Chapter 8

Linear Programming with Matlab

The Matlab function LINPROG can be used to solve a linear programming problem with the following syntax (help LINPROG):

X=LINPROG(f, A, b) solves the linear programming problem:

$$\min f'^*x \quad \text{subject to:} \quad A^*x <= b$$

X=LINPROG(f, A, b, Aeq, beq) solves the problem above while additionally satisfying the equality constraints Aeq*x = beq.

X=LINPROG(f, A, b, Aeq, beq, LB, UB) defines a set of lower and upper bounds on the design variables, X, so that the solution is in the range LB <= X <= UB. Use empty matrices for LB and UB if no bounds exist. Set LB(i) = -Inf if X(i) is unbounded below; set UB(i) = Inf if X(i) is unbounded above.

Since LINPROG minimizes an objective function, the example 3.2-1 is reformulated as

$$\text{Minimize } z = -150x_1 - 175x_2$$

Note: Maximizing ($150x_1 + 175x_2$) is equivalent to minimizing ($-150x_1 - 175x_2$)

Subject to

$7x_1 + 11x_2 \le 77$	(material constraint)
$10x_1 + 8x_2 \le 80$	(time constraint)
$x_1 \le 9$	(storage constraint of regular heating gas)
$x_2 \le 6$	(storage constraint of premium heating gas)
$x_1, x_2 \ge 0$	(positive production constraint)

The coefficient vector f for the objective function is

$$f = \begin{bmatrix} -150 \\ -175 \end{bmatrix}$$

The matrix coefficient A for the inequality constraints is

$$A = \begin{bmatrix} 7 & 11 \\ 10 & 8 \end{bmatrix}$$

The right hand vector b for the inequality constraints is

$$b = \begin{bmatrix} 77 \\ 80 \end{bmatrix}$$

Since there are no equality constraints in this example, *Aeq* and *beq* are zeros.

$$Aeq = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } beq = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The lower and upper bounds vectors are given by

$$\mathrm{LB} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ and } \mathrm{UB} = \begin{bmatrix} 9 \\ 6 \end{bmatrix}$$

The following Matlab statements are used to solve this linear programming problem.

Matlab Example ---

```
% Example 8.3-1
f=[-150;-175];
A=[7 11;10 8];
b=[77;80];
Aeq=[0 0;0 0];
beq=[0;0];
LB=[0;0];UB=[9;6];
x=linprog(f,A,b,Aeq,beq,LB,UB)

>> e3d2d1
Optimization terminated successfully.
x =
   4.8889
   3.8889
```

Example 8.3-2 --

Solve the following linear programming problem using Matlab LINPROG

Maximize $y = x_1 + 2x_2$

Subject to

$$2x_1 + x_2 \le 10$$
$$x_1 + x_2 \le 6$$
$$-x_1 + x_2 \le 2$$
$$-2x_1 + x_2 \le 1$$
$$x_1, x_2 \ge 0$$

Solution ---

The problem is reformulated using slack variable S_1, S_2, S_3, and S_4

Minimize $y = -x_1 - 2x_2$

Subject to

$$
\begin{aligned}
&2x_1 + x_2 + S_1 = 10 \\
&x_1 + x_2 + S_2 = 6 \\
&-x_1 + x_2 + S_3 = 2 \\
&-2x_1 + x_2 + S_4 = 1 \\
&x_1, x_2, S_1, S_2, S_3, \text{ and } S_4 \geq 0
\end{aligned}
$$

The arguments for the Matlab command X=LINPROG(f, A, b, Aeq, beq, LB, UB) are given as

$$
f = \begin{bmatrix} -1 \\ -2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, A = 0, b = 0, Aeq = \begin{bmatrix} 2 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ -2 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, beq = \begin{bmatrix} 10 \\ 6 \\ 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \text{LB} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \text{UB} = \begin{bmatrix} \infty \\ \infty \\ \infty \\ \infty \\ \infty \\ \infty \end{bmatrix}
$$

The following Matlab statements are used to solve this linear programming problem.

Matlab Example ---

```
% Example 8.3-2
f=[-1;-2;0;0;0;0];
A=zeros(6,6);
b=zeros(6,1);
Aeq=[2 1 1 0 0 0;1 1 0 1 0 0; -1 1 0 0 1 0;-2 1 0 0 0 1;0 0 0 0 0 0;0 0 0 0 0 0];
beq=[10;6;2;1;0;0];
LB=[0;0;0;0;0;0];UB=[inf;inf;inf;inf;inf;inf];
x=linprog(f,A,b,Aeq,beq,LB,UB)

>> e8d3d2
Optimization terminated successfully.
x =
   2.0000
   4.0000
   2.0000
   0.0000
   0.0000
   1.0000
```

The solution from the Matlab program is

$x_1 = 2$

$x_2 = 4$

$S_1 = 2$

$S_2 = 0$

$S_3 = 0$

$S_4 = 1$

This solution is verified with the graphical solution shown in Figure 8.3-2. The vertex D is the basic feasible solution obtained graphically.

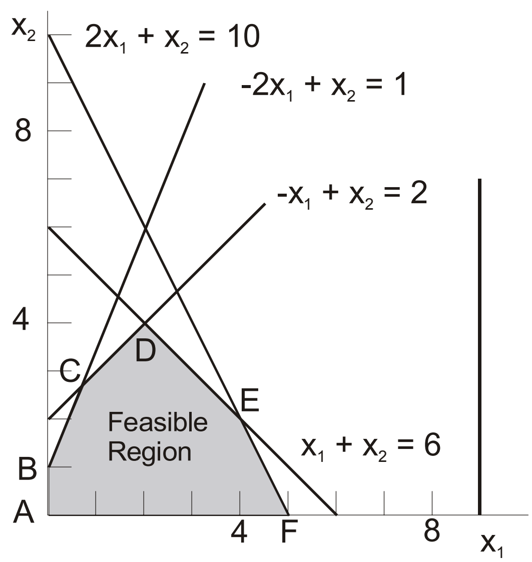

Figure 8.3-2 Geometric representation of the linear programming problem.

The problem can also be reformulated using only one slack variable S_1

Minimize $y = - x_1 - 2x_2$

Subject to

$2x_1 + x_2 + S_1 = 10$
$x_1 + x_2 \le 6$
$- x_1 + x_2 \le 2$
$- 2x_1 + x_2 \le 1$
x_1, x_2, and $S_1 \ge 0$

Consider the Matlab command X=LINPROG(*f*, *A*, *b*, *Aeq*, *beq*, LB, UB). The coefficient vector *f* for the objective function is

$$f = \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix}$$

The matrix coefficient *A* for the inequality constraints is

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ -2 & 1 & 0 \end{bmatrix}$$

The right hand vector *b* for the inequality constraints is

$$b = \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}$$

The equality constraints *Aeq* and *beq* are given as

$$Aeq = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } beq = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

The lower and upper bounds vectors are given by

$$\text{LB} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ and UB} = \begin{bmatrix} \infty \\ \infty \\ \infty \end{bmatrix}$$

The following Matlab statements are used to solve this linear programming problem.

```
% Example 8.3-2b
f=[-1;-2;0];
A=[1 1 0;-1 1 0;-2 1 0];
b=[6;2;1];
Aeq=[2 1 1; 0 0 0;0 0 0];
beq=[10;0;0];
LB=[0;0;0];UB=[inf;inf;inf];
x=linprog(f,A,b,Aeq,beq,LB,UB)

>> e8d3d2b
```

```
Optimization terminated successfully.
x =
   2.0000
   4.0000
   2.0000
```

The solution from the Matlab program is

$$x_1 = 2$$

$$x_2 = 4$$

$$S_1 = 2$$
