1

Numerical Solution Table:

	y=0.00	y=0.31	y=0.63	y=0.94	y=1.26	y=1.57
x=0.00	1.000000e+00	0.951057	0.809017	0.587785	0.309017	0.0
x=0.31	9.510565e-01	0.753226	0.564622	0.368085	0.172806	0.0
x=0.63	8.090170e-01	0.555909	0.347641	0.176348	0.053087	0.0
x=0.94	5.877853e-01	0.333232	0.132648	-0.004975	-0.058882	0.0
x=1.26	3.090170e-01	0.085807	-0.086862	-0.182347	-0.166748	0.0
x=1.57	6.123234e-17	-0.173196	-0.305632	-0.353912	-0.269910	0.0
x=1.88	-3.090170e-01	-0.424257	-0.511151	-0.511645	-0.364162	0.0
x=2.20	-5.877853e-01	-0.645220	-0.686178	-0.642021	-0.441313	0.0
x=2.51	-8.090170e-01	-0.814475	-0.809947	-0.724390	-0.486321	0.0
x=2.83	-9.510565e-01	-0.915787	-0.858894	-0.725487	-0.467875	0.0
x=3.14	-1.000000e+00	-0.951057	-0.809017	-0.587785	-0.309017	0.0

2a

=== (a) Forward Difference Method ===

	r=0.5	r=0.6	r=0.7	r=0.8	r=0.9	r=1.0	r=1.1
t=0.0	0.000000e+00	2.000000e+01	4.000000e+01	6.000000e+01	8.000000e+01	1.000000e+02	120.0
t=0.5	2.820513e+01	3.666667e+01	5.428571e+01	7.250000e+01	9.111111e+01	1.100000e+02	120.0
t=1.0	7.178548e+01	9.332112e+01	7.005952e+01	8.599206e+01	1.029167e+02	7.277778e+01	140.0
t=1.5	-1.010648e+02	-1.313842e+02	2.634127e+02	1.012202e+02	-1.360714e+02	5.688542e+02	160.0
t=2.0	1.650818e+03	2.146063e+03	-2.438461e+03	-3.991146e+02	4.704913e+03	-4.926027e+03	180.0
t=2.5	-1.919743e+04	-2.495665e+04	2.977190e+04	1.715660e+04	-7.022740e+04	6.762758e+04	200.0
t=3.0	2.291432e+05	2.978861e+05	-2.919070e+05	-3.879366e+05	1.069987e+06	-9.411784e+05	220.0
t=3.5	-2.470691e+06	-3.211898e+06	1.931974e+06	7.807423e+06	-1.642914e+07	1.355420e+07	240.0
t=4.0	2.157534e+07	2.804794e+07	9.525331e+06	-1.484905e+08	2.562667e+08	-2.000249e+08	260.0
t=4.5	-7.842228e+07	-1.019490e+08	-7.509901e+08	2.742481e+09	-4.063292e+09	3.017493e+09	280.0
t=5.0	-2.699814e+09	-3.509758e+09	2.097744e+10	-4.978884e+10	6.544589e+10	-4.645807e+10	300.0
t=5.5	1.021857e+11	1.328415e+11	-4.718182e+11	8.941126e+11	-1.069322e+12	7.289906e+11	320.0
t=6.0	-2.525310e+12	-3.282904e+12	9.653016e+12	-1.593944e+13	1.769355e+13	-1.164020e+13	340.0
t=6.5	5.404536e+13	7.025896e+13	-1.875090e+14	2.827004e+14	-2.959459e+14	1.888061e+14	360.0
t=7.0	-1.077151e+15	-1.400297e+15	3.528250e+15	-4.995465e+15	4.994964e+15	-3.104998e+15	380.0
t=7.5	2.059776e+16	2.677708e+16	-6.501705e+16	8.803360e+16	-8.493186e+16	5.167106e+16	400.0
t=8.0	-3.836639e+17	-4.987630e+17	1.181084e+18	-1.548270e+18	1.452809e+18	-8.684659e+17	420.0
t=8.5	7.021498e+18	9.127947e+18	-2.123975e+19	2.718881e+19	-2.497013e+19	1.471704e+19	440.0
t=9.0	-1.269372e+20	-1.650183e+20	3.791919e+20	-4.769145e+20	4.307961e+20	-2.510615e+20	460.0
t=9.5	2.274866e+21	2.957325e+21	-6.733782e+21	8.358296e+21	-7.454307e+21	4.305834e+21	480.0
t=10.0	-4.051086e+22	-5.266411e+22	1.191111e+23	-1.463903e+23	1.292837e+23	-7.416047e+22	500.0

2b

=== (b) Backward Difference Method ===

	r=0.5	r=0.6	r=0.7	r=0.8	r=0.9	r=1.0	r=1.1
t=0.0	0.000000	20.000000	40.000000	60.000000	80.000000	100.000000	120.0
t=0.5	25.968074	33.758496	45.967212	61.339438	79.047041	98.636088	120.0
t=1.0	30.981170	40.275521	52.681651	68.442602	87.820356	111.335723	140.0
t=1.5	35.548889	46.213556	60.112825	77.751003	99.726730	126.809187	160.0
t=2.0	40.384455	52.499792	68.189459	88.032505	112.714676	143.051022	180.0
t=2.5	45.482742	59.127564	76.699323	98.807322	126.161683	159.581805	200.0
t=3.0	50.755723	65.982440	85.473260	109.852439	139.838467	176.248524	220.0
t=3.5	56.134624	72.975011	94.402938	121.050786	153.640659	192.987453	240.0
t=4.0	61.575864	80.048623	103.423399	132.337163	167.513855	209.766836	260.0
t=4.5	67.053414	87.169439	112.496554	143.674368	181.427827	226.569357	280.0
t=5.0	72.552036	94.317647	121.600251	155.040975	195.365338	243.385216	300.0
t=5.5	78.062870	101.481730	130.721639	166.424601	209.316465	260.208787	320.0
t=6.0	83.580776	108.655009	139.853272	177.818081	223.275474	277.036820	340.0
t=6.5	89.102779	115.833612	148.990838	189.217267	237.239046	293.867437	360.0
t=7.0	94.627153	123.015299	158.131840	200.619757	251.205260	310.699550	380.0
t=7.5	100.152902	130.198772	167.274832	212.024160	265.173004	327.532529	400.0
t=8.0	105.679446	137.383279	176.418975	223.429671	279.141635	344.366010	420.0
t=8.5	111.206450	144.568385	185.563786	234.835823	293.110778	361.199781	440.0
t=9.0	116.733721	151.753838	194.708983	246.242347	307.080219	378.033721	460.0
t=9.5	122.261147	158.939491	203.854404	257.649087	321.049831	394.867758	480.0
t=10.0	127.788662	166.125261	212.999955	269.055950	335.019543	411.701851	500.0

2c

```
=== (c) Crank-Nicolson Method ===
           r=0.5        r=0.6        r=0.7        r=0.8        r=0.9         r=1.0   r=1.1
t=0.0    0.000000    20.000000    40.000000    60.000000    80.000000    100.000000   120.0
t=0.5   19.324707    25.122120    37.845077    54.674062    74.141177     95.770098   120.0
t=1.0   29.430795    38.260034    48.556026    62.901316    82.275340    107.502161   140.0
t=1.5   33.371336    43.382737    57.588317    75.404905    97.815873    125.871798   160.0
t=2.0   39.535546    51.396210    66.536223    86.434416   111.472791    142.232592   180.0
t=2.5   44.771420    58.202846    75.923654    97.985609   125.430576    159.240162   200.0
t=3.0   50.460676    65.598879    84.869537   109.343655   139.485669    175.990749   220.0
t=3.5   55.889701    72.656611    94.171733   120.765230   153.373053    192.881858   240.0
t=4.0   61.483410    79.928433   103.200573   132.174031   167.409847    209.671280   260.0
t=4.5   66.965534    87.055194   112.433903   143.573475   181.329521    226.542620   280.0
t=5.0   72.525561    94.283230   121.514506   154.990409   195.336458    243.346286   300.0
t=5.5   78.029967   101.438957   130.708958   166.387524   209.279308    260.205207   320.0
t=6.0   83.574500   108.646849   139.818240   177.803739   223.268814    277.019665   340.0
t=6.5   89.089793   115.816731   148.991357   189.202757   237.224415    293.869344   360.0
t=7.0   94.626521   123.014478   158.116472   200.616605   251.204593    310.691462   380.0
t=7.5  100.147438   130.191670   167.277763   212.017980   265.166975    327.534920   400.0
t=8.0  105.680072   137.384094   176.411721   223.429656   279.142215    344.361991   420.0
t=8.5  111.203984   144.565179   185.566373   234.832927   293.108185    361.201556   440.0
t=9.0  116.734412   151.754736   194.705326   246.242994   307.080817    378.031651   460.0
t=9.5  122.259955   158.937942   203.856208   257.647601   321.048679    394.868887   480.0
t=10.0 127.789171   166.125923   212.998011   269.056574   335.019940    411.700763   500.0
```

3

```
        θ=0.00  θ=0.10  θ=0.21  θ=0.31  θ=0.42  θ=0.52  θ=0.63  θ=0.73  θ=0.84  θ=0.94  θ=1.05
r=0.50     0.0   50.00   50.00   50.00   50.00   50.00   50.00   50.00   50.00   50.00     0.0
r=0.60     0.0   21.32   33.75   41.07   44.98   46.21   44.98   41.07   33.75   21.32     0.0
r=0.70     0.0   17.86   31.92   41.51   46.98   48.75   46.98   41.51   31.92   17.86     0.0
r=0.80     0.0   21.49   38.07   49.05   55.15   57.10   55.15   49.05   38.07   21.49     0.0
r=0.90     0.0   31.27   51.02   62.24   67.94   69.67   67.94   62.24   51.02   31.27     0.0
r=1.00     0.0   52.32   71.68   79.95   83.59   84.64   83.59   79.95   71.68   52.32     0.0
r=1.10     0.0  100.00  100.00  100.00  100.00  100.00  100.00  100.00  100.00  100.00     0.0
```

4

```
      x=0.0      x=0.1      x=0.2      x=0.3      x=0.4      x=0.5      x=0.6      x=0.7      x=0.8      x=0.9  x=1.0
t=0.0   1.0   0.809017   0.309017  -0.309017  -0.809017  -1.000000  -0.809017  -0.309017   0.309017   0.809017    2.0
t=0.1   1.0   1.023825   0.847566   0.347566  -0.285192  -0.809017  -1.023825  -0.847566  -0.347566   0.285192    2.0
t=0.2   1.0   1.038549   1.062374   0.871391   0.347566  -0.309017  -0.847566  -1.062374  -0.871391   0.843417    2.0
t=0.3   1.0   1.038549   1.062374   1.062374   0.847566   0.309017  -0.347566  -0.871391   0.128609   0.843417    2.0
t=0.4   1.0   1.023825   1.038549   1.038549   1.023825   0.809017   0.285192   0.843417   0.843417   1.285192    2.0
t=0.5   1.0   1.000000   1.000000   1.000000   1.000000   1.000000   2.000000   2.000000   2.000000   2.000000    2.0
t=0.6   1.0   0.976175   0.961451   0.961451   0.976175   2.190983   2.714808   3.156583   3.156583   2.714808    2.0
t=0.7   1.0   0.961451   0.937626   0.937626   2.152434   2.690983   3.347566   3.871391   3.871391   3.156583    2.0
t=0.8   1.0   0.961451   0.937626   2.128609   2.652434   3.309017   3.847566   4.062374   3.871391   3.156583    2.0
t=0.9   1.0   0.976175   2.152434   2.652434   3.285192   3.809017   4.023825   3.847566   3.347566   2.714808    2.0
```