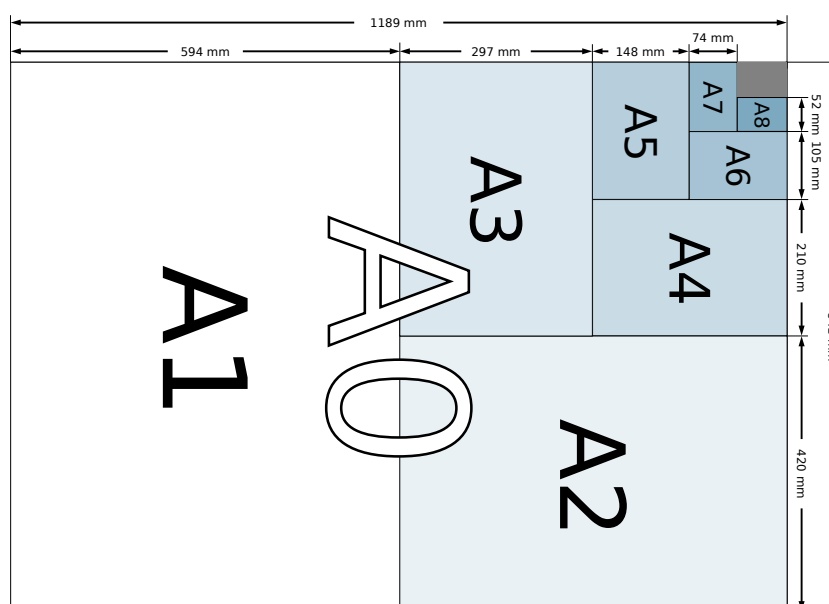


# THE MATH BEHIND PAPER

## Investigating Geometric Sequences Behind the ISO 216 Series-A Paper Format

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May 20, 2019



## Introduction

Understanding the properties of arithmetic and geometric sequences are essential for quantifying the world, and for computing in general. In this paper, I will investigate the different sizes of ISO 216 standard A-series paper, and quantify their properties using my knowledge of sequences.

# The Properties of ISO 216 A-Series Paper

Let us begin by quantifying the properties of ISO 216 A-series paper (henceforth referred to as A-paper). It is given that the largest A-paper size,  $A_0$ , has a total area of  $1 \text{ m}^2$ . Likewise, we know that each successive smaller paper is the previous paper folded in half.

## Formalizing the Area

We can formalize this property as the following geometric sequence:

$$\begin{aligned}a_n &= a \times r^{n-1} \\A_n &= A \times \left(\frac{1}{2}\right)^{n-1} \\&= 2^{-n}\end{aligned}$$

By plugging in the the numbers, it is trivial to generate a table of area for each successive A-series paper:

$n$	Area (fractional $\text{m}^2$ )	Area (decimal $\text{m}^2$ )
0	1	1
1	$\frac{1}{2}$	0.5
2	$\frac{1}{4}$	0.25
3	$\frac{1}{8}$	0.125
4	$\frac{1}{16}$	0.0625
5	$\frac{1}{32}$	0.03125
6	$\frac{1}{64}$	0.015625

Table 1: List of A-series paper areas for  $0 \leq n \leq 6$

## Formalizing the Length and Width

The geometric sequence behind the area of the A-series paper is trivial to discover, but what about the length and width of the paper for a given  $n$  in  $A_n$ ? Let us begin first by listing the sequence of dimensions for both the length  $L_n$  and the width  $W_n$  of  $A_n$ . In order to work with exact values, we will use variables for the starting values rather than their decimal approximations.

$$\begin{aligned} L_n &= L_0, \frac{L_0}{2}, \frac{L_0}{2}, \frac{L_0}{4}, \frac{L_0}{4}, \frac{L_0}{8}, \frac{L_0}{8}, \\ W_n &= W_0, W_0, \frac{W_0}{2}, \frac{W_0}{2}, \frac{W_0}{4}, \frac{W_0}{4}, \frac{W_0}{8}, \end{aligned}$$

## The Scaling Factor of Conversions

## Image credit

Cover page illustration is a diagram illustrating ISO 216 A-series paper sizes, sourced from Wikipedia under Creative Commons (CC BY-SA 3.0) license.

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