1/f dephasing.

- (a) We may compute the 1/f dephasing times from Eq. (13). Write a function that does that for the tunable transmon for flux and charge noise. We should choose some values for the infrared and ultraviolet frequency cutoffs. Typically  $\omega_{uv} \gg \omega_{ge}$  and  $\omega_{ir} \ll 1/t \sim 1/T_2$  so if we expect  $\omega_{ge} \sim 1 \text{GHz}$  and  $T_2 \sim 10 \mu \text{s}$ , then we might choose  $\omega_{uv} = 10^3 \text{GHz}$ ,  $\omega_{ir} = 10 \text{Hz}$  and  $t \sim T_2 \sim 10 \mu \text{s}$ . The dependence on these parameters is very weak, so their exact values are not so important.
- (b) Plot the 1/f flux dephasing time as a function of external flux. Do you see the sweet-spots? Should you?
- (c) Plot the 1/f charge dephasing time as a function of  $E_J/E_C$ . Do you see the exponential dependence on  $E_J/E_C$ ? Should you?

### $\mathbf{E4}$

Imagine you are writing a research paper and you want to describe the following question:

Consider a tunable (i.e. split) transmon with Josephson energies  $E_J$  and  $0.8E_J$  with  $E_C$  set by  $E_J/E_C = 50$  and that you operate it at zero external flux bias. How will the coherence of the qubit change, if you increase the flux in the loop by a quarter of a flux quantum?

This question is intentionally vague, tricky and open-ended to emulate the kind of questions you might pose to yourself when doing research. You may of course use any numerical and analytical tool to help yourself analyze the problem before writing an answer.

#### References

- [1] Philipp Aumann, Tim Menke, William D. Oliver, and Wolfgang Lechner. CircuitQ: An open-source toolbox for superconducting circuits. arXiv:2106.05342 [quant-ph], June 2021. arXiv: 2106.05342.
- [2] Philip Krantz, Morten Kjaergaard, Fei Yan, Terry P. Orlando, Simon Gustavsson, and William D. Oliver. A Quantum Engineer's Guide to Superconducting Qubits. Applied Physics Reviews, 6(2):021318, June 2019. arXiv: 1904.06560.
- [3] Peter Groszkowski, A Di Paolo, A L Grimsmo, A Blais, D I Schuster, A A Houck, and Jens Koch. Coherence properties of the 0- π qubit. New Journal of Physics, 20(4):043053, April 2018.

[4] Fei Yan, Simon Gustavsson, Archana Kamal, Jeffrey Birenbaum, Adam P Sears, David Hover, Ted J. Gudmundsen, Danna Rosenberg, Gabriel Samach, S Weber, Jonilyn L. Yoder, Terry P. Orlando, John Clarke, Andrew J. Kerman, and William D. Oliver. The flux qubit revisited to enhance coherence and reproducibility. *Nature Communications*, 7(1):12964, December 2016.