# W = emgr(f,g,s,t,w,pr,nf,ut,us,xs,um,xm,dp)

emgr - EMpirical GRamian Framework (Version 5.8)

## **Mandatory Arguments**

	· · · · · · · · · · · · · · · · · · ·					
f	System Vector Field	(Handle)	x = f(x,u,	<b>p,t)</b> i.e.: <b>f</b>	= @(x,u,p,t)	A*x+B*u+F*p
g	Output Functional	(Handle) <b>1</b>	y = g(x,u, y = x	<b>p,t)</b> i.e.: <b>g</b>	= @(x,u,p,t)	C*x+D*u
S	System Dimensions	(Vector)	s = [M,N,Q]	] (Input	ts, States, Outpu	uts)
t	Time Discretization	(Vector)	t = [dt,Tf	] (Time	Step, Time Hori	izon)
W	Gramian Type	(Char) 'c' 'o' 'x' 'y' 's' 'i'	Empirical Controllability Gramian (returns $\mathbf{W_c}$ ) Empirical Observability Gramian (returns $\mathbf{W_o}$ ) Empirical Cross Gramian (returns $\mathbf{W_x}$ ) Empirical Linear Cross Gramian (returns $\mathbf{W_y}$ ) Empirical Sensitivity Gramian (returns $\mathbf{W_c}$ , $\mathbf{W_s}$ )			

### **Optional Arguments**

- p						
pr	Parameters		Column vector of parameters (default: $pr = 0$ ) Set of parameter columns ( $\mathbf{W_s}$ , $\mathbf{W_l}$ , $\mathbf{W_l}$ require min & max)			
nf	Options Flags	(Vector)	Thirteen components (default: nf = 0)			
ut	Input Function	(Handle)	Input function $\mathbf{u_t} = \mathbf{ut(t)}$ or char (default: $\mathbf{ut} = 'i'$ )  Delta impulse input (default)			
		's'	Step input			
		'c'	Decaying exponential chirp input			
		'a'	Sinc input			
		'r'	Pseudo-random binary input			
us	Steady-State Input	(Scalar)	Uniform steady-state input (default: us = 0)			
		(Vector)	Individual steady-state input (M x 1)			
xs	Steady-State	(Scalar)	Uniform steady-state (default: xs = 0)			
		(Vector)	Individual steady-states (N x 1)			
um	Input Scales	(Scalar)	Uniform max input scales (default: um = 1)			
		(Vector)	Individual max input scales (M x 1)			
		(Matrix)	Custom input scales (M x *)			
xm	Steady-State Scales	(Scalar)	Uniform max steady-state scales (default: xm = 1)			
		(Vector)	Individual max steady-state scales (N x 1)			
		(Matrix)	Custom steady-state scales (N x *)			
dр	Dot Product	(Handle)	Custom Inner product / kernel $xy = dp(x,y)$ , (default: $dp = []$ )			

Optio	on Flags	
nf(1)	Trajectory Centering	nf(8)
	None (default)	
	1 Steady state	
	2 Final state	nf(9)
	3 Arithmetic average	
	4 Root-mean-squared	
	5 Mid-range	
nf(2)	Input Scale Sequence	nf(10) <sup>F</sup>
	0 Single (default)	$\mathbf{W}_{s}$
	1 Linear	$\mathbf{W}_{s}$
	2 Geometric	$\mathbf{W}_{_{\mathbf{I}'}}\mathbf{W}_{_{\mathbf{J}}}$
	<b>3</b> Logarithmic	$\mathbf{W}_{l}, \mathbf{W}_{j}$
	4 Sparse	nf(11) F
nf(3)	State Scale Sequence	
	<b>0</b> Single (default)	
	<b>1</b> Linear	nf(12) <sup>F</sup>
	<b>2</b> Geometric	
	<b>3</b> Logarithmic	
	4 Sparse	nf(13) 7
nf(4)	Input Transformation	
	<pre>0 ± Unit (default)</pre>	
	1 + Unit	
nf(5)	State Transformation	
	0 ± Unit (default)	
	1 + Unit	
nf(6)	Normalizing	Custo
	<b>0</b> None (default)	Global var
	1 Steady state	y = ODE(
	<b>2</b> Jacobi	
nf(7)	• •	
	0 Regular (default)	
W <sub>c</sub>	1 Output controllability (	
$W_o, W_i$	1 Averaged observability	Gramian

 $\mathbf{W}_{x}, \mathbf{W}_{y}, \mathbf{W}_{J}$  1 Non-symmetric cross Gramian

nf(8)	Extra Input ( $\mathbf{W_o}$ , $\mathbf{W_x}$ , $\mathbf{W_s}$ , $\mathbf{W_I}$ , $\mathbf{W_J}$ only)					
	0	No (default)				
	1	Yes				
nf(9)	Cen	Center Param. Scales ( <b>W<sub>s</sub>, W<sub>ı</sub>, W<sub>ı</sub></b> only)				
	0	None (default)				
	1	Linear mean centering				
	2	Logarithmic mean centering				
nf(10)	Parameter Gramian Type ( <b>W</b> <sub>s</sub> , <b>W</b> <sub>I</sub> , <b>W</b> <sub>I</sub> only)					
$\mathbf{W}_{s}$	0	Input-state average (default)				
$\mathbf{W}_{s}$	1	Input-output average				
$\mathbf{W}_{i}, \mathbf{W}_{j}$	0	Approx. Schur-complement (default)				
$\mathbf{W}_{l}, \mathbf{W}_{j}$	1	Coarse Schur-complement				
nf(11)	Part	Partitioned Cross Gramian ( <b>W</b> <sub>x</sub> , <b>W</b> <sub>J</sub> only)				
	0	Full cross Gramian (default)				
	<n< th=""><th>Cross Gramian partition size</th></n<>	Cross Gramian partition size				
nf(12)	Part	itioned Cross Gramian ( <b>W<sub>x</sub>, W<sub>J</sub></b> only)				
	0	Full cross Gramian (default)				
	>0	Partition running index				
nf(13)	Trajectory Weighting					
	0	None (default)				
	1	Linear time-weighting				
	2	Quadratic time-weighting				
	3	State weighting				
	4	Scale weighting				

#### m Solver

ariable **ODE** is a handle with signature: (f,g,t,x0,u,p) (default: RK - SSP32)

About Info: V = emgr('version')

More info at: https://gramian.de

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