

# W = emgr(f, g, s, t, w, pr, nf, ut, us, xs, um, xm);

## emgr – Empirical Gramian Framework ( Version 3.8 )

### Mandatory Arguments

<b>f</b>	System Vector Field	(Function Handle)	$\mathbf{x} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{p})$	i.e.: $\mathbf{f} = @(\mathbf{x}, \mathbf{u}, \mathbf{p}) \mathbf{A}*\mathbf{x} + \mathbf{B}*\mathbf{u} + \mathbf{F}*\mathbf{p}$ ;
<b>g</b>	Output Functional	(Function Handle)	$\mathbf{y} = \mathbf{g}(\mathbf{x}, \mathbf{u}, \mathbf{p})$	i.e.: $\mathbf{g} = @(\mathbf{x}, \mathbf{u}, \mathbf{p}) \mathbf{C}*\mathbf{x} + \mathbf{D}*\mathbf{u}$ ;
		1	$\mathbf{y} = \mathbf{x}$	
<b>s</b>	System Dimensions	(Vector)	$\mathbf{s} = [\mathbf{J}, \mathbf{N}, \mathbf{o}]$	(Inputs, States, Outputs)
<b>t</b>	Time	(Vector)	$\mathbf{t} = [\mathbf{h}, \mathbf{T}]$	(Step, Stop)
<b>w</b>	Gramian Type	(Character)		
		'c'	Empirical Controllability Gramian (returns <b>WC</b> )	
		'o'	Empirical Observability Gramian (returns <b>WO</b> )	
		'x'	Empirical Cross Gramian (returns <b>WX</b> )	
		'y'	Empirical Linear Cross Gramian (returns <b>WY</b> )	
		's'	Empirical Sensitivity Gramian (returns <b>WS</b> = { <b>WC</b> , <b>WS</b> })	
		'i'	Empirical Identifiability Gramian (returns <b>WI</b> = { <b>WO</b> , <b>WI</b> })	
		'j'	Empirical Joint Gramian (returns <b>WJ</b> = { <b>WX</b> , <b>WI</b> })	

### Optional Arguments

<b>pr</b>	Parameters	(Vector)	Column Vector of System Parameters (Default: <b>pr</b> = <b>0</b> )	
		(Matrix)	Set of Parameter Column Vectors ('s', 'i', 'j' requires two)	
<b>nf</b>	Options	(Vector)	Ten Components (Default: <b>nf</b> = <b>0</b> ), see Option Flags	
<b>ut</b>	Input Function	(Scalar)	Uniformly Scaled Impulse Input (Default: <b>ut</b> = <b>1</b> )	
		(Vector)	Individual Scaled Impulse Input ( <b>J x 1</b> )	
		(Matrix)	Discrete Input Function ( <b>J x (T-S)/h</b> )	
		(Function Handle)	Function Handle ( <b>u</b> = <b>ut(t)</b> )	
		$\infty$	Chirp Function	
<b>us</b>	Steady-State Input	(Scalar)	Uniform Steady-State Input (Default: <b>us</b> = <b>0</b> )	
		(Vector)	Individual Steady-State Input ( <b>J x 1</b> )	
<b>xs</b>	Steady State	(Scalar)	Uniform Steady State (Default: <b>xs</b> = <b>0</b> )	
		(Vector)	Individual Steady States ( <b>N x 1</b> )	
<b>um</b>	Input Scales	(Scalar)	Uniform Maximum Input Scales (Default: <b>um</b> = <b>1</b> )	
		(Vector)	Individual Maximum Input Scales ( <b>J x 1</b> )	
		(Matrix)	Custom Input Scales ( <b>J x *</b> )	
<b>xm</b>	Steady-State Scales	(Scalar)	Uniform Maximum Steady State Scales (Default: <b>xm</b> = <b>1</b> )	
		(Vector)	Individual Maximum Steady-State Scales ( <b>N x 1</b> )	
		(Matrix)	Custom Steady-State Scales ( <b>N x *</b> )	

### Option Flags

<b>nf (1)</b>	Trajectory Centering				<b>nf (7)</b>	Non-Symmetric Cross Gramian			
	0	None (Default)				0	Off (Default)		
	1	Initial State				1	Non-Sym. Cross-Gramian ( <b>WX</b> , <b>WJ</b> only)		
	2	Final Steady State			<b>nf (8)</b>	Robust Parameters			
	3	Arithmetic Average				0	Off (Default)		
	4	Median				1	Treat Parameters as Inputs		
	5	Midrange			<b>nf (9)</b>	Parameter Action			
	6	Root-Mean-Squared				0	Active Parameters		
<b>nf (2)</b>	Input Scale Sequence					1	Passive Parameters		
	0	Linear (Default)			<b>nf (10)</b>	Center Parameter Scales			
	1	Logarithmic				0	No Centering		
	2	Geometric				1	Mean Centered Parameters		
	3	Single				2	Logarithmic Centered Parameters		
	4	Sparse			<b>nf (11)</b>	Exclusive Options			
<b>nf (3)</b>	State Scale Sequence					0	None (Default)		
	0	Linear (Default)				1	Root-Mean-Square-Centering ( <b>WS</b> only)		
	1	Logarithmic				1	Schur Complement ( <b>WI</b> only)		
	2	Geometric				1	Detailed Schur Complement ( <b>WJ</b> only)		
	3	Single			<b>nf (12)</b>	Gramian Symmetry			
	4	Sparse				0	Assume Symmetry		
<b>nf (4)</b>	Input Transformation					1	Enforce Symmetry		
	0	Unit (Default)							
	1	Inverse							
	2	Dyadic							
	3	Single							
<b>nf (5)</b>	State Transformation								
	0	Unit (Default)							
	1	Inverse							
	2	Dyadic							
	3	Single							
<b>nf (6)</b>	Preconditioning								
	0	None (Default)							
	1	Jacobi (Double Run)							
	2	Steady-State Scaled							

### Custom Solver

Set global variable **ODE** to solver function handle with signature: **y = solver(f, g, h, T, x, u, p)**;  
default solver: 2nd Order Ralston's Runge-Kutta

**Minimal Usage: W = emgr(f, g, s, t, w);**

**About Info: V = emgr('version');**

More info at: <http://gramian.de>