EDHEC Business School

Master in Management Finance Economics

Understanding the Bond Excess Return: A Practitioner's Review

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Contents

1. Introduction

We study the time-varying bond risk premia in US zero-coupon bonds from 1971 to 2018. By the definition of carry strategy, we buy the long-term bond by selling same amount of currency of one-year maturity bond and unwind the position after holding one year.

In the unconditional study, bond excess returns can be acquired by the US zero-coupon bonds, we summary and analyze results in terms of annualized mean returns, volatility and Sharp ratio, minimum and maximum excess returns obtained by different carry strategies from 1971 to 2018.

In the conditional study. We introduce two return-predicting factors, ***Slope*** factor and **Cochrane-Piazzesi** (***CP*** in short) factor**,** and compare the results obtained by two factors and contrast the predicting ability between two factors. To the US zero-coupon bond yield curve, ***CP*** have better and more stable predicting ability to ***Slope***, while ***Slope*** can provide predicting with relatively bigger indication deviation to the carry strategies with shorter long legs and can better predict the returns by the longer legs carry strategies.

In the procedure to ***Slope*** and ***CP***, we acknowledge the look-ahead bias and propose a practical method to adjusted this issue and reducing the impact. We test the feasibility of this solution and compare the results of look-ahead adjusted with look-ahead bias ones. The applicable feature of the solution to ***CP*** and ***Slope*** is different, but the look-ahead bias solution is approved on our analysis.

In the last part, we conduct our empirical study on another physical universe by the methodology of unconditional and conditional study, we check the robustness of the carry strategy on the UK government bonds to see whether the bond risk premia can be secured and test the return-predicting ability of ***Slope*** and ***CP*** to find whether we can draw conclusions similar to the ones in the conditional study of US zero-coupon bond about which return-predicting factor have a more precise indication. To the UK government bond, similar conclusion but with difference can be reached. While it is hard to come to a conclusion that which one is better, ***CP*** is a more stable return-predicting factor, ***Slope*** approximately perform the same as it does in conditional study on the US zero-coupon bond yield curve.

2. Empirical protocol

2.1. The definition of the carry strategy and the bond excess return

The carry strategy of the bond to capture risk premium is as the following: selling 1 unit of currency of the 1-year maturity zero-coupon bond at time t, which is defined as short leg, then using this 1 unit of currency to buy one N-year maturity zero-coupon bond, and holding this bond to time $t+1$ when this bond has $(N-1)$- years left to maturity, which is defined as the long leg. At time $t+1$, unwinding the position.

The excess bond return can be deducted as the difference between long leg and short leg to capture the long term risk premium, the interest rate risk of investor's position is only exposed to the long leg. If the expectation hypothesis is true, no excess return can be secured. The short leg can be seen as the benchmark or the financing cost.

By the definition of the carry strategy, we can derivate the bond risk premia as following:

$$Exet_{t\to t+1}^{N} = ln\left(\frac{p_{t+1}^{N-1}\cdot\frac{1}{p_t^N}}{p_t^N\cdot\frac{1}{p_t^N}}\right) - ln\left(\frac{1\cdot\frac{1}{p_t^1}}{p_t^1\cdot\frac{1}{p_t^1}}\right)$$

$$= ln\left(\frac{e^{y_t^N\cdot N}}{e^{y_{t+1}^{N-1}\cdot(N-1)}}\right) + ln\left(\frac{1}{e^{y_t^1\cdot 1}}\right)$$

$$= (y_t^N - y_{t+1}^{N-1})(n-1) + (y_t^N - y_t^1) \quad \mathbf{(1)}$$

p_t^N denotes the price of a N-year maturity bond at time t, and y_t^N is its yield.

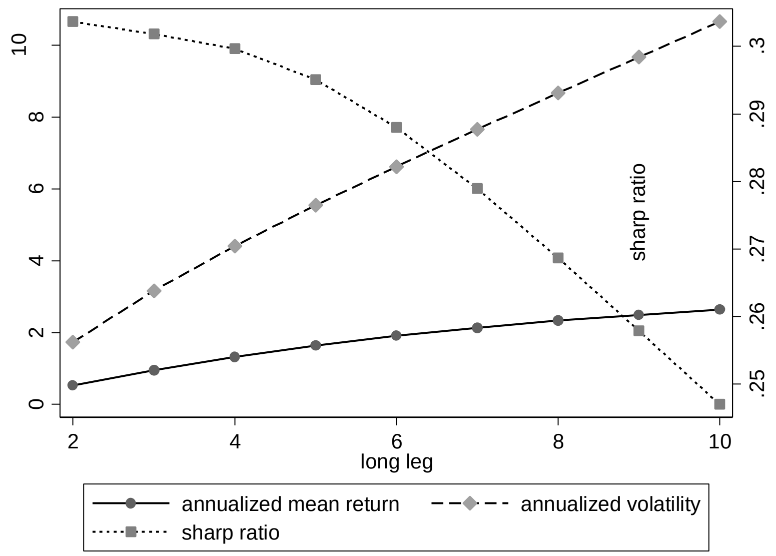
2.2. Unconditional study: excess returns by different carry strategies with long legs from 2- to 10- years with one-year zero-coupon bond

The U.S. Treasury yield curve is of tremendous importance both in concept and in practice. From a conceptual perspective, the yield curve determines the value that investors place today on nominal payment at all future dates, which is a fundamental determinant of almost all asset price and economic decisions. From the practical perspective, the U.S. bond market is the largest and most liquid bond market in the world. U.S. treasuries are extensively used to manage interest rate risk and provide a benchmark for the pricing of the other assets.

The yield curve we use is an off-the-run Treasury yield curve based on a large set of outstanding Treasury notes and bonds. The yield curve is present as the daily estimates from 1961 to 2019 for the entire maturity range spanned by outstanding treasury securities.

To compute the bond excess returns times series, we extract the sample data which is the yield curve of the zero-coupon bond with continuous compounding convention with different maturities of from 1- to 10- years from 1971 to 2018 by a monthly overlap.

By the ***equation (1)***, we can compute 9 sets time series of bond excess return by different carry strategies when the long legs equal from 2 to 10, starting from 1971 to 2018. To put it more precisely, we buy 1 dollar of N-years maturity zero-coupon bond ($N = 2, 3, \ldots, 10$) by selling the same amount of money of 1-year maturity zero-coupon bond. The overall result is as following:



Notes 1 The overall results of excess bond return of 9 long legs of different maturities bonds on 1-year maturity bond

As figure 1 above, by different carry strategy, the longer leg is, the higher the annualized mean return and the annualized volatility are, leading to the lower the Sharp ratio is, which means that investors will be exposed to the relatively higher long term duration risk. As the long leg duration increase, volatility increase faster than mean returns do, so that Sharp ratio decrease more quickly when the long leg maturity increases. In summary, the longer leg is, the more excess returns can be secured with higher volatility.

2.3. Two return-predicting factors

In the former part, we have compute the excess returns, in this part, we will calculate the return-predicting factors, which are ***Slope*** and ***Cochrane and Piazzesi*** (***CP*** in short) factor, to predict the excess return in the future, compare and analyze the results of using different factors, and contrast the predicting ability between them.

2.3.a. Cochrane and Piazzesi factor

To compute the ***CP*** factor across maturities, which is defined as the linear combination of 5 forward rates. First we run 9 sets regression of the excess returns time-series on 5 forward rates time-series as following:

$$Xret_{t\to t+1}^{N} = \beta_0 + \beta_1 f_t^2 + \beta_2 f_t^4 + \beta_3 f_t^6 + \beta_4 f_t^8 + \beta_5 f_t^{10} + \varepsilon$$

f_t^i refers to the 1-year forward rates with i years since time t. ($i = 2, 4, 6, 8, 10$)

To accomplish that, we need to calculate the forward rate time-series by the current yield curve. The yield curve can also be expressed in terms of forward rates rather than yields. A forward rate is the yield that an investor would agree to today to make an investment over a specified period in the future. These forward rate can be synthesized from the yield curve. Similar to expectation hypothesis, the long-term interest rate can be substituted by the spot short-term rate and expected interest rate. For the continuous compounded return, the forward rate can be given by the following formula:

$$f_t^i = ln\left(\frac{e^{y_t^{i+1}\cdot(i+1)}}{e^{y_t^i \cdot i}}\right) = y_t^{i+1}\cdot(i+1) - y_t^i \cdot i$$

Through the regression, we can get 9 sets of coefficients marked by different long legs maturities:

$$\boldsymbol{\gamma}^{(\boldsymbol{n})} = (\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5)^n$$

Then, we defined the vector x_t as the repressors time series:

$$\boldsymbol{x_t} = \begin{bmatrix} 1 \\ f_t^2 \\ f_t^4 \\ f_t^6 \\ f_t^8 \\ f_t^{10} \end{bmatrix}^T$$

After obtaining the corresponding individual maturity dependent return-predicting factor time-series as $\boldsymbol{\gamma}^{(\boldsymbol{n})} \cdot \boldsymbol{x_t}$, the ***Cochrane and Piazzesi*** factor is finally acquired by the averaging the 9 sets of $\boldsymbol{\gamma}^{(\boldsymbol{n})} \cdot \boldsymbol{x_t}$ time-series across long legs maturities.

2.3.b. *Slope* factor

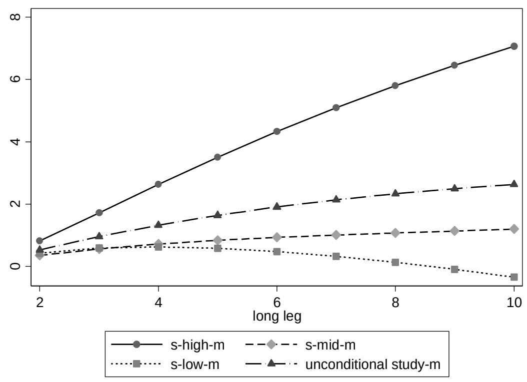
The ***Slope*** is one of the three principal yield curve factors, which is simple defined as the long term yield minus the short term yield. To capture the long term duration risk, we defined ***Slope*** factor as the difference between the yield of 10-years maturities bond and the yield of 2-year maturities bond and obtain the time-series of the ***Slope*** factor:

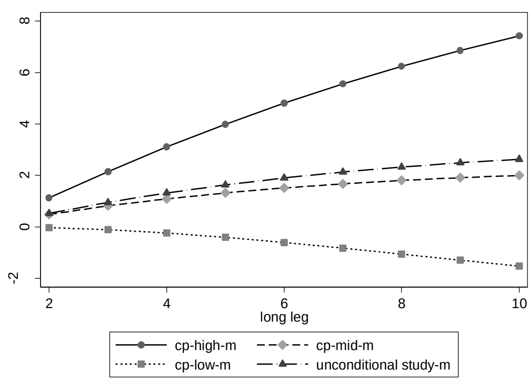
$$S_t = y_t^{10} - y_t^2$$

2.4. Conditional Study: A practical method of applying the return-predicting factors

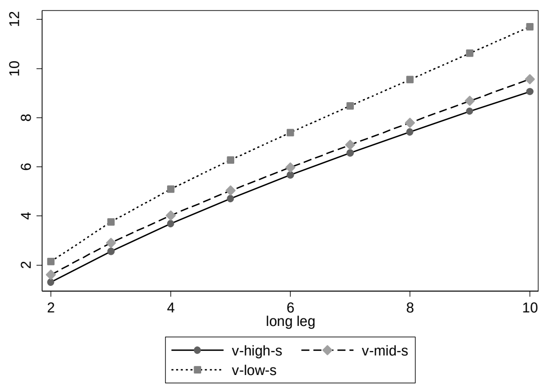
To apply the factor to predict the excess return, we allocate the excess returns series into 3 subsets, which is “low”, “middle” and “high” regimes, by the position of the factors in their time-series.

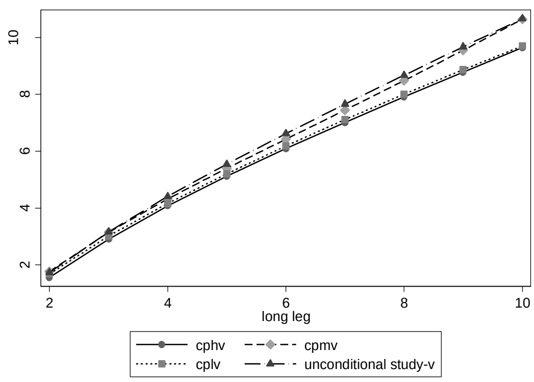
To put it more precisely, we calculate the 33th percentile and the 67th percentile value of the ***Slope*** time-series and ***Cochrane and Piazzesi*** factor time-series to split them into three tercile, if the factor is in the first tercile, then labeled the corresponding excess return as “low”, so as to divide the excess return time-series into 3 different subsets. We can simply obtain the annualized mean return, volatility and Sharp ratio of the bond excess returns time series of different carry strategies in 3 regimes assigned by ***CP*** and ***Slope***.



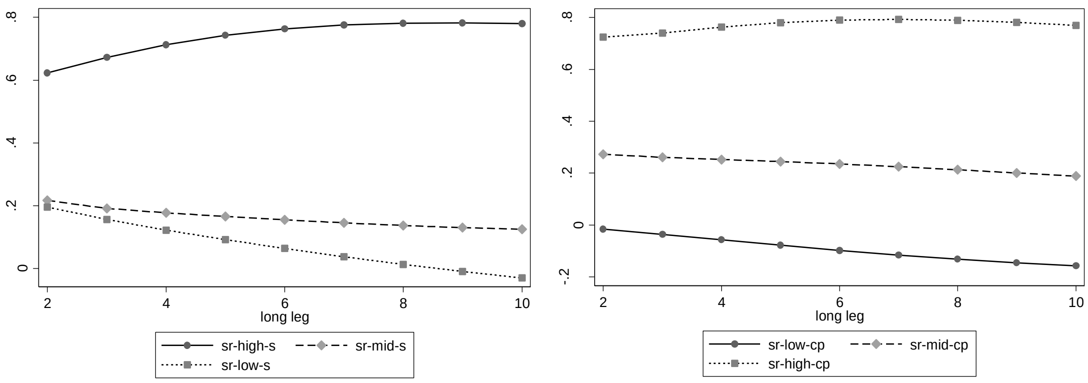


By computing annualized mean returns, volatility and Sharp ratio in each regime obtained by Slop and ***CP*** factor, the results are similar. We can see that, annualized mean returns all diverse in different regimes and mean returns in "high" regime is larger than those in the other two regimes, mean returns in "lower" regime are smaller, showing same distributing pattern. In this way, to each carry strategy, for example, the average of annualized mean returns in "low", "mid" and "high" regimes are approximately equal to the annualized mean return in the whole time series in unconditional study. Mean returns absolute value increase in all three regimes as the long leg duration increase.





As to the volatility, results obtained by different factors are very similar and increase as the long leg maturity increase.



Sharp ratio shows a dramatically difference between each regime. Sharp ratio in high regime is higher than those in the other two, “low” regime’s Sharp ratio is the lowest. The Sharp ratio in the “mid” regime obtained by ***CP*** is approximately in the middle position between “high” and “low” regime, while Sharp ratio in “mid” regime tend to be closer to those in “low” regime.

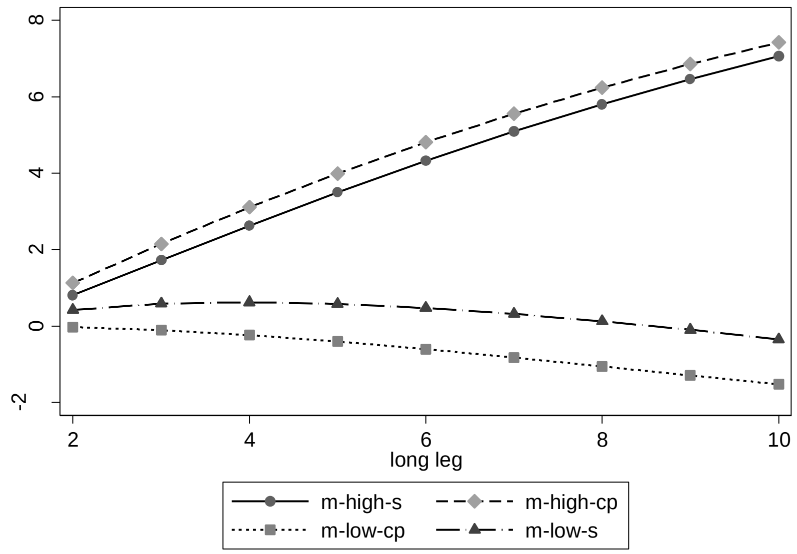
Next part we will analyze the results above obtained by different factors and compare the indicative ability of both factors.

2.5. Results analysis of conditional and unconditional study

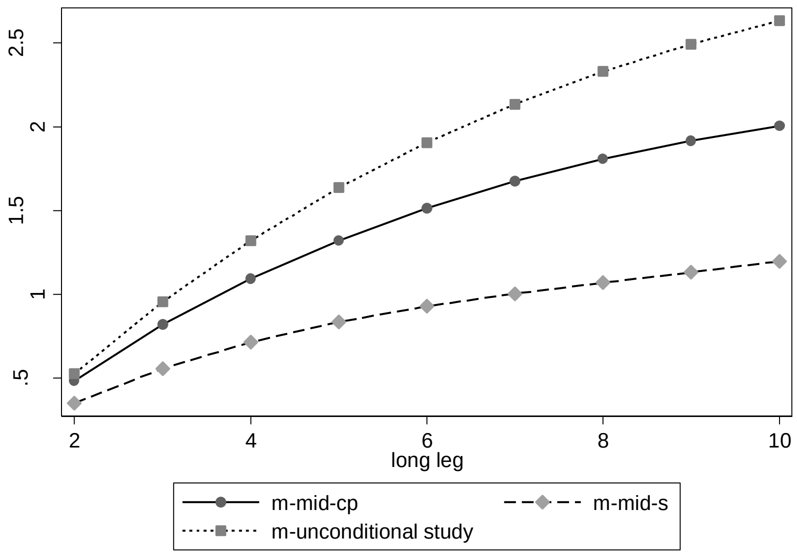
In this part, we will analyze and compare the annualized mean returns, maximum excess returns, minimum excess returns, volatility and Sharp ratio in unconditional study with those in conditional study obtained by 2 return-predicting factors.

2.5.a. ***Slope*** vs ***CP***

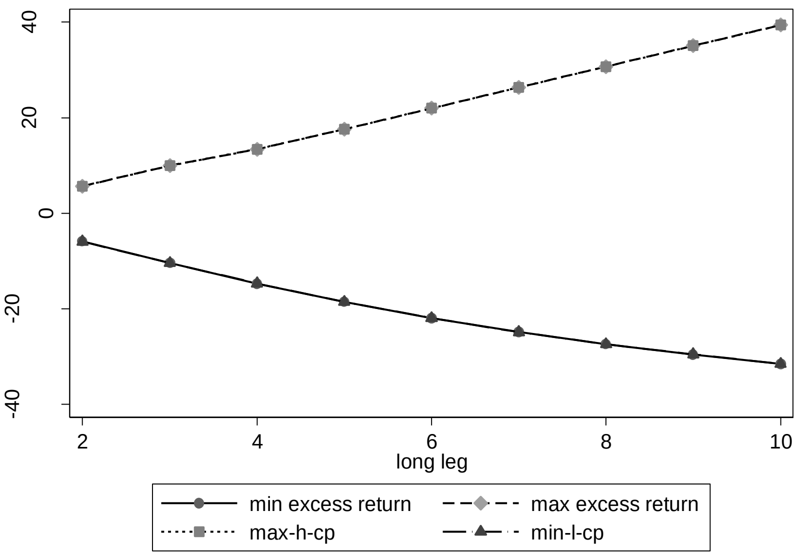
Firstly, we compare annualized mean returns, minimum and maximum excess returns of different carry strategies in different regimes assigned by ***Slope*** and ***CP*** with those in unconditional study.



In terms of predicting annualized mean returns of different carry strategies in different regimes, the indicative ability of predicting using ***CP*** factor is stronger, the ***Slope*** could always underestimate the annualized mean returns in 3 regimes than using ***CP*** factor. The mean returns of different carry strategies in the "high" regime assigned by ***CP*** is overall above those assigned by ***Slope***, meanwhile, mean returns assigned by ***CP*** into "low" regime is below those by ***Slope***.

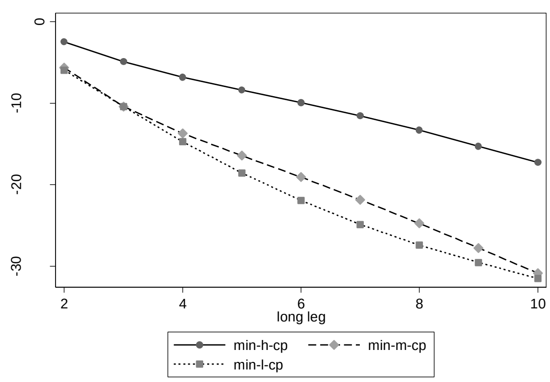


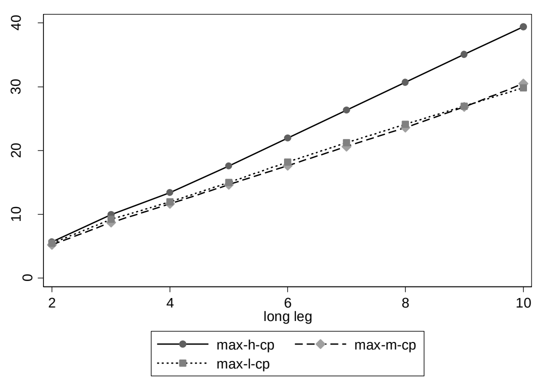
Mean returns of different carry strategies in the "mid" regime assigned by ***CP*** are much closer to the unconditional study mean returns in the whole time series than those assigned by ***Slope*** are. It seems ***CP*** factor can have a more explicit and accurate predicting ability than ***Slope*** in different regimes, which will give investors more distinct indication of returns they may obtain after 1 year if investor executes the carry strategy and acquired the factor, put it more straight forward, if both factors are in the "high" regime, "***Slope***" will tell investors that, on average, their excess returns after 1 year will be "2%", but "***CP***" will tell investors that they can get more.



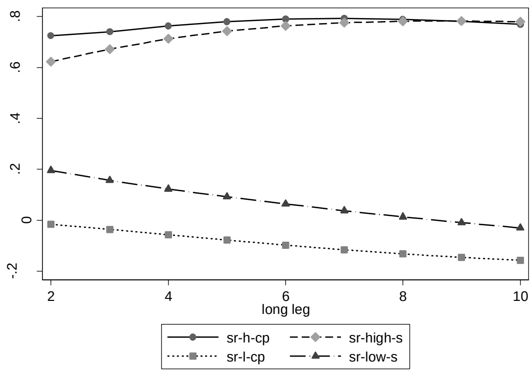
In terms of the minimum or maximum excess returns, indication of ***CP*** factor is more obvious. The maximum excess returns across different carry strategies in unconditional study are totally coincided with maximum excess returns occur in the "high" regime predicted by ***CP*** in the conditional study, same as the minimum excess returns do.

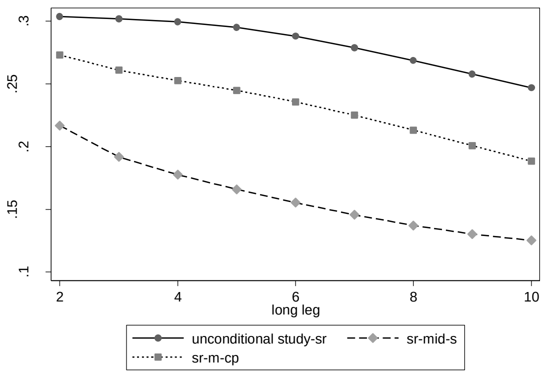
While by ***Slope*** factor, results are totally different. Maximum excess returns of the unconditional study are not superposition to the maximum excess returns in the "high" regime but occur in the "mid" regime. On the other hand, minimum excess returns randomly occur in the "low" and "mid" regime, with more frequency in the "mid" regime.



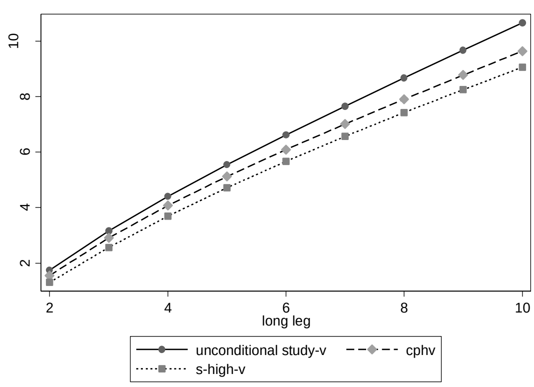


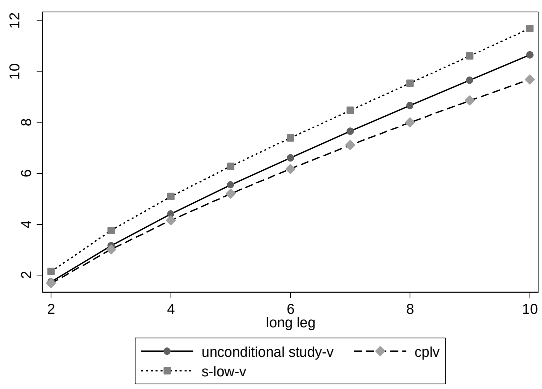
Besides, assigned by ***CP*** factor, the maximum or minimum excess returns in "high" regime are above those in the other two regimes in the same time. While by ***Slope*** factor, the results are disorder, for example, the maximum excess returns across different carry strategies in “high” regime become to the “middle” position between those in “low” and “mid” regimes.





Secondly, when comparing the Sharp ratio of different carry strategies in unconditional study with those in different regimes across different carry strategies that is assigned by ***CP*** and by ***Slope*** in conditional study, we can obtain the same conclusion as in the analysis of the annualized mean returns. The Sharp ratio are underestimated when using ***Slope***. More precisely, Sharp ratio of different carry strategies in “high” regime assigned by ***CP*** is overall higher than those by ***Slope***. Sharp ratio in “low” regimes by ***CP*** are lower than those by ***Slope***. Sharp ratio in “mid” regimes by ***CP*** is closer to those in unconditional study than Sharp ratio in “mid” regimes by ***Slope***.

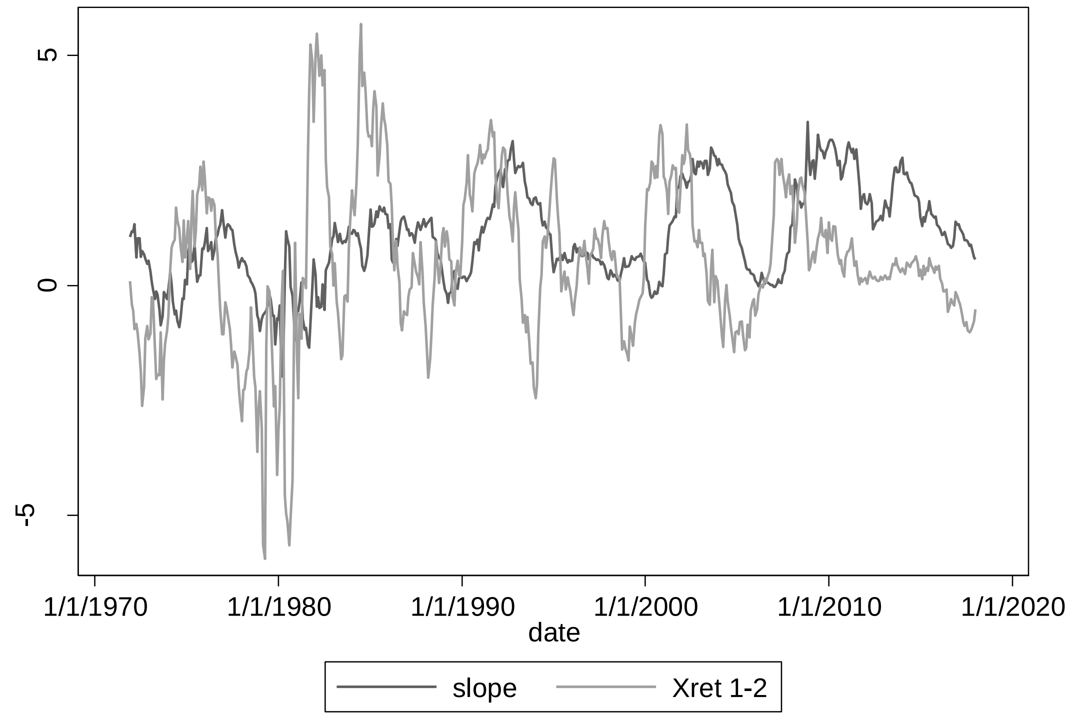


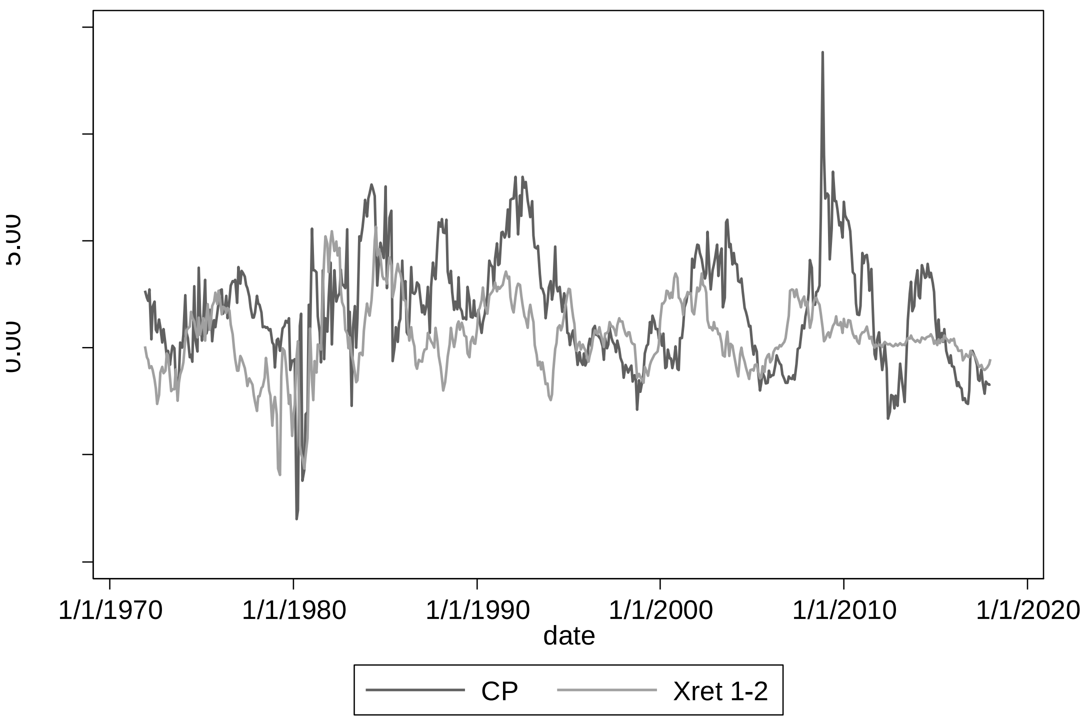


Thirdly, for volatility, the results obtained by ***Slope*** and ***CP*** share a same pattern with that in unconditional study, but the results obtained by ***CP*** in “low”, “mid” and “high” regimes are all closer to the volatility in unconditional study than the volatility obtained by ***Slope*** in 3 regimes.

Above results all show that ***CP*** factor has a more precise indicative ability to the Us zero-coupon bond yield curve.

2.5.b. Reasons behind the results





Notes 2 Xret1-2 refers to buy 2 year maturity bond and sell the same amount of 1 year maturity bond, unwind the position after holding 1 year. Two figures are fluctuation pattern of Xret1-2 with two return-predicting factors.

As the figure above, taking time series of $Xret^2_{t\to t+1}$ as an example, excess returns occur earlier than ***Slope*** but share relatively common fluctuation cycle with ***CP***.

CP factor is linear combination of 5 forward rate, which can better capture the change of excess returns than ***Slope*** does. The excess return time series of different carry strategies share a very similar fluctuation cycle and is approximately synchronized with ***CP*** time series in a same time period, with high correlation amounting to 0.32, therefore, for example, when ***CP*** is in its first tercile, the corresponding excess return is also relatively small, we allocate these small excess returns to the "low" regime so that the small excess returns which should be in the "low" regime are exactly in the "low" regime. Annualized mean returns in "low" regime obtained by ***CP*** is lower than those obtained by ***Slope***, which is much closer to the actual situation.

Contrasting with ***CP***, ***Slope*** factor may have a relatively bigger indicative deviation, more precisely, when ***Slope*** is small, the corresponding excess returns could be very large, but they are mistakenly assigned to the "low" regime, which could increase the annualized mean returns in the "low" regime, leading to the underestimated results of different carry strategies in each regime. The reason behind this situation is that ***Slope*** does not share a common fluctuation with the excess returns of different carry strategies, correlation between time series of $Xret^2_{t\to t+1}$ from 1971 to 2018 and the ***Slope*** in a same time period is 0.11, ***Slope*** tend to have a relatively bigger indicative error.

On the other hand, the correlation between time series of $Xret^2_{t\to t+1}$ from 1971 to 2017 and the ***Slope*** time series from 1972 to 2018 can be increased to 0.6, which proofs that ***Slope*** is a return-predicting factor with larger predicting-error than ***CP***. Excess return is more synchronized with the ***Slope*** after 12-month in the fluctuation pattern terms, and this property is also find in the conditional study on the UK government bond yield curve.

In conclusion, by contrast with ***CP***, ***Slope*** has a relative bigger indication-error predicting ability.

By analysis the whole sets of carry strategies, a more representative results and conclusion can be reached. In this way.

Table 1 correlation of return-predicting factor with excess returns of different carry strategies

	xret1-2	xret1-3	xret1-4	xret1-5	xret1-6	xret1-7	xret1-8	xret1-9	xret1-10
CP	0,32	0,35	0,38	0,40	0,41	0,42	0,42	0,42	0,42
Slope	0,11	0,18	0,23	0,26	0,29	0,31	0,33	0,34	0,34
Future-***Slope***	0,60	0,60	0,59	0,56	0,53	0,51	0,48	0,45	0,43

*notes 3 "Future-**Slope**" refers to the correlation between excess returns from 1971 to 2017 and **Slope** from 1972 to 2018, the **Slope** are in 12-month future time period.*

As long leg increases, the correlation between ***CP*** and excess returns of different carry strategies increases with narrow range, leading to a more stable predicting ability for ***CP*** to excess returns of different carry strategies.

While as long leg increases, correlation between ***Slope*** and excess return increases, showing that indicative deviation is becoming small and ***Slope*** can better capture the excess returns of carry strategies with longer leg, but still lower than those of ***CP***. One is worth to be mentioned is that as long leg increases, future-***Slope*** correlation decreases, showing that the property that 12-month-later synchronization is weakened, in other words, the more obvious the future synchronization is, the relatively larger predicting-error ***Slope*** can provide now. In predictive terms, if ***Slope*** want to chase ***CP***, this property is considerable.

Table 2 critical values of each tercile by different factors

	Slope	***CP***
"1st"-"2nd" tercile critical value	0,55	0,43
"2nd"-"3rd" tercile critical value	1,39	2,75

Besides, critical values of 3 terciles of ***Slope*** is included by ***CP*** as the above table shows, which also relates to the difference results obtained above by ***CP*** or by ***Slope***. For example, at same date t, ***Slope*** may be in the first tercile, while ***CP*** can be in the second tercile, so that the corresponding excess returns can be assigned to the "low" regime by ***Slope***, or assigned to the "mid" regime by ***CP***. In this way, the ***Slope*** mistakenly assigns the medium excess return, which is corresponded by ***CP*** in second tercile, to the "low" regime, so as to raise the annualized mean returns in the "low" regime. We can conduct the same analysis of the other two regimes.

In summary, ***CP*** has more stable and precise return-predicting ability to excess returns across different carry strategies, while ***Slope*** has relatively bigger indication error and can better to predict returns of carry strategy with longer legs.

2.6. Interpretation the return-predicting factors

This part is to state the interpretation of ***Slope*** and ***CP*** factor, and to reveal the economic information they may contain.

2.6.a. Cochrane and Piazzesi factor

The bond excess returns are highly correlated to the business cycle and macro economy, when the overall economy environment is in a good state, relatively larger excess returns can be secured. During the recession, overall level of interest rate tends to drop and yield curve often flattens, leading to less possibility to acquire the excess returns.

A good way to find the economic interpretation of the return-predicting factor is to build the link with the macro economy indicator such as inflation index. Since the expected inflation measures the expectations of real cash flows to bond holders, it can be a priory information to consider that contains useful information about the bond excess return, and also a significant indicator to the economy state of a country and contains very much useful information.

In recent literatures, ***Radwanski (2010)*** firstly gives an economic interpretation to the ***CP*** factor. It proposes that ***CP*** factor contains the information of inflation and the level of the forward rate of a country. More precisely, the same linear combination of five forward rate can be used to explain the inflation, and he defines it as a new factor, which is IE(inflation expectation). There are striking similarities between these IE and ***CP*** factors. The shape of the time series of ***CP*** factors is almost the mirror image of the shape of IE factors time series and their time series properties are closely related at the business-cycle frequency. Besides, the tent-shape of IE factors appears an inverted version of the tent-shape of ***CP*** factors.

By construction, ***CP*** factor can be explained by the level-of-nominal-forward rates and IE factor with high R square. In summary, the **Cochrane-Piazzesi** return-predicting factor is shown to contain the information of expected inflation and level of yield curve of a country.

2.6.b. ***Slope*** **factor**

Following the idea above, we try to find the relationship between economy and ***Slope*** trying to extract information economically it contains.

There are large body of research in this area. Professor Renonato states ***Slope*** has relatively higher correlation with output gap of a country. Output gap is defined as the estimated difference between the potential and the actual GDP. If the gap is positive, inflation intensified and economy is becoming strong, monetary authorities tend to increase rates to cool the economy, reducing the circulation of the currency, in this way, short-term interest rate will be relatively higher, the ***Slope*** will be lower. By constructing, there is 52% positive correlation between output gap time series and minus the ***Slope*** time series from 1970 to 2010 in US.

3. A practical solution to look ahead bias

We acknowledge of a look-ahead bias in the procedure of applying **Cochrane-Piazzesi and *Slope* factor**. This part raises a solution to deal with the look-ahead bias and compare and analyze the results.

3.1. look-ahead bias adjusted results to *CP* procedure and the look-ahead bias ones

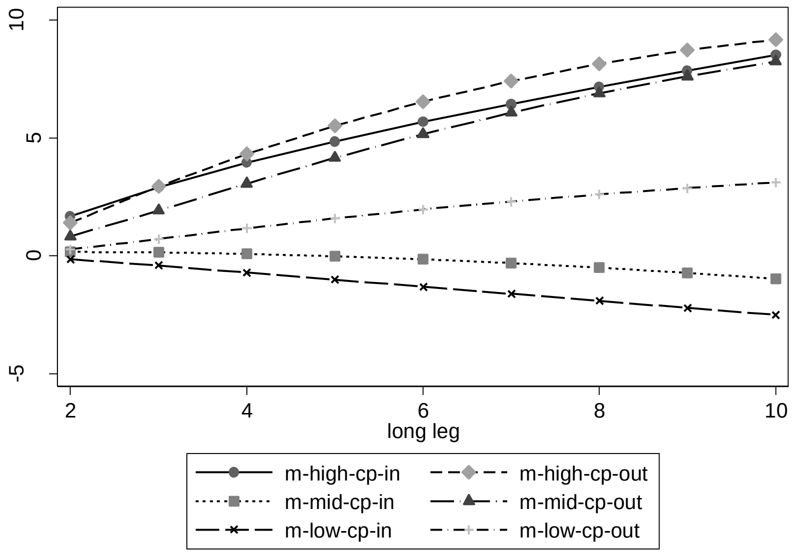
The whole time periods of 1971 to 2018 include many recessions, where the interest rates were decreased, so as to have precise and representative results, the in-sample should take a bigger window. To solve the look-ahead bias to apply ***CP*** factor, we can divide the whole time series of forward rates and bond excess returns of different carry strategies into two parts, the in-sample covers the 1971 to 2001 period and to test the out of sample that covers the time periods of 2002 to 2018. Then utilizing the out of sample ***CP*** factors to assign the corresponding excess returns to different regimes by the critical value obtained in the in-sample procedure. Finally, we will compare and analyze the in-sample and out of sample results to assess the look ahead-bias solution.

More precisely, the detailed procedure is as following.

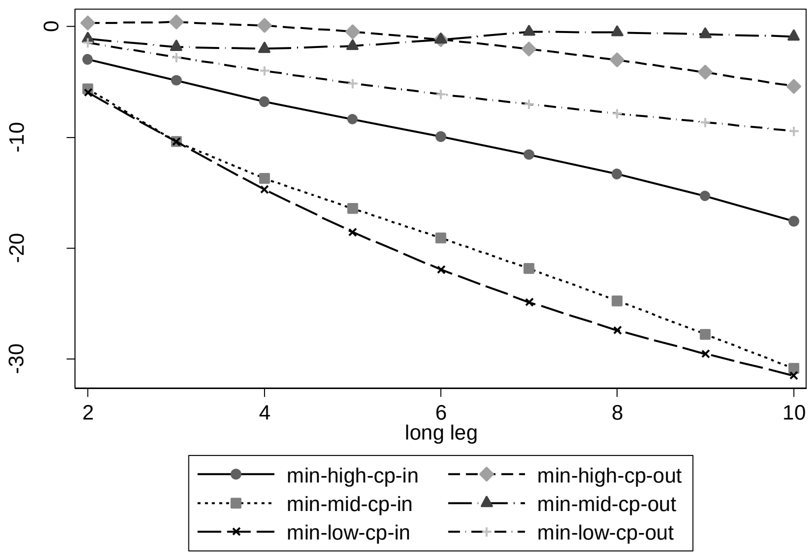
By running the regression of 9 sets of bond excess returns time series of different carry strategies of the in-sample time period on the forward rate time series during a same time period, we can obtain the 9 sets of coefficients. To our solution, the R square is 0.18. Then calculate critical value of different tercile of the in-sample ***CP*** factors and assign the corresponding excess returns to "low", "mid" and "high" regimes as us in-sample results.

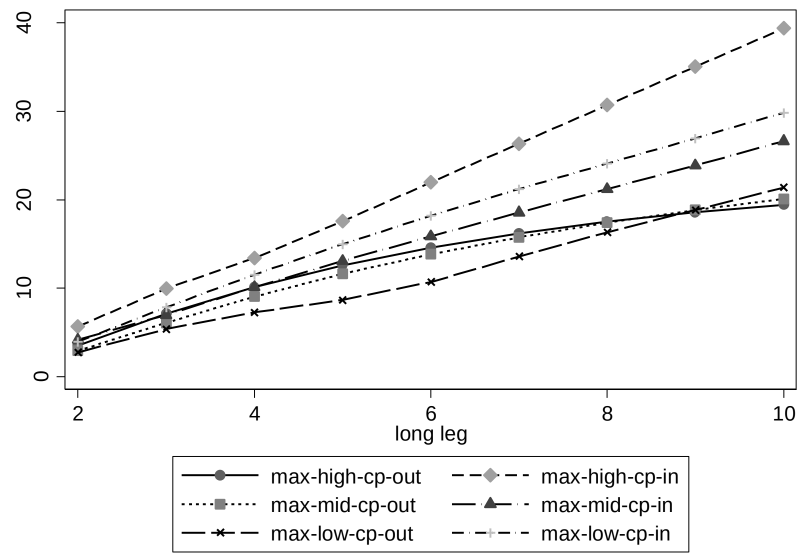
Then compute the out of sample ***CP*** factors by using the coefficients obtained in the in-sample, divide the out of sample ***CP*** factors into 3 terciles by the critical value of in-sample, finally assign the out of sample corresponding bond excess returns into three regimes as our out of sample results.

With look-ahead bias adjusted to ***CP*** procedure, the out of sample results are approximately same as the results for the in-sample results with look-ahead bias. Mean returns as well as volatility and Sharp ratio in different regimes of in-sample share a same distribution pattern with those in the out of sample, the values in the "high" regime are overall above those in "mid" regime, which are above those occurs in the "low" regime. (please go to appendix to see figures)



However, overall the mean returns in out of sample are higher than those of in-sample in all three regimes, particularly, mean returns of out of sample in "low" and "mid" regime tend to increase as the long leg maturity increase, while mean returns of in-sample in "low" and "mid" regime tend to decrease, which can be explained by the better economy condition of out of sample period. In sample time period, from 1971 to 2001, contains the period where the interest rate is decreased, while macro economy is warming and more stable, and interest rate is increased during out of sample time period. On the contrary, for volatility, the results of out-sample in three regimes are all lower than those of in-sample regimes. In terms of Sharp ratio, out of sample results are higher than in-sample results.



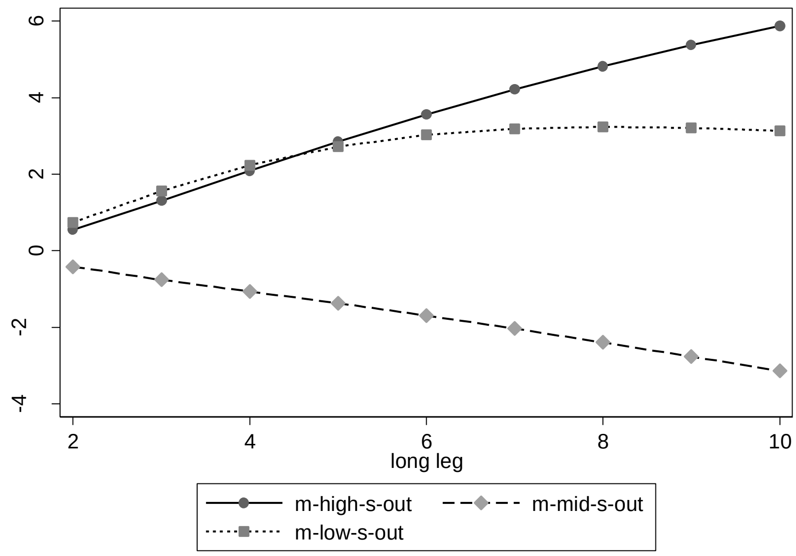


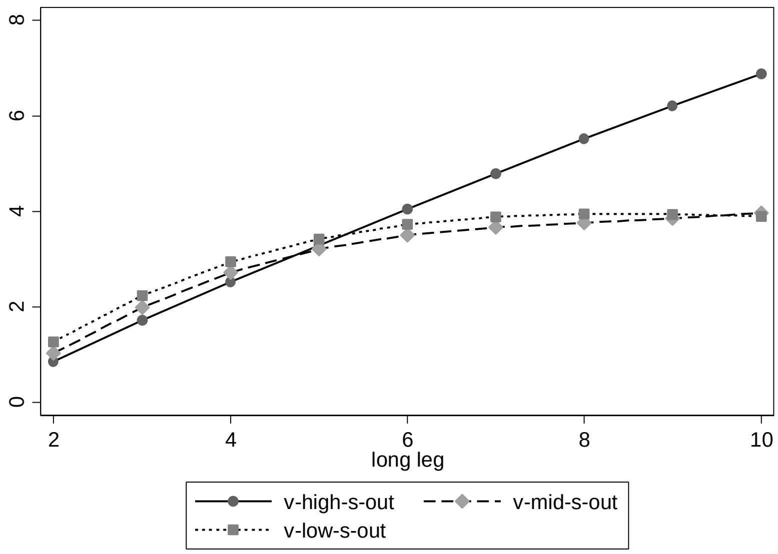
Out of sample can avoid larger loss for having a smaller minimum excess returns and has lower maximum excess returns than in-sample in three regimes. In summary, results are highly related to the business cycle and out of sample economy situation is better than in-sample, so investor can be exposed less volatility and higher Sharp ratio, but could obtain lower maximum return and face to less maximum loss- higher minimum returns. The look-ahead bias solution successfully works to the ***CP*** procedure.

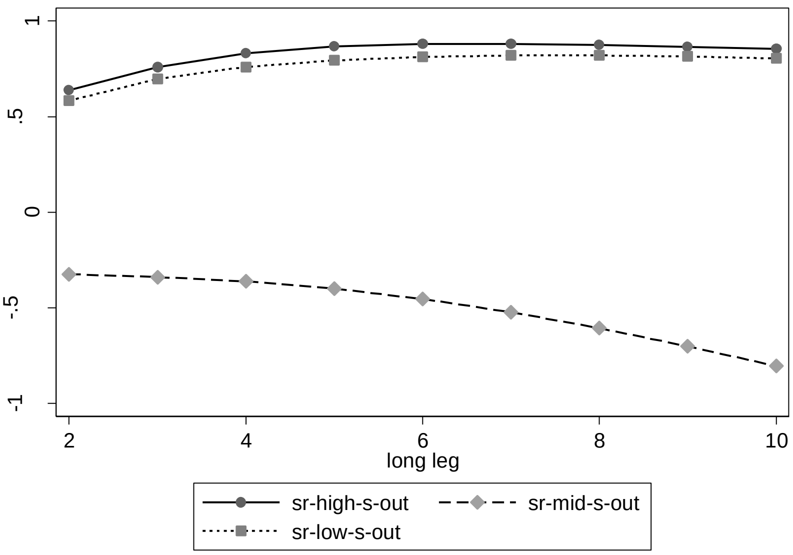
3.2. look-ahead bias adjusted results to *Slope* procedure and the look-ahead bias ones

Similar to the above solution, we also divide the whole time series into two parts, in-sample period starting from 1971 to 2001 and out of sample from 2002 to 2018. We calculate the critical values of the tercile of ***Slope*** in the in-sample, and assign the corresponding in-sample excess returns into “low”, “mid” and “high” regimes. Then by the critical value of ***Slope*** of in-sample, we allocate the out of sample ***Slope*** into three terciles and assign the out of sample corresponding excess returns to “low”, “mid” and “high” regime.

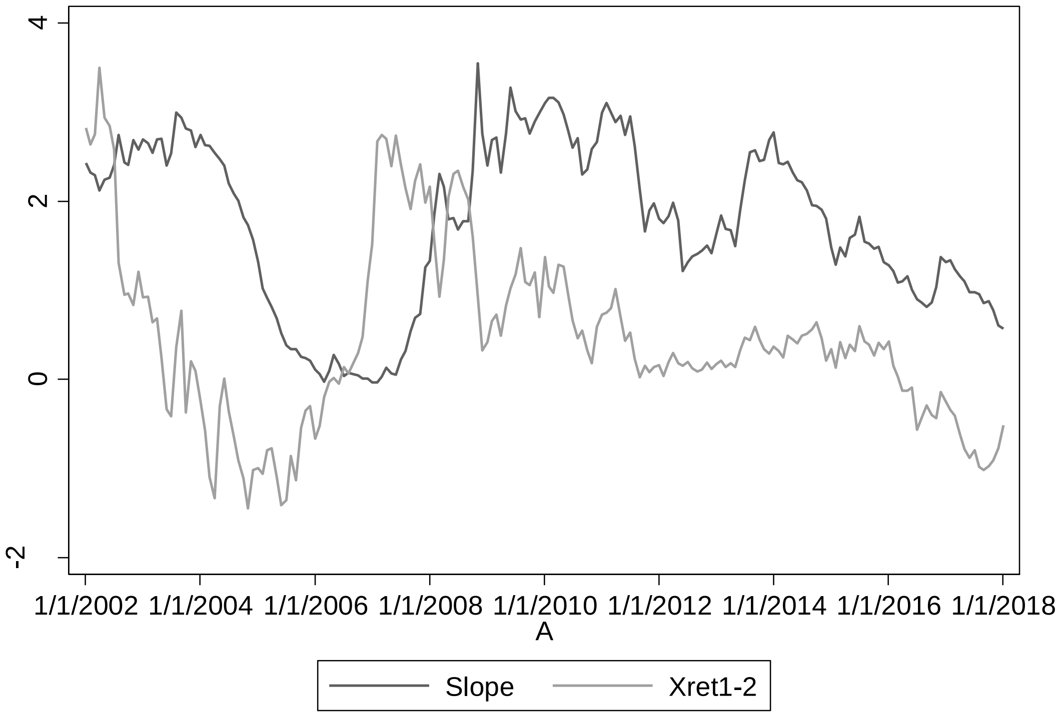
By comparing look-ahead bias adjusted results to ***Slope*** procedure, out of sample, with look-ahead bias ones, in-sample, the former is disordered in terms of the mean returns, Sharp ratio and volatility of out of sample and dramatically different to the results with look-ahead bias.







For out of sample, mean returns in "high" regime is above those in other two regimes, but mean returns in the "low" regime is overall larger than those in the "mid" regime, the same as the volatility do in the "low" regime to those in the "mid" regime. Sharp ratio in "low" regime are closer to those in "high" regime and larger than those in "mid" regime. These results are different from the former results obtained in unconditional and conditional study.



The reason is highly related to the future synchronization property and the relatively bigger indication error of ***Slope*** factor as discussed above, this impact to return-predicting is enlarged in the out of sample. During out of sample period, when ***Slope*** is in its first tercile, taking $Xret_{t\rightarrow t+1}^{2}$ as an example, the corresponding excess returns could be large, but we assign these excess returns to the "low" regimes, it would be better if they are in the "high" regime, leading to increase mean returns in the "low" regime, much larger than those in "mid" regime.

But why the mean returns of in-sample are not influenced by this property? During in-sample period, higher and lower values of excess returns corresponded by the ***Slope*** in first tercile are offset with each other and by averaging, the influence of ***Slope***'s indicative deviation is eliminated or weakeded, which leads to in-sample results more "normal".

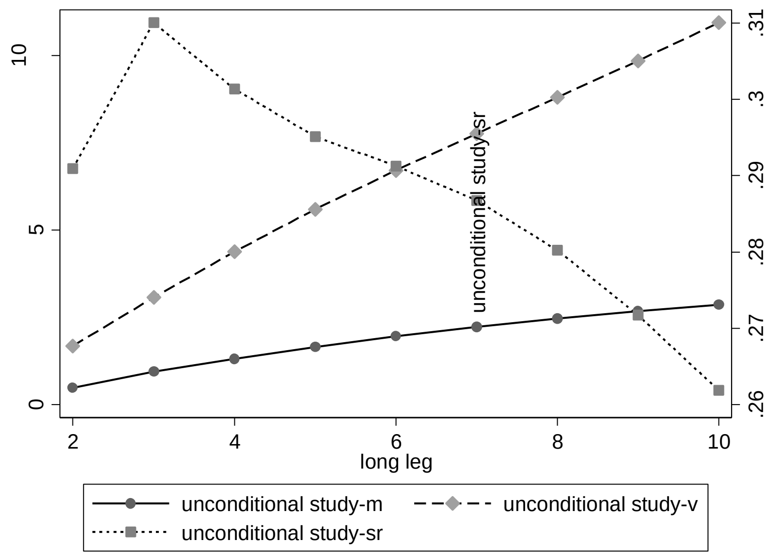
In summary, the unusual look-ahead bias adjusted results is linked to the inherent vice of ***Slope***, not resulting from the solution itself, we should not deny the look-ahead bias solution.

4. Robustness check on the UK government bond yield curve from 1970 to 2019

This part is to check the robustness of the indicative ability of ***Slope*** and ***CP*** in another universe, the datasets selected are the UK government bond yield curve with maturity starting from 1 to 10. We do the conditional and unconditional study again to our dataset and compare the results and discuss the former conclusion whether is applicable and representative to the new universe.

4.1. Unconditional study

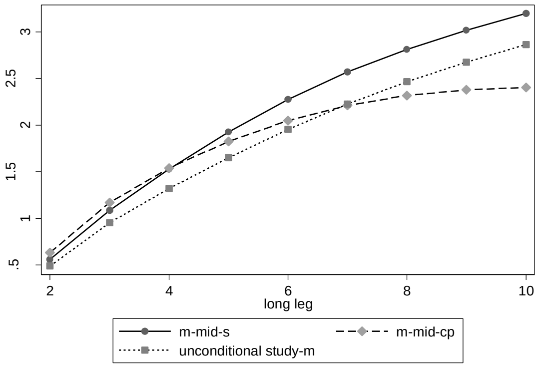
In the unconditional study on the UK government bond yield curve, we obtain the similar results and pattern as the unconditional study on the US zero-coupon bond yield curve.

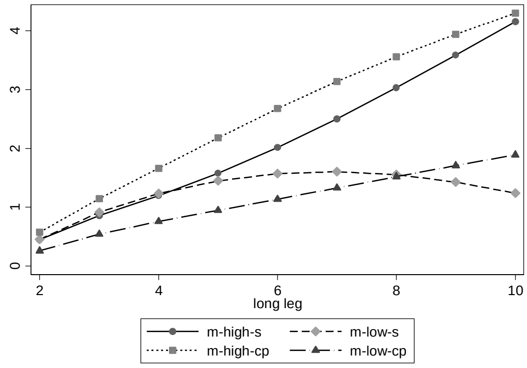


Bond excess premia can be obtained. Mean returns of different carry strategies increase as the long leg increase with volatility increasing faster than mean returns, leading to the decreasing Sharp ratio as long leg increasing.

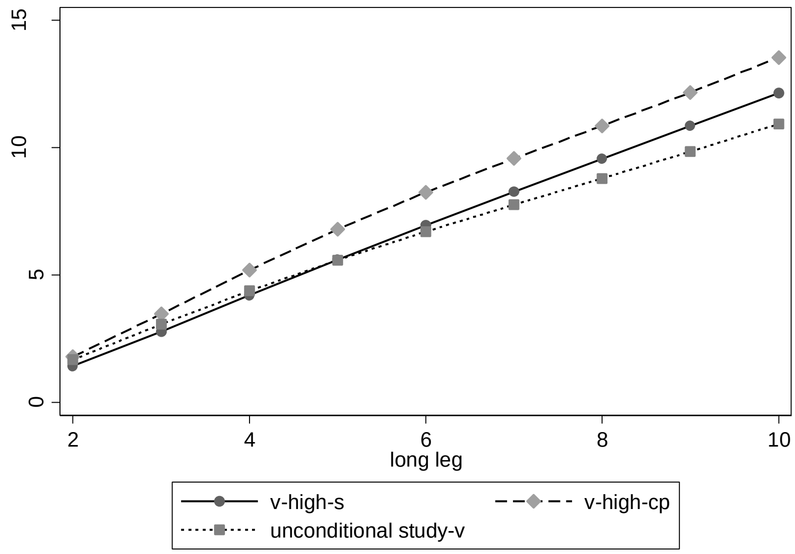
4.2. Conditional study

In conditional study, results and distribution pattern of mean returns, volatility and Sharp ratio obtained by ***CP*** and ***Slope*** are also similar to those in the conditional study on US zero-coupon bond yield curve. (please go to appendix and see details)





By comparing the result obtained by ***CP*** and ***Slope*** conditionally, we can draw similar but with difference conclusion for the predicting ability of each factor. For mean returns of different carry strategies, ***Slope*** still underestimate the mean returns in the “high” and “low” regimes than ***CP*** do in the same regimes. Mean returns in the “high” regimes obtained by ***CP*** are overall higher than those in the “high” regime obtained by ***Slope***. While mean returns in the “mid” obtain by ***Slope*** and ***CP*** are both close to the unconditional mean returns. Unconditional maximum excess returns again occur in the “high” regime obtained by ***CP***, while the unconditional minimum excess returns occur in both “high” and “mid” regimes, with more frequency in “mid” regime. By ***Slope***, unconditional maximum and minimum excess returns occur in the “mid” regime, which is the same as the conditional study of US zero-coupon bond yield curve. In terms of Sharp ratio, it is hard to give a description for which factor has a more precise indication, because the results obtained by either factor are disordered simultaneously.



For volatility, either by ***CP*** and ***Slope***, the volatilities in three regimes are all very close to the unconditional volatility, while the volatility obtained by ***Slope*** in three regimes is much closer to those in unconditional study.

Table 3 the coefficients of different carry strategies and return-predicting factors

	xret1-2	xret1-3	xret1-4	xret1-5	xret1-6	xret1-7	xret1-8	xret1-9	xret1-10
CP	0,09	0,10	0,11	0,12	0,13	0,13	0,13	0,13	0,13
Slope	0,01	0,01	0,01	0,03	0,06	0,08	0,11	0,14	0,16
future ***Slope***	0,56	0,50	0,44	0,38	0,34	0,30	0,26	0,23	0,20

The correlation between ***CP*** and ***Slope*** and $Xret_{t \to t+1}^{2}$ from 1970 to 2019 is 0.09 and 0.01 respectively, although both increase as long leg increases from 2 to 10, they do not exceed 0.13 and 0.16, showing that both factors have relatively larger indicative deviation than they perform to the US zero-coupon bond yield curve.

The correlation between $Xret_{t \to t+1}^{2}$ from 1970 to 2019 and ***Slope*** in the same time period is 0.01, while the correlation between $Xret_{t \to t+1}^{2}$ from 1970 to 2018 and the ***Slope*** time series from 1971 to 2019 can be increased to 0.56, which is the same case as it does in the US sphere, ***Slope*** has a 12-month-later synchronization with excess return. As long leg increases, the future-***Slope*** correlation decreases but the smallest is close to 0.2.

As the table above, the predicting ability of ***CP*** is more stable than ***Slope*** across different carry strategies, and ***Slope*** can better predict excess returns from longer leg carry strategies, as long leg increases, the future synchronization of ***Slope*** is weakened. Above are the same results as they show in the US sphere.

In summary, to the inactive ability of each factor to the UK government bond yield curve, it is hard to draw a conclusion which is better and which has a more precise predicting ability. For predicting mean returns, ***CP*** can be a more precise indicator than ***Slope***, for predicting volatility, ***Slope*** can be a better predictor. In both US and UK sphere, ***CP*** and ***Slope*** show similar features.

5. Conclusion

In the empirical study on the US zero-coupon bond, for different carry strategies where long leg equals from 2 to 10, bond excess returns can be harvested and return-return-predicting factors has significant indication. To the look-ahead bias, we should take a bigger window for in-sample to test robustness of the ***Slope*** and ***CP*** in the out of sample sphere.

Both conditional study by ***CP*** on US and UK government bond yield curve, we obtain the bat-shaped coefficients, rather than the tent-shaped coefficients shown in Cochrane and Piazzesi (2005). Villegas (2015) explained that "choppy" forward rate produces a tent, while the smooth ones produce a bat-like pattern, and he conduct a constrained regression methodology and lead to a conclusion that despite tent-and bat-shaped coefficients are optically very different, they are in predictive terms very similar.

CP and ***Slope*** in terms of predicting ability perform differently on US and UK zero-coupon bond yield curve. Overall ***Slope*** has a bigger indicative deviation than ***CP*** does, in both sphere, ***Slope*** having a 12-month-later synchronization property is found. We will continue to check on the other physical universe.

6. References

Cochrane JH, Piazzesi M. (2005). *Bond Risk Premia*.

Radwanski JF. (2010). *Understanding Bond Risk Premia Uncovered by the Term Structure*. Vienna Graduate School of Finance, Vienna Univer- sity of Economics, (VGSF).

Villegas D. (2015). *Bond Excess Returns Predictability and the Expectation Hypothesis.*

Antti Ilmanen. (1997). *Forcasing U.S. Bond Returns.*

George Mylnikov. (2014). *Forecasting U.S. Bond Returns: A practitioner's Perspective*. Institutional Investor Journals.

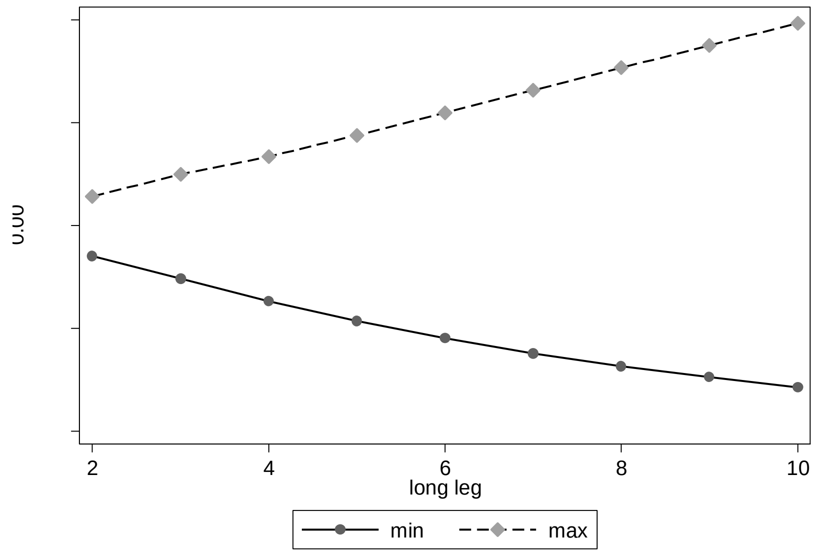
Appendix

A.1. Unconditional study results

Table 4: unconditional excess returns with different carry strategy time-series from 1971 to 2018 results

Carry Strategy	Long Leg	Mean Returns	Volatility	Sharp Ratio	Min	Max
xret1-2	2	0,53	1,74	0,30	-5,93	5,67
xret1-3	3	0,95	3,16	0,30	-10,39	9,96
xret1-4	4	1,32	4,41	0,30	-14,71	13,41
xret1-5	5	1,64	5,54	0,30	-18,56	17,56
xret1-6	6	1,91	6,62	0,29	-21,93	21,97
xret1-7	7	2,14	7,66	0,28	-24,87	26,35
xret1-8	8	2,33	8,67	0,27	-27,41	30,71
xret1-9	9	2,49	9,67	0,26	-29,54	35,06
xret1-10	10	2,63	10,66	0,25	-31,52	39,39

Notes 4: Xret1-x refers to buying 1 dollars of x-years maturity bond and selling same amounts of 1-year maturity bond and unwind the position after holding 1 year.



Notes 5 the minimum and maximum value of the unconditional excess return with different carry strategy

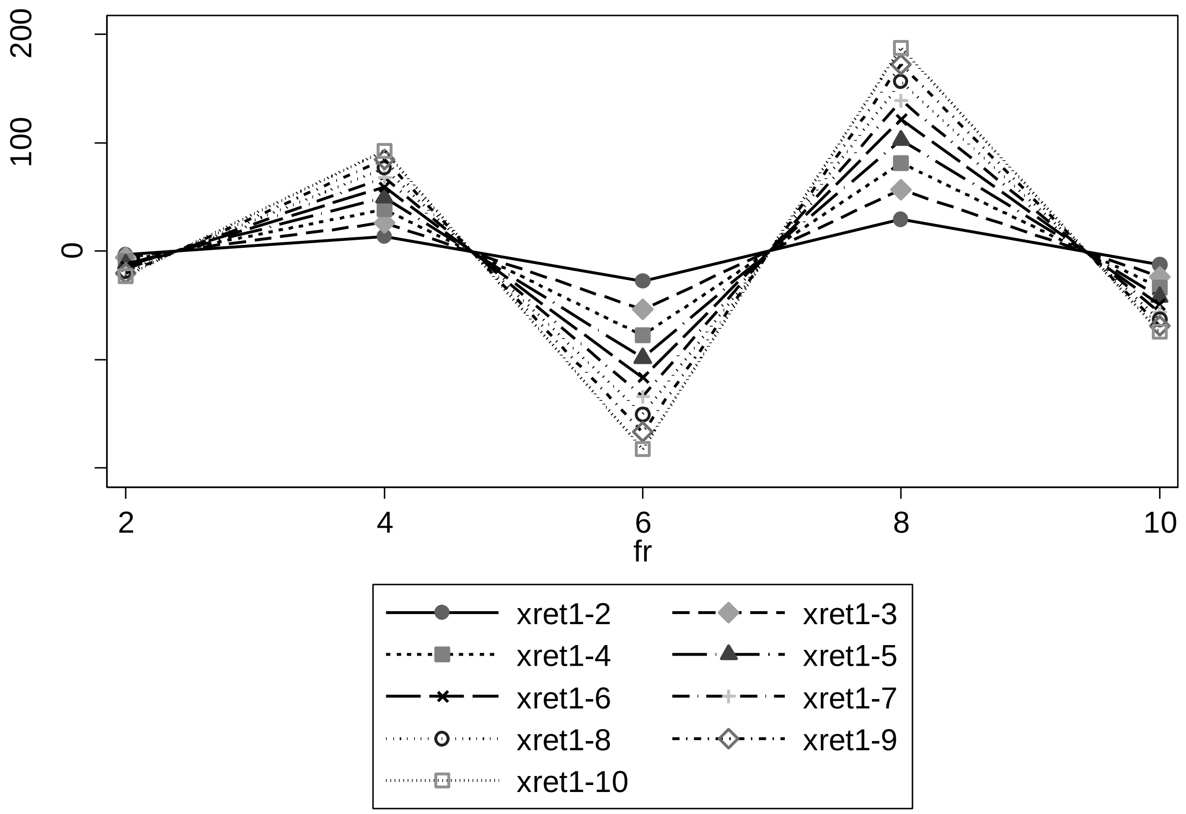
A.2. Conditional study results

A.2.i. Regression

Table 5: the regression results of Cochrane and Piazzesi factor

	xret1-2	xret1-3	xret1-4	xret1-5	xret1-6	xret1-7	xret1-8	xret1-9	xret1-10
f2	-2.797***	-5.797***	-8.758***	-11.488***	-13.985***	-16.309***	-18.516***	-20.639***	-22.685***
	(0.609)	(1.106)	(1.531)	(1.913)	(2.271)	(2.617)	(2.957)	(3.294)	(3.632)
f4	13.270***	26.419***	38.684***	49.474***	59.071***	67.947***	76.459***	84.762***	92.838***
	(3.150)	(5.726)	(7.925)	(9.903)	(11.756)	(13.544)	(15.303)	(17.052)	(18.799)
f6	-27.532***	-53.838***	-77.680***	-98.323***	-116.658***	-133.776***	-150.388***	-166.703***	-182.535***
	(6.656)	(12.097)	(16.744)	(20.922)	(24.837)	(28.616)	(32.333)	(36.027)	(39.719)
f8	29.283***	56.821***	81.420***	102.678***	121.594***	139.213***	156.131***	172.436***	187.822***
	(6.516)	(11.844)	(16.393)	(20.483)	(24.316)	(28.016)	(31.654)	(35.271)	(38.886)
f10	-12.054***	-23.331***	-33.304***	-41.886***	-49.465***	-56.407***	-62.899***	-68.941***	-74.395***
	(2.448)	(4.449)	(6.159)	(7.695)	(9.135)	(10.525)	(11.892)	(13.251)	(14.609)
_cons	-0.779***	-1.296**	-1.853***	-2.539***	-3.375***	-4.352***	-5.444***	-6.625***	-7.866***
	(0.279)	(0.507)	(0.702)	(0.877)	(1.041)	(1.200)	(1.356)	(1.510)	(1.665)
Obs.	554	554	554	554	554	554	554	554	554
R-squared	0.131	0.135	0.147	0.158	0.167	0.174	0.178	0.179	0.179

*Notes 6: Standard errors are in parenthesis, *** p<0.01, ** p<0.05, * p<0.1*



Notes 7 the coefficients of different carry strategies regression on five forward rate

A.2.ii. Three regimes result obtained by *CP* factor

Table 6 critical values of tercile by ***CP***

	CP
1^{st} and 2^{nd} tercile critical value	0,43
2^{nd} and 3^{rd} tercile critical value	2,75

Table 7 "high" regime results obtained by ***CP*** *factors*

Carry strategy	Long Leg	Mean Return	Volatility	Sharp Ratio	Min	Max
xret1-2	2	1,13	1,55	0,72	-2,45	5,67
xret1-3	3	2,15	2,91	0,74	-4,88	9,96
xret1-4	4	3,11	4,07	0,76	-6,78	13,41
xret1-5	5	3,99	5,12	0,78	-8,38	17,56
xret1-6	6	4,81	6,09	0,79	-9,93	21,97
xret1-7	7	5,56	7,01	0,79	-11,55	26,35
xret1-8	8	6,24	7,91	0,79	-13,31	30,71
xret1-9	9	6,86	8,78	0,78	-15,26	35,06
xret1-10	10	7,42	9,64	0,77	-17,27	39,39

Table 8 "mid" regime results obtained by ***CP***

Carry strategy	Long Legs	Mean Return	Volatility	Sharp Ratio	Min	Max
xret1-2	2	0,48	1,77	0,27	-5,63	5,23
xret1-3	3	0,82	3,15	0,26	-10,39	8,72
xret1-4	4	1,09	4,32	0,25	-13,71	11,64
xret1-5	5	1,32	5,40	0,24	-16,42	14,64
xret1-6	6	1,51	6,42	0,24	-19,07	17,64
xret1-7	7	1,68	7,44	0,23	-21,84	20,63
xret1-8	8	1,81	8,48	0,21	-24,75	23,61
xret1-9	9	1,92	9,55	0,20	-27,77	26,85
xret1-10	10	2,01	10,64	0,19	-30,86	30,47

Table 9 "low" regime results obtained by ***CP***

carry strategy	long legs	mean returns	volatility	Sharp ratio	min	max
xret1-2	2	-0,03	1,68	-0,02	-5,93	5,47
xret1-3	3	-0,11	3,01	-0,04	-10,39	9,21
xret1-4	4	-0,24	4,16	-0,06	-14,71	11,94
xret1-5	5	-0,40	5,20	-0,08	-18,56	14,99
xret1-6	6	-0,60	6,18	-0,10	-21,93	18,20
xret1-7	7	-0,82	7,11	-0,12	-24,87	21,22
xret1-8	8	-1,06	8,01	-0,13	-27,41	24,11
xret1-9	9	-1,29	8,87	-0,15	-29,54	26,96
xret1-10	10	-1,52	9,70	-0,16	-31,52	29,85

A.2.iii. Three regimes result obtained by *Slope*

*Table 10 critical values of tercile by **Slope***

	Slope
1st and 2nd tercile critical value	0,55
2nd and 3rd tercile critical value	1,39

*Table 11 "high" regime results obtained by **CP** factors*

carry strategy	long legs	mean returns	volatility	Sharp ratio	min	max
xret1-2	2	0,81	1,30	0,62	-2,45	3,95
xret1-3	3	1,72	2,56	0,67	-4,85	7,84
xret1-4	4	2,63	3,69	0,71	-6,74	11,81
xret1-5	5	3,50	4,71	0,74	-8,29	15,85
xret1-6	6	4,33	5,67	0,76	-9,99	19,93
xret1-7	7	5,09	6,57	0,78	-11,59	24,00
xret1-8	8	5,81	7,43	0,78	-13,22	28,06
xret1-9	9	6,46	8,25	0,78	-14,79	32,10
xret1-10	10	7,06	9,06	0,78	-16,30	36,09

*Table 12 "mid" regime results obtained by **Slope***

carry strategy	long legs	mean returns	volatility	Sharp ratio	min	max
xret1-2	2	0,35	1,61	0,22	-5,65	5,67
xret1-3	3	0,56	2,91	0,19	-10,39	9,96
xret1-4	4	0,71	4,02	0,18	-14,71	13,41
xret1-5	5	0,84	5,03	0,17	-18,56	17,56
xret1-6	6	0,93	5,98	0,16	-21,93	21,97
xret1-7	7	1,00	6,90	0,15	-24,87	26,35
xret1-8	8	1,07	7,80	0,14	-27,41	30,71
xret1-9	9	1,13	8,69	0,13	-29,54	35,06
xret1-10	10	1,20	9,57	0,13	-31,23	39,39

*Table 13 "low" regime results obtained by **Slope***

carry strategy	long legs	mean returns	volatility	Sharp ratio	min	max
xret1-2	2	0,42	2,15	0,20	-5,93	5,47
xret1-3	3	0,59	3,76	0,16	-10,39	9,21
xret1-4	4	0,62	5,09	0,12	-13,71	11,94
xret1-5	5	0,58	6,28	0,09	-16,45	14,72
xret1-6	6	0,47	7,40	0,06	-20,08	17,64
xret1-7	7	0,32	8,48	0,04	-23,46	20,63
xret1-8	8	0,13	9,55	0,01	-26,51	23,61
xret1-9	9	-0,10	10,62	-0,01	-29,20	26,85
xret1-10	10	-0,35	11,70	-0,03	-31,52	30,47

A.3. Look-ahead bias to *CP* procedure

A.3. i. In-sample results

Table 14 regression results

	xret12	xret13	xret14	xret15	xret16	xret17	xret18	xret19	xret110
f2	-3.795***	-7.654***	-11.469***	-15.036***	-18.368***	-21.541***	-24.618***	-27.632***	-30.590***
	(0.825)	(1.480)	(2.040)	(2.549)	(3.031)	(3.498)	(3.956)	(4.411)	(4.862)
f4	19.444***	38.223***	56.350***	73.146***	88.957***	104.312***	119.556***	134.792***	149.927***
	(4.406)	(7.907)	(10.898)	(13.619)	(16.195)	(18.689)	(21.139)	(23.565)	(25.980)
f6	-39.910***	-77.473***	-113.479***	-147.145***	-179.512***	-211.724***	-244.376***	-277.462***	-310.519***
	(9.361)	(16.799)	(23.154)	(28.936)	(34.408)	(39.708)	(44.913)	(50.068)	(55.198)
f8	40.975***	78.849***	114.650***	148.164***	180.625***	213.125***	246.103***	279.364***	312.261***
	(9.071)	(16.278)	(22.436)	(28.039)	(33.341)	(38.476)	(43.520)	(48.515)	(53.486)
f10	-16.417***	-31.432***	-45.349***	-58.243***	-70.629***	-82.900***	-95.180***	-107.350***	-119.134***
	(3.354)	(6.019)	(8.296)	(10.368)	(12.328)	(14.227)	(16.092)	(17.939)	(19.778)
_cons	-1.888***	-3.401***	-4.845***	-6.329***	-7.936***	-9.706***	-11.642***	-13.724***	-15.918***
	(0.469)	(0.841)	(1.159)	(1.448)	(1.722)	(1.988)	(2.248)	(2.506)	(2.763)
Obs.	361	361	361	361	361	361	361	361	361
R-squared	0.171	0.179	0.186	0.190	0.191	0.192	0.192	0.193	0.193

*Table 15 tercile critical value of **CP***

	CP
1st and 2nd tercile critical value	-0,09
2nd and 3rd tercile critical value	1,93

Table 16 "high" regime

Carry Strategy	Long Legs	m-high-*CP*-in	v-high-*CP*-in	sr-high-*CP*-in	min-high-*CP*-in	max-high-*CP*-in
xret1-2	2	1.67	1.89	0.88	-2.96	5.67
xret1-3	3	2.92	3.43	0.85	-4.88	9.96
xret1-4	4	3.96	4.74	0.83	-6.78	13.41
xret1-5	5	4.86	5.92	0.82	-8.38	17.56
xret1-6	6	5.68	7.03	0.81	-9.93	21.97
xret1-7	7	6.44	8.12	0.79	-11.55	26.35
xret1-8	8	7.17	9.21	0.78	-13.31	30.71
xret1-9	9	7.86	10.32	0.76	-15.29	35.06
xret1-10	10	8.53	11.44	0.75	-17.56	39.39

Table 17 "mid" regime

Carry Strategy	Long Legs	m-mid-*CP*-in	v-mid-*CP*-in	sr-mid-*CP*-in	min-mid-*CP*-in	max-mid-*CP*-in
xret1-2	2	0,17	1,69	0,10	-5,63	4,19
xret1-3	3	0,15	3,03	0,05	-10,39	6,99
xret1-4	4	0,08	4,17	0,02	-13,71	10,13
xret1-5	5	-0,01	5,20	0,00	-16,42	13,08
xret1-6	6	-0,14	6,18	-0,02	-19,07	15,87
xret1-7	7	-0,30	7,14	-0,04	-21,84	18,56
xret1-8	8	-0,50	8,09	-0,06	-24,75	21,21
xret1-9	9	-0,72	9,05	-0,08	-27,77	23,88
xret1-10	10	-0,97	10,02	-0,10	-30,86	26,63

Table 18 "low" regime

Carry Strategy	Long Legs	m-low-*CP*-in	v-low-*CP*-in	sr-low-*CP*-in	min-low-*CP*-in	max-low-*CP*-in
xret1-2	2	-0,14	1,99	-0,07	-5,93	3,95
xret1-3	3	-0,41	3,55	-0,11	-10,39	7,84
xret1-4	4	-0,71	4,92	-0,14	-14,71	11,54
xret1-5	5	-1,01	6,18	-0,16	-18,56	14,99
xret1-6	6	-1,31	7,37	-0,18	-21,93	18,20
xret1-7	7	-1,61	8,50	-0,19	-24,87	21,22
xret1-8	8	-1,91	9,58	-0,20	-27,41	24,11
xret1-9	9	-2,21	10,63	-0,21	-29,54	26,96
xret1-10	10	-2,50	11,64	-0,21	-31,52	29,85

A.3. ii. Out-of-sample results

Table 19 “high”regime

Carry Strategy	Long Legs	m-high-*CP*-out	v-high-*CP*-out	sr-high-*CP*-out	min-high-*CP*-out	max-high-*CP*-out
xret1-2	2	1,40	0,99	1,41	0,32	3,50
xret1-3	3	2,95	2,09	1,41	0,40	7,11
xret1-4	4	4,33	3,03	1,43	0,09	10,12
xret1-5	5	5,52	3,83	1,44	-0,45	12,56
xret1-6	6	6,55	4,54	1,44	-1,16	14,56
xret1-7	7	7,42	5,18	1,43	-2,02	16,19
xret1-8	8	8,15	5,79	1,41	-3,00	17,52
xret1-9	9	8,73	6,37	1,37	-4,13	18,58
xret1-10	10	9,18	6,92	1,33	-5,38	19,43

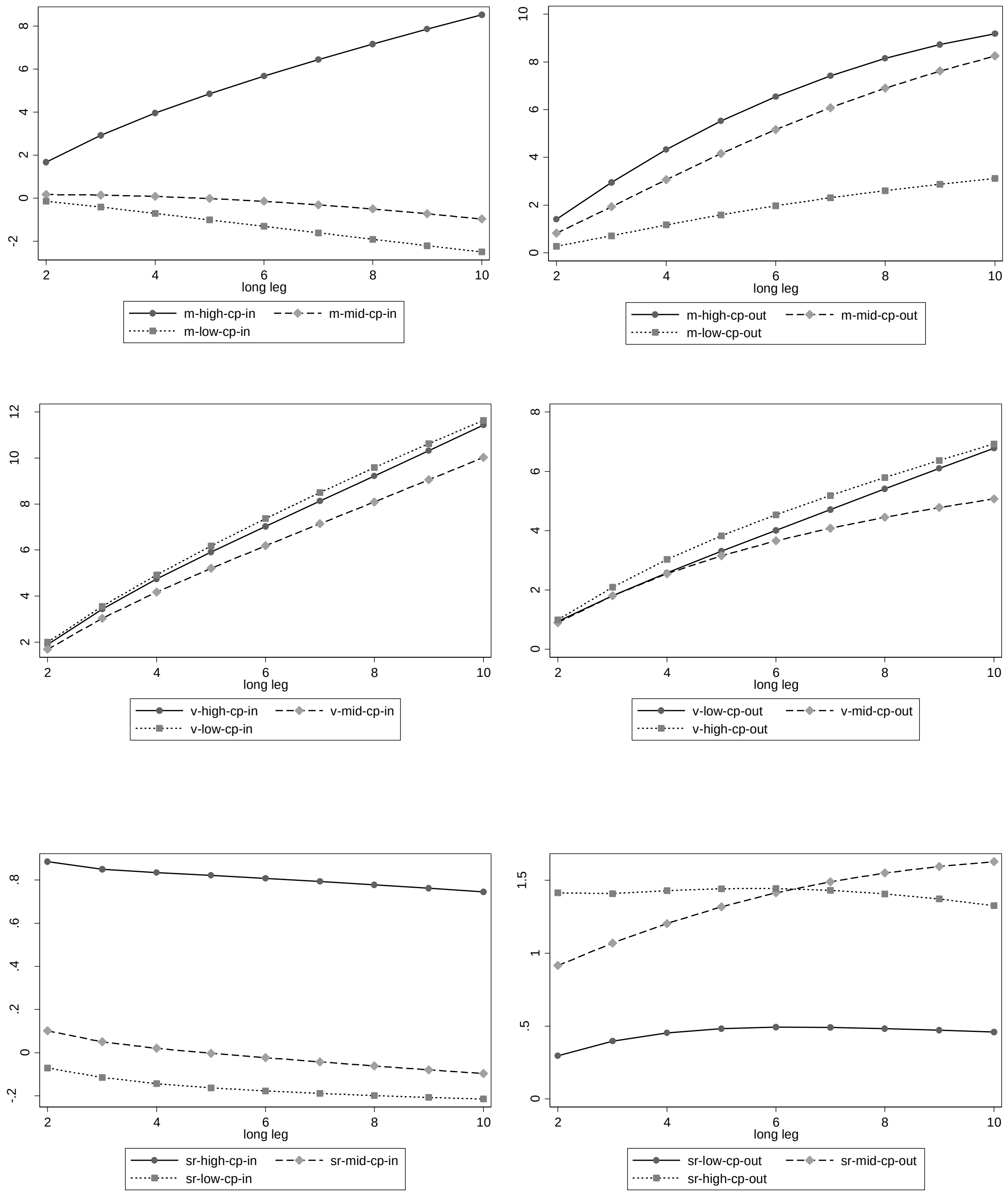
Table 20 “mid” regime

Carry Strategy	Long Legs	m-mid-*CP*-out	v-mid-*CP*-out	sr-mid-*CP*-out	min-mid-*CP*-out	max-mid-*CP*-out
xret1-2	2	0,83	0,90	0,91	-1,10	2,94
xret1-3	3	1,93	1,80	1,07	-1,83	6,10
xret1-4	4	3,07	2,55	1,20	-2,01	9,05
xret1-5	5	4,16	3,15	1,32	-1,74	11,63
xret1-6	6	5,17	3,66	1,41	-1,19	13,85
xret1-7	7	6,08	4,08	1,49	-0,46	15,76
xret1-8	8	6,90	4,45	1,55	-0,52	17,41
xret1-9	9	7,62	4,78	1,59	-0,70	18,85
xret1-10	10	8,25	5,07	1,63	-0,89	20,10

Table 21 "low" regime

Carry Strategy	Long Legs	m-low-*CP*-out	v-low-*CP*-out	sr-low-*CP*-out	min-low-*CP*-out	max-low-*CP*-out
xret1-2	2	0,28	0,94	0,30	-1,45	2,75
xret1-3	3	0,71	1,81	0,40	-2,76	5,39
xret1-4	4	1,17	2,58	0,45	-4,00	7,29
xret1-5	5	1,59	3,31	0,48	-5,11	8,67
xret1-6	6	1,97	4,01	0,49	-6,11	10,70
xret1-7	7	2,31	4,71	0,49	-7,01	13,58
xret1-8	8	2,61	5,41	0,48	-7,84	16,31
xret1-9	9	2,88	6,10	0,47	-8,63	18,85
xret1-10	10	3,12	6,78	0,46	-9,43	21,39

A.3.iii. comparison of in-sample and out of sample



A.4. Look-ahead bias to *Slope* procedure

A.4. i. In-sample results

*Table 22 critical values for tercile by **Slope***

	Slope
1st and 2nd tercile critical value	0,34
2nd and 3rd tercile critical value	1,05

Table 23 "high" regime

Carry Strategy	Long Legs	m-high-s-in	v-high-s-in	sr-high-s-in	min-high-s-in	max-high-s-in
xret1-2	2	0,93	1,79	0,52	-4,95	4,22
xret1-3	3	1,64	3,46	0,47	-8,43	8,66
xret1-4	4	2,28	4,96	0,46	-11,25	13,13
xret1-5	5	2,86	6,33	0,45	-13,52	17,56
xret1-6	6	3,39	7,59	0,45	-15,41	21,97
xret1-7	7	3,87	8,77	0,44	-17,03	26,35
xret1-8	8	4,32	9,91	0,44	-18,47	30,71
xret1-9	9	4,73	11,01	0,43	-19,79	35,06
xret1-10	10	5,12	12,09	0,42	-21,04	39,39

Table 24 "mid" regime

Carry Strategy	Long Legs	m-mid-s-in	v-mid-s-in	sr-mid-s-in	min-mid-s-in	max-mid-s-in
xret1-2	2	0,37	1,78	0,21	-5,65	5,67
xret1-3	3	0,58	3,13	0,18	-10,39	9,96
xret1-4	4	0,73	4,27	0,17	-14,71	13,41
xret1-5	5	0,85	5,31	0,16	-18,56	16,45
xret1-6	6	0,96	6,29	0,15	-21,93	19,33
xret1-7	7	1,06	7,25	0,15	-24,87	22,21
xret1-8	8	1,17	8,20	0,14	-27,41	25,14
xret1-9	9	1,28	9,12	0,14	-29,54	28,13
xret1-10	10	1,41	10,01	0,14	-31,23	31,14

Table 25 "low" regime

Carry Strategy	Long Legs	m-low-s-in	v-low-s-in	sr-low-s-in	min-low-s-in	max-low-s-in
xret1-2	2	0,40	2,39	0,17	-5,93	5,47
xret1-3	3	0,44	4,16	0,11	-10,39	9,21
xret1-4	4	0,32	5,61	0,06	-13,71	11,94
xret1-5	5	0,12	6,91	0,02	-16,45	14,72
xret1-6	6	-0,13	8,14	-0,02	-20,08	17,64
xret1-7	7	-0,42	9,35	-0,04	-23,46	20,63
xret1-8	8	-0,74	10,56	-0,07	-26,51	23,61
xret1-9	9	-1,10	11,79	-0,09	-29,20	26,85
xret1-10	10	-1,49	13,04	-0,11	-31,52	30,47

A.4. ii. Out-of-sample results

Table 26"high" regime

Carry Strategy	Long Legs	m-high-s-out	v-high-s-out	sr-high-s-out	min-high-s-out	max-high-s-out
xret1-2	2	0,55	0,86	0,64	-1,45	3,50
xret1-3	3	1,31	1,72	0,76	-2,51	7,11
xret1-4	4	2,09	2,52	0,83	-3,00	10,12
xret1-5	5	2,85	3,29	0,87	-3,41	12,56
xret1-6	6	3,56	4,05	0,88	-4,30	14,56
xret1-7	7	4,22	4,79	0,88	-5,13	16,19
xret1-8	8	4,82	5,52	0,87	-6,65	17,52
xret1-9	9	5,37	6,22	0,86	-8,10	18,85
xret1-10	10	5,88	6,88	0,85	-9,43	21,39

Table 27 "mid" regime

Carry Strategy	Long Legs	m-mid-s-out	v-mid-s-out	sr-mid-s-out	min-mid-s-out	max-mid-s-out
xret1-2	2	-0,41	1,03	-0,32	-1,41	2,42
xret1-3	3	-0,76	1,99	-0,34	-2,76	4,87
xret1-4	4	-1,06	2,72	-0,36	-4,00	6,52
xret1-5	5	-1,37	3,21	-0,40	-5,11	7,12
xret1-6	6	-1,69	3,50	-0,45	-6,11	7,26
xret1-7	7	-2,03	3,67	-0,52	-7,01	7,40
xret1-8	8	-2,39	3,76	-0,61	-7,84	7,67
xret1-9	9	-2,76	3,85	-0,70	-8,63	7,87
xret1-10	10	-3,14	3,97	-0,80	-9,40	7,90

Table 28 "low" regime

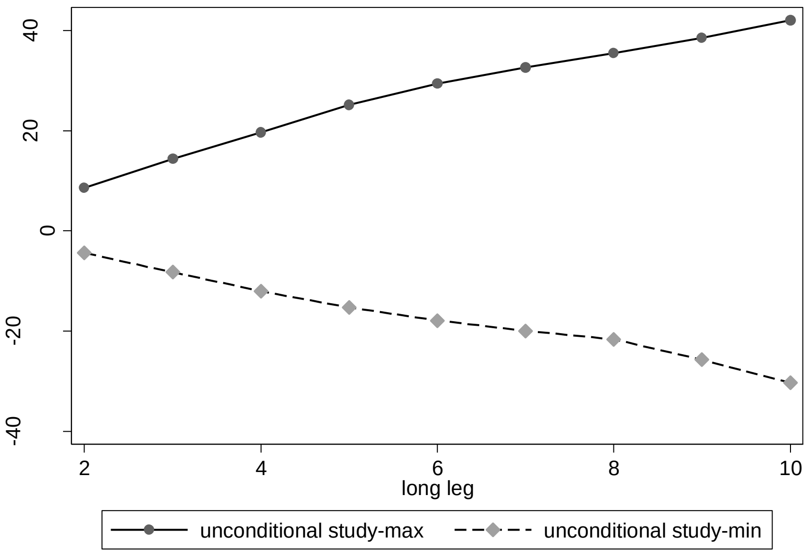
Carry Strategy	Long Legs	m-low-s-out	v-low-s-out	sr-low-s-out	min-low-s-out	max-low-s-out
xret1-2	2	0,74	1,27	0,58	-1,13	2,75
xret1-3	3	1,56	2,24	0,70	-1,84	5,39
xret1-4	4	2,24	2,94	0,76	-2,46	7,29
xret1-5	5	2,72	3,43	0,79	-3,05	8,67
xret1-6	6	3,03	3,73	0,81	-3,62	9,53
xret1-7	7	3,19	3,89	0,82	-4,15	9,96
xret1-8	8	3,24	3,95	0,82	-4,64	10,08
xret1-9	9	3,21	3,94	0,81	-5,07	10,01
xret1-10	10	3,14	3,90	0,80	-5,47	9,82

A.5. Robustness check on the UK government bond yield curve

A.5. i. Unconditional study

Table 29 unconditional study results of UK government bond yield curve from 1970 to 2019

Carry Strategy	Long Legs	Mean Returns	Volatility	Sharp Ratio	Min	Max
xret1-2	2	0,49	1,68	0,29	-4,41	8,56
xret1-3	3	0,95	3,07	0,31	-8,26	14,41
xret1-4	4	1,32	4,38	0,30	-12,03	19,70
xret1-5	5	1,65	5,59	0,30	-15,28	25,16
xret1-6	6	1,95	6,71	0,29	-17,89	29,39
xret1-7	7	2,23	7,76	0,29	-19,96	32,67
xret1-8	8	2,47	8,80	0,28	-21,64	35,49
xret1-9	9	2,68	9,85	0,27	-25,72	38,56
xret1-10	10	2,86	10,94	0,26	-30,35	42,06



Notes 8 maximum and minimum excess returns of carry strategies on UK government bond yield curve

A.5. ii. Conditional study

A.5.ii.a. Regression

Table 30 regression results for ***CP*** *in the conditional study on UK government bond*

	xret12	xret13	xret14	xret15	xret16	xret17	xret18	xret19	xret110
f2	-0.185	-1.068*	2.106***	-3.040***	-3.712***	4.093***	4.210***	4.107**	-3.822*
	(0.297)	(0.546)	(0.781)	(1.001)	(1.205)	(1.396)	(1.581)	(1.766)	(1.953)
f4	1.133	4.980**	8.844***	11.613***	12.872***	12.769**	11.655**	9.858	7.621
	(1.069)	(1.962)	(2.809)	(3.600)	(4.331)	(5.019)	(5.686)	(6.349)	(7.023)
f6	-1.087	-7.378**	-13.579**	-17.607**	-18.995**	-18.269*	-16.214	-13.477	-10.464
	(2.031)	(3.729)	(5.338)	(6.842)	(8.232)	(9.540)	(10.807)	(12.068)	(13.349)
f8	-0.459	4.398	9.576*	13.061*	14.472*	14.299	13.218	11.740	10.142
	(1.976)	(3.627)	(5.191)	(6.653)	(8.006)	(9.278)	(10.510)	(11.736)	(12.981)
f10	0.658	-0.843	-2.625	-3.895	-4.487	-4.547	-4.288	-3.858	-3.328
	(0.729)	(1.338)	(1.915)	(2.455)	(2.954)	(3.423)	(3.878)	(4.330)	(4.790)
_cons	0.167	0.447	0.593	0.615	0.578	0.524	0.472	0.429	0.395
	(0.164)	(0.301)	(0.430)	(0.552)	(0.664)	(0.769)	(0.871)	(0.973)	(1.076)
Obs.	591	591	591	591	591	591	591	591	591
R-squared	0.052	0.042	0.034	0.027	0.021	0.018	0.019	0.024	0.031

*Notes 9 Standard errors are in parenthesis, *** p<0.01, ** p<0.05, * p<0.1*

A.5.ii.b. Three regimes results obtained by *CP* factor

*Table 31 critical values of tercile by **CP***

	CP
1st and 2nd tercile critical value	1,42
2nd and 3rd tercile critical value	2.27,

Table 32 "high" regime

Carry Strategy	Long Legs	Mean Returns	Volatility	Sharp Ratio	Min	Max
xret1-2	2	0,58	1,80	0,32	-3,98	8,56
xret1-3	3	1,15	3,47	0,33	-7,12	14,41
xret1-4	4	1,66	5,20	0,32	-9,85	19,70
xret1-5	5	2,18	6,80	0,32	-12,06	25,16
xret1-6	6	2,68	8,25	0,32	-15,37	29,39
xret1-7	7	3,14	9,58	0,33	-18,33	32,67
xret1-8	8	3,56	10,86	0,33	-21,55	35,49
xret1-9	9	3,94	12,17	0,32	-25,72	38,56
xret1-10	10	4,30	13,54	0,32	-30,35	42,06

Table 33 "mid" regime

Carry Strategy	Long Legs	Mean Returns	Volatility	Sharp Ratio	Min	Max
xret1-2	2	0,63	1,68	0,38	-4,41	7,13
xret1-3	3	1,17	3,01	0,39	-8,26	11,51
xret1-4	4	1,54	4,22	0,37	-12,03	15,31
xret1-5	5	1,82	5,31	0,34	-15,28	18,87
xret1-6	6	2,05	6,32	0,32	-17,89	22,27
xret1-7	7	2,21	7,26	0,30	-19,96	25,47
xret1-8	8	2,32	8,15	0,28	-21,64	28,51
xret1-9	9	2,38	9,04	0,26	-23,12	31,45
xret1-10	10	2,40	9,94	0,24	-28,36	34,37

Table 34 "low" regime

Carry Strategy	Long Legs	Mean Returns	Volatility	Sharp Ratio	Min	Max
xret1-2	2	0,26	1,53	0,17	-3,81	4,55
xret1-3	3	0,54	2,63	0,21	-6,93	8,48
xret1-4	4	0,76	3,50	0,22	-9,70	11,39
xret1-5	5	0,95	4,30	0,22	-12,42	13,64
xret1-6	6	1,14	5,08	0,22	-15,21	15,50
xret1-7	7	1,33	5,89	0,23	-18,08	17,10
xret1-8	8	1,52	6,75	0,23	-21,03	19,89
xret1-9	9	1,71	7,64	0,22	-24,02	23,65
xret1-10	10	1,89	8,56	0,22	-27,01	27,56

A.5.ii.c. Three regimes results obtained by *Slope* factor

Table 35 critical values of tercile by ***Slope***

	Slope
1st and 2nd tercile critical value	0,18
2nd and 3rd tercile critical value	1,31

Table 36 "high" regime

Carry Strategy	Long Legs	Mean Returns	Volatility	Sharp Ratio	Min	Max
xret1-2	2	0,45	1,43	0,32	-2,99	6,48
xret1-3	3	0,86	2,78	0,31	-6,14	10,94
xret1-4	4	1,20	4,21	0,28	-9,08	14,31
xret1-5	5	1,58	5,61	0,28	-11,85	17,41
xret1-6	6	2,02	6,96	0,29	-14,45	20,58
xret1-7	7	2,50	8,27	0,30	-17,67	23,93
xret1-8	8	3,03	9,56	0,32	-21,55	27,50
xret1-9	9	3,58	10,85	0,33	-25,72	31,31
xret1-10	10	4,16	12,15	0,34	-30,19	35,39

Table 37 "mid" regime

Carry Strategy	Long Legs	Mean Returns	Volatility	Sharp Ratio	Min	Max
xret1-2	2	0,56	1,89	0,30	-4,41	8,56
xret1-3	3	1,09	3,40	0,32	-8,26	14,41
xret1-4	4	1,53	4,81	0,32	-12,03	19,70
xret1-5	5	1,93	6,06	0,32	-15,28	25,16
xret1-6	6	2,28	7,14	0,32	-17,89	29,39
xret1-7	7	2,57	8,12	0,32	-19,96	32,67
xret1-8	8	2,81	9,07	0,31	-21,64	35,49
xret1-9	9	3,02	10,07	0,30	-23,12	38,56
xret1-10	10	3,20	11,17	0,29	-30,35	42,06

Table 38 "low" regime

Carry Strategy	Long Legs	Mean Returns	Volatility	Sharp Ratio	Min	Max
xret1-2	2	0,46	1,70	0,27	-3,81	4,55
xret1-3	3	0,91	2,99	0,31	-6,93	8,48
xret1-4	4	1,23	4,08	0,30	-9,70	11,39
xret1-5	5	1,45	5,05	0,29	-12,42	13,64
xret1-6	6	1,57	5,94	0,26	-15,37	15,50
xret1-7	7	1,60	6,78	0,24	-18,33	17,10
xret1-8	8	1,55	7,56	0,21	-21,03	18,51
xret1-9	9	1,43	8,32	0,17	-24,02	19,75
xret1-10	10	1,24	9,05	0,14	-27,01	20,83