Differentiation

Differentiation Rules

$$\frac{d}{dx}x^n = nx^{n-1} \qquad \frac{d}{dx}e^x = e^x \qquad \frac{d}{dx}b^x = b^x \ln b \qquad \frac{d}{dx}\ln x = \frac{1}{x} \qquad \frac{d}{dx}\log_b x = \frac{1}{x \ln b}$$

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x) \qquad \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2} \qquad \frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Derivatives of Trigonometric Functions (and their Inverses)

$$\begin{aligned}
&\frac{d}{dx}\sin x = \cos x && \frac{d}{dx}\cos x = -\sin x && \frac{d}{dx}\tan x = \sec^2 x \\
&\frac{d}{dx}\csc x = -\csc x \cot x && \frac{d}{dx}\sec x = \sec x \tan x && \frac{d}{dx}\cot x = -\csc^2 x \\
&\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}} && \frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}} && \frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}
\end{aligned}$$

Integration

Remember to always include the constant of integration $+\,C$ when evaluating indefinite integrals.

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}\ (n \neq -1) \qquad \int e^x\,dx = e^x \qquad \int \frac{1}{x}\,dx = \ln|x| \qquad \int \ln x\,dx = x\ln x - x \qquad \int b^x\,dx = \frac{b^x}{\ln b}$$

Integration by Parts

For choice of u: use the mnemonic LIATE: Logarithmic, Inverse-trigonometric, Algebraic, Trigonometric, Exponential.

$$\int u\,dv = uv - \int v\,du \qquad \int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$$

Integrals of Trigonometric Functions (and Hyperbolic Functions)

$$\begin{aligned}
&\int \sin x\,dx = -\cos x + C && \int \cos x\,dx = \sin x + C && \int \sec^2 x\,dx = \tan x + C \\
&\int \csc^2 x\,dx = -\cot x + C && \int \sec x \tan x\,dx = \sec x + C && \int \csc x \cot x\,dx = -\csc x + C \\
&\int \sec x\,dx = \ln|\sec x + \tan x| + C && \int \tan x\,dx = \ln|\sec x| + C && \int \sinh x\,dx = \cosh x + C \\
&\int \csc x\,dx = \ln|\csc x - \cot x| + C && \int \cot x\,dx = \ln|\sin x| + C && \int \cosh x\,dx = \sinh x + C
\end{aligned}$$

Integrals that yield Inverse Trigonometric Functions

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C \qquad \int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C \qquad \int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$$

Reduction Formulas for $\sin$ **and** $\cos$

$$\int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx \qquad \int \cos^n x \, dx = +\frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx$$

Half-Angle Identities

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x) \qquad \sin^2 x = \frac{1}{2}(1 - \cos 2x) \qquad \sin x \cos x = \frac{1}{2} \sin 2x$$

Trigonometric Identities for Evaluating Integrals of Products of Trigonometric Powers

$\cos^n$ **is odd, use** $u = \sin x$	$\sin^n$ **is odd, use** $u = \cos x$	$\sec^n$ **is even, use** $u = \tan x$	$\tan^n$ **is odd, use** $u = \sec x$
$\cos^2 x = (1 - \sin^2 x)$	$\sin^2 x = (1 - \cos^2 x)$	$\sec^2 x = (1 + \tan^2 x)$	$\tan^2 x = (\sec^2 x - 1)$

Product Identities for Trigonometric Products of $\sin$, $\cos$

$$\sin A \cos B = \frac{\sin(A-B) + \sin(A+B)}{2} \qquad \cos A \cos B = \frac{\cos(A-B) + \cos(A+B)}{2} \qquad \sin A \sin B = \frac{\cos(A-B) - \cos(A+B)}{2}$$

Trigonometric Substitutions

Expression	**Substitution**	**Domain**	**Identity**
$\int \sqrt{a^2 - x^2} \, dx$	$x = a \sin \theta,$	$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$	$1 - \sin^2 \theta = \cos^2 \theta$
$\int \sqrt{a^2 + x^2} \, dx$	$x = a \tan \theta,$	$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$1 + \tan^2 \theta = \sec^2 \theta$
$\int \sqrt{x^2 - a^2} \, dx$	$x = a \sec \theta,$	$0 \leq \theta < \frac{\pi}{2}$	$\sec^2 \theta - 1 = \tan^2 \theta$