Convex Optimization for Extreme Locomotion

Implementation Details

Conventions

N_J Number of Actuated Joints

N_S Number of Bodies in Support

N_F Number of Faces on Linearized Friction Cone

N_P Number of Points in Contact Lattice Under Each Foot

$$\boldsymbol{\tau} \in \mathbb{R}^{N_J}$$

$$\ddot{\boldsymbol{q}} \in \mathbb{R}^{N_J+6}$$

$$\boldsymbol{F}_S \in \mathbb{R}^{6N_S}$$

$$\boldsymbol{\lambda} \in \mathbb{R}^{N_S N_P N_F}$$

Objective

Assume we have a task Jacobian J_t:

$$\min_{\ddot{\boldsymbol{q}},\boldsymbol{\tau},\boldsymbol{\lambda}} \frac{1}{2}||\boldsymbol{J}_t\ddot{\boldsymbol{q}} - \boldsymbol{b}_t||^2$$

$$= \min \frac{1}{2}\ddot{\boldsymbol{q}}^T\boldsymbol{J}_t^T\boldsymbol{J}_t\ddot{\boldsymbol{q}} - \boldsymbol{b}_t^T\boldsymbol{J}_t\ddot{\boldsymbol{q}} + \frac{1}{2}\boldsymbol{b}_t^T\boldsymbol{b}_t$$

Constraints

Dynamics $(\boldsymbol{N}_J + 6 \text{ Constraints})$

$$\boldsymbol{H}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{G} = \boldsymbol{S}^T\boldsymbol{\tau} + \sum_i \boldsymbol{J}_{s_i}^T \mathbf{f}_{s_i}$$

Contact Forces $(6N_S \text{ Total Constraints})$

$$\mathbf{f}_{s_i} = \sum_j {}^{s_i}\boldsymbol{X}_{p_{ij}} \boldsymbol{V} \boldsymbol{\lambda}_{ij}$$

p_ij is the jth contact point on support body i
V is the basis of the linearized friction code

Higher-Priority Tasks $(\text{Number of constraints Varies} := N_{hpt})$

$$\boldsymbol{J}_c \ddot{\boldsymbol{q}} = \boldsymbol{b}_c^*$$

The * denotes an optimal value found at higher levels

Summary

- Nominal Params
  - 30 Joints
  - 2 Supports
  - 4 Verticies
  - 4 faces
- Variables
  - $2N_J + 6 + 6N_S + N_S N_P N_F$
  - Nominally: 78 +32= 110
- Constraints
  - $N_J + 6 + 6N_S + N_{hpt}$
  - Nominally: 48 + N_hpt

Bounds

$$\underline{\boldsymbol{\tau}} \leq \boldsymbol{\tau} \leq \overline{\boldsymbol{\tau}}$$

$$\mathbf{0} \leq \boldsymbol{\lambda}$$

Whole Problem

$$\min_{\ddot{\boldsymbol{q}},\boldsymbol{\tau},\boldsymbol{\lambda}} \quad \frac{1}{2}||\boldsymbol{J}_t\,\ddot{\boldsymbol{q}} - \boldsymbol{b}_t||^2$$

$$\text{s.t.} \quad \boldsymbol{H}\,\ddot{\boldsymbol{q}} + \boldsymbol{C}\,\dot{\boldsymbol{q}} + \boldsymbol{G} = \boldsymbol{S}^T\boldsymbol{\tau} + \sum_{i=1}^{N_S} \boldsymbol{J}_{s_i}^T \mathbf{f}_{s_i}$$

$$\mathbf{f}_{s_i} = \sum_{j=1}^{N_P} {}^{s_i}\boldsymbol{X}_{p_{ij}}\,\boldsymbol{V}\,\boldsymbol{\lambda}_{ij} \qquad i \in \{1,\ldots,N_S\}$$

$$\boldsymbol{J}_c\,\ddot{\boldsymbol{q}} = \boldsymbol{b}_c^*$$

$$\underline{\boldsymbol{\tau}} \leq \boldsymbol{\tau} \leq \overline{\boldsymbol{\tau}}$$

$$\mathbf{0} \leq \boldsymbol{\lambda}$$

Variable Indices

- \vtau = 0 to (N_J-1)
- \vqdd = N_J to $(2\ N_J+5)$
- \vF_s = $(2\ N_J + 6)$ to $(2\ N_J+5 + 6\ N_S)$
  – vF_s_i = $(2\ N_J +6 + 6i)$ – $(2\ N_J + 11 + 6i)$
- \vlambda = $(2\ N_J + 6 + 6\ N_S)$ to
  $(2\ N_J + 5 + 6\ N_S + N_S\ N_P\ N_F)$
  – \vlambda_i = $(2\ N_J + 6 + 6\ N_S +i\ N_P N_F)$ to
  $(2\ N_J + 5 + 6\ N_S +(i+1)\ N_P N_F)$
  – \vlambda_ij =$(2\ N_J + 6 + 6\ N_S +i\ N_P N_F+j\ N_F)$ to
  $(2\ N_J + 5 + 6\ N_S +i\ N_P N_F+(j+1)\ N_F)$

Constraint Indices

- Dynamics = 0 to (N_J+5)
- Contact Forces = (N_J+6) to (N_J+5+6N_S)
- High Priority Tasks
  - Problem Dependent (N_J+5+6N_S) to ?