

A TOURIST'S JOURNAL OF THE CRITIQUE: Adventures in the Land of Transcendental Metaphysics

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Introduction

How is mathematics possible? How is physics possible? Is metaphysics as a science possible at all? These are some of the questions which Immanuel Kant's *Critique of Pure Reason* attempts to answer, through a systematic, and entirely novel presentation of a *transcendental* metaphysics. Over the course of some eight hundred-odd pages, Kant examines the human faculties of *sensibility*, *understanding*, and *reason*, culminating in a complete metaphysical system with the ambition to serve as a foundation to all physics and a posteriori knowledge. Just exactly how successful is Kant's endeavour, which even he himself likens to a 'Copernican Revolution?' What *exactly* is the 'synthetic unity of apperception?' What do the dialectical 'illusions' of pure reason hold, for the nature of truth in Philosophy? And are the mathematical and physical sciences truly *empirical*, under the terms of Kant's transcendental foundation? This paper seeks to explore these questions, and more, in a critical discourse of Kant's Critique. Thus, one may call this essay a Critique of the Critique of Pure Reason: a paper that seeks not to refute Kant's thesis, but rather more of a tourist's journal. A journal that documents our expedition through this strange new country, that asks both the tourist's questions – as well as record all the sights and marvels of the trip.

We will begin our metaphysical adventure with the Critique's *Transcendental Doctrine of the Elements*, where we walk along with Kant on his journey to discover the a priori foundations (i.e. the epitomous *elements*) of all human knowledge. Starting at the trailhead of *Transcendental Aesthetics*, we will explore the human faculty of sensibility, to find the a priori forms which underpin all sensate intuition. Moving on to the *Transcendental Logic*, we will attempt the very same campaign upon the faculty of understanding, making the demonstrative leap from *judgements* to *categories*, all in pursuit of the *pure concepts of understanding*. With our prize in hand, we will follow Kant as he unifies the two faculties, and explore *cognition* – the only heir

to *sensibility* and *understanding*'s union. And finally, once we have summited the peaks of this Analytic mountain, we will at last be able to survey the island of Kant's *transcendental metaphysics*. We will see both the beaches of a priori certainty – as well as the farther fog-banks of the stormy, dialectical sea.

The Transcendental Aesthetic

Our expedition begins with the *transcendental aesthetics* – that philosophical trailhead, which separates the orderly boulevards of physics from the uncharted wilderness of metaphysics. For what better way there is to explore beyond the physical – μετὰ τὰ φυσικά – than by starting with the very foundation of our means of physical perception? Indeed through Kant's introductory division of knowledge as *a priori* and *a posteriori*, we are already driven to aesthetics, as it is the very means in which we know things *a posteriori*.¹ The faculty of sensibility is shown to us as a means in which we (i.e. the subject, the reasoning human), have the power to intuit *sensations*, which are presented to us by objects that are external to us. The study of sensations, sensibility, and the intuitions which we yield is the science of aesthetics. And it is from this springboard of aesthetics, where Kant begins the Copernican leap that upturns our world.

For we ask ourselves the question: “can there be any a priori knowledge?” To which Kant responds with the demonstration that: although cognitions are composed of concepts which are in reference to objects², all concepts by necessity do not directly refer to their underlying object, but rather only to a sensible *intuition*. However, our faculty of sensibility is never an active one. We never reach out and ‘grasp’ the objects that we wish to sense with some sort of sensory ectoplasm. Rather, the faculty of sensibility is entirely passive – it is solely our capacity for the *receptivity* of sensation, our ability to be *affected* by objects that are external to us:

The capacity (a receptivity) to acquire presentations as a result of the way in which we are affected by objects is called **sensibility**. Hence by means of sensibility objects are *given* to us, and it alone supplies us with *intuitions*.

(Kant et al. B33)

The objective, material study of our sensibility (i.e. the means in which we are receptive to

¹A priori knowledge comes *prior* to experience, lit. ‘from earlier.’ A posteriori knowledge comes *posterior* to experience, lit. ‘from later.’

²Cognitions arise from the union of understanding with sensibility. Kant explores this in further detail starting at the transcendental analytic.

sensation) is an empirical science, one that is perhaps closer kin to ophthalmology³, than that of any rank befitting a philosopher. Hence, it is no surprise that Kant dusts away the empirical trappings of a mundane aesthetic science, to ask: “when we abstract away all matter of intuition, what is left there to remain?” For the *matter* of intuition is always an object (and hence, objective, and therefore, empirical) – to strip intuitions of their matter is to leave only their *form*. There can be only two possibilities regarding the *formal* nature of intuition. Either we are to deny its existence altogether – as to say that the evaporation of intuitive matter yields only a bare, deposit-less vapour – or we are to acknowledge that there is indeed some metaphysical residue, a crystallisation of formal structure that underlies all human intuition.

The first case is an apparent impossibility to Kant, one that even a tourist can understand on grounds both metaphysical and mundane. For to argue that there is no formal nature behind human intuition is to accept that all intuitions are inherently structureless. It is to accept that there is no deep, underlying relationship between sensory intuitions – a nihilistic capitulation so craven that Kant devotes the majority of the Critique’s introduction (as well as a good part of his *Prolegomena*) in refuting. He argues that the lack of an a priori, formal structure behind intuition will prevent the apodeictic⁴ certainty of mathematics and geometry. Likewise, to use a more mundane analogy – the mundane, material nature of the human sensory organs by necessity create some formal structure which will underpin our intuition. It is no great leap to take such mundane, aesthetic a prioriae, and to make a further jump that abstracts away even the material nature of our sensory biology – and ask the *transcendental* question, of what are the a priori forms behind intuition:

There must, therefore, be a science of all principles of a priori sensibility; I call such a science *transcendental aesthetic*. It constitutes the first part of the transcendental doctrine of elements, and stands in contrast to that [part of the] transcendental doctrine of elements which contains the principles of pure thought and is called transcendental logic.

(B36)

So what do we find, in this new-found, metaphysical science, of the transcendental aesthetic? What are the underlying a priori forms behind pure intuition? Kant, the ever-obliging tour guide, is happy to answer us, as we begin our hike within the boundaries of the

³The medical science of eyeballs.

⁴necessarily or self-evidently true.

transcendental aesthetic itself. “In the course of that inquiry it will be found that there are two pure forms of sensible intuition, which are principles for a priori cognition: viz., space and time. We now proceed to the task of examining these” (B37). We are guided to a view of the twin peaks of space (which founds our *outer sense*) and time (which founds our *inner sense*). These mountains serve as the core geology of our transcendental landscape – we are cautioned against thinking that there are any other – the lesser distractions of colour, extension, or motion are but superficial molehills which take no part in the a priori bedrock of transcendental aesthetics:

Transcendental aesthetic cannot contain more than these two elements, i.e., space and time. This is evident from the fact that all other concepts belonging to sensibility presuppose something empirical.

(B58)

To the untrained glaze of the tourist, it is reasonable for us to ask: “just how firm are the grounds upon which such mountains protrude?” Unable to tell treacherous marsh from wholesome grassland, it is easy for us to assume that Kant’s thesis has become an Idealistic one. For surely, should space and time serve as the synthetic a prioriae which underlie all sensibility – then all of our worldly intuitions are founded upon a mirage – some dreamy, subjective precept that is real only within our minds. Troubled by such thoughts, we would have been lost in the bogs of Idealism forever, should we not be rescued by Kant’s ready alpenstock⁵, ever on the watch against Idealism. Space and time are *empirically real*, but *transcendentally ideal*. As the necessary a priori which preconditions intuition, space and time are both universal to all objects of intuition, regardless of subject (i.e. not *subjective*). Likewise, by not being present in the object (i.e. not a property, accident, or attribute) – but only in the formal nature of our intuition, space and time are *transcendentally ideal*. To use Kant’s words in this matter:

Hence we assert that space is empirically real (as regards all possible outer experience), despite asserting that space is transcendently ideal, i.e., that it is nothing as soon as we omit [that space is] the condition of the possibility of all experience and suppose space to be something underlying things in themselves.

(B45)

Thus concludes the presentation (or to use Euclidean language, the *enunciation*) of the two transcendental elements of space and time, from the metaphysical science of the transcendental

⁵a shepherd’s trekking pole, commonly used to navigate hazardous snowfields and wetlands.

aesthetic. The proper, demonstrative proof of these elements are naturally contained within Kant's *Critique* itself, to which this simple tourist's journal dares not to re-attempt. However, naturally it is within the mete of every foreign visitor to question their guide. Hence we will ask (with the characteristic brazenness of a tourist): "to what is the industry of such metaphysical mountains?" Are these mountains mere wastes, suitable for no further purpose – or do they yield productive quarries, that lead us to further knowledge in metaphysics? These transcendental elements of space and time are ultimately useful, for in their nature as the a priori forms that necessitate all intuition, they define the boundaries for what intuition can yield.

For one, it is readily apparent that should space and time be the necessary condition for intuition, it is impossible for there to be any intuitive knowledge of objects without the forms of space and time. Such knowledge of the *thing-in-itself*, the cognition of an object without the formal conditions of space and time, is impossible. This impossibility of knowing the thing-in-itself serves as a hard boundary for what intuition is capable of, marking all of our intuitive knowledge as knowledge of appearances alone:

[space and time] being merely conditions of sensibility, these a priori sources of cognition determine their own bounds; viz., they determine that they apply to objects merely insofar as these are regarded as appearances, but do not exhibit things in themselves. Appearances are the sole realm where these a priori sources of cognition are valid; if we go outside that realm, there is no further objective use that can be made of them.

(B56)

And yet, such a border is not a destructive one – these constraints of the sensible condition are not the concessions of foreign occupiers, partitioning what is rightfully ours. But much like the demarcations of a city's walls, this limit is a productive one – which allows wealth and industry to flourish. Our empirical world is of appearances – appearances of worldly objects given to the human subject, through the faculty of sensibility. As all of appearance partakes in the shared a prioriae of space and time, there is now a common *synthetic* denominator, universal to all that is which we perceive. Any cognition is always the union of a piece of a posteriori matter, synthetically mated to an a priori form. Now we are able to make broad, general propositions about the empirical objects of sensation, by using the necessary a priori conditions for their appearance as the foundation for our reasoning.

It is this very generality, to which Kant takes as the foundation for mathematical intuition. Does

it now follow that the universal applicability of mathematics rest in the foundation of these a priori forms behind intuition? Although some may think that the foundation of mathematics is secure now, to which we may close Kant's *Critique* forever – I caution that the mere existence of a priori forms behind intuition is not enough to sufficiently explain mathematical reasoning. For intuition serves only as the means in which we perceive empirical objects, not as a means in which we understand them. Mathematics deals not just with the intuitive process of appearances (i.e. the appearance of a triangle, that of a derivative), but also a discursive process of seeking the possibility of appearance (i.e. the creation of proofs, application of theorems). Such an act of seeking the possibility of appearance relies not merely on the a priori forms behind possible intuition, but also upon certain acts of understanding, which can only be found in the *transcendental logic*. Hence, the transcendental aesthetic by itself is insufficient to explain the universal applicability of mathematics.

Kant's transcendental treatment of aesthetics is a unique thesis, and indeed a marvellous one. It's metaphysical duality: the synthetic a prioriae both empirically real and transcendently ideal, is a spectacular marriage that bridges the best parts of Scepticism and Idealism, while avoiding the nihilistic excesses of both. Such a treatment on intuition is already a foundation to mathematical intuition, which is indeed highly spatial and temporal (think of all the figures of geometry, or the integrals of calculus!). Furthermore, as far as intuition serves for the purpose of cognition (where cognition is the union of sensibility and understanding) – Kant teases us with one final afterthought, which may prove hauntingly prescient in our age of artificial minds:

There is, moreover, no need for us to limit this kind of intuition – intuition in space and time – to the sensibility of man. It may be (though we cannot decide this) that any finite thinking being must necessarily agree with man in this regard.

(B72)

Ultimately, as a work of metaphysical knowledge, the transcendental aesthetics is important to us, because it gives us two important transcendental elements that work with the pure elements of understanding as the basis for cognition. Through this process of analysing the material of sensibility (sensation), we are able to find the a priori concepts of space and time. However, we must not rest on our laurels yet – for although we have visited this one landmark, there are further mountains ahead, which we must conquer in order to view the entirety of the Kantian island. For intuition alone does not cognition make – the scion of the union of sensibility and understanding demands equal contribution from both parents. To understand the metaphysical groundwork for

cognition, we must next examine the *transcendental logic*. It is only in light of both the *logic* and the *aesthetic*, could we come to a proper assessment of Kant's *Critique*. Our guide hurries us forth – eager to apply his transcendental method of analysis, to the faculty of understanding. We will not make him wait.

The Transcendental Logic

As the faculty of sensibility deals with sensations, the faculty of understanding deals with concepts. Sensibility without understanding is only reaction, while understanding without sensation is but nothing but computation. It is the union of both sensibility and understanding which yields cognition, that fruit of the mind so characteristic of the human animal. As the science of understanding is named logic, Kant brings us to this aptly named summit of the transcendental logic. The journey takes place in two steps. We first acknowledge that the characteristic nature of any concept is that of *function*: “the unity of the act of arranging various presentations under one common presentation” (B93). Given a raw and unorganised manifold of intuitive presentations, the functional process is one in which such a manifold is unified under a singular concept. That is how a whiff of asphalt, the roar of an engine, and the sparkle of sun-shined chrome all combine to form the singular concept of ‘race car.’

But yet all such unity is also difference – for every way in which various presentations are combined, we also perform the negative act of excluding other, unrelated presentations. Only in such a manner do our concepts have any order. This means that concepts are also *judgements* – acts of discernment and differentiation. Thus lies the basis step of Kant's inductive proof.⁶ As all acts of understanding are concepts, and all concepts judgement... therefore all understanding is judgement. “Now since all acts of the understanding can be reduced to judgments, the understanding as such can be presented as a power of judgment” (B94). This reductive method in which we reduce the problem of finding the *a priorie* of concepts to that of judgements is an indispensable one – as a complete, transcendental analysis of the different modes of judgement is a far easier task, than the analysis of infinite unbounded concepts. We are able to start our analysis by drawing upon the prior scholarship of mundane logic, much like beginning a difficult ascent from a pre-established base camp.

⁶A mathematical analogy. Inductive proofs are completed in two steps, a *basis* step, and an *inductive* step.

Judgement of Quantity	Judgement of Relation
<ul style="list-style-type: none"> • Universal • Particular • Singular 	<ul style="list-style-type: none"> • Categorical • Hypothetical • Disjunctive
Judgement of Quality	Judgement of Modality
<ul style="list-style-type: none"> • Affirmative • Negative • Infinite 	<ul style="list-style-type: none"> • Problematic • Assertoric • Apodeictic

Figure 1: Kant's Table of Judgements

However, upon what grounds do we have to reduce the power of understanding, into a power of judgement? Unlike a mathematical proof that may demonstrate equivalency through formal means (e.g. using the side-angle-side postulate to prove the similarity of two different triangles), this leap from concepts to judgements is an unintuitive⁷ one. As tourists unfamiliar to the customs of this land, we are forgiven to voice our suspicions to our guide. There are two problems in Kant's approach. On one hand, the given space of all possible concepts (a concept-space, to borrow a topological vocabulary) may be larger than judgement-space. On the other hand, the four logical judgements that Kant provides may be incomplete. The first question he answers on the basis that the functional nature of concepts makes them formally analogous to judgement (the enunciation which we have walked through in the above paragraph). It is the latter question – that of whether or not the judgements provided are complete – which serves as the essential linchpin that holds the Kantian wagon together. Kant himself recognises the dependency of his transcendental logic on furnishing a complete set of judgements:

These concepts [judgements] can be collected in an essay that will be more or less comprehensive ... but by this – as it were, mechanical – procedure, we can never reliably determine at what point that inquiry will be completed.

(B92)

Alas, Kant himself never clearly elucidates the reasons why his table of judgements is

⁷This is a colloquial usage of the word, no relation to Kantian intuition

apodeictically complete. Although it is clear that he borrows the technical apparatus of classical logic, he modifies said apparatus to suit his transcendental use, justifying only his modifications, and not the choice of the apparatus itself. Whether this is because the justification was already apparent to him, or simply an oversight is unknown to us. Kant's judgements hold a certain resemblance to modern fields of mathematical logic. However, there is no single axiomatic formalisation that neatly corresponds to all four judgements which he lists. The judgements of quality are analogous to propositional calculus, while the judgements of relation are valid forms of logical inference within propositional calculus. Judgements of quantity are analogous to predicate logic, while the judgements of modality bear the closest resemblance to modal logic. The fact that these Kantian forms of judgement do not correspond neatly to any axiomatically-complete system of logic (for indeed, modal logic as a field has only had axiomatic formalisations in this century) makes it difficult to demonstrate apodeictically that such a table of judgement is complete. Hence, the transcendental analysis of these judgements may not rest upon as certain of a foundation as Kant implies. However, assuming that these judgements are indeed complete, we may move on forth with our journey and not embarrass our guide any further. Let us delve into the pure elements of understanding which we seek.

Using the four logical judgements (or the 'logical function in judgements', as per Kant's typical verbosity), we next perform the inductive or transcendental step – of abstracting away from the content of judgement, to look at it's form. Mired in the difficulty of a rocky ascent, we are forced to take a brief detour to examine the byway of *synthesis*, before the path forward becomes clear. Recall that the characteristic nature of concepts is their unifying function – that act of uniting presentations. However the very act of judgement (i.e. unifying act) presupposes a priori the existence of a further concept, which gives unity to the synthesis of said disparate presentations. Hence, for all four formal logical judgement, there exists an associated a priori concept which gives synthetic unity to the presentations which underlie the judgement. These transcendental concepts – now fully abstracted away from any material content of judgement, are the *categories*: the *pure elements of understanding* of the transcendental logic.

<p>Category of Quantity</p> <ul style="list-style-type: none"> • Unity • Plurality • Allness 	<p>Category of Relation</p> <ul style="list-style-type: none"> • Inherence & Subsistence • Casuality & Dependence • Community & Interaction
<p>Category of Quality</p> <ul style="list-style-type: none"> • Reality • Negation • Limitation 	<p>Category of Modality</p> <ul style="list-style-type: none"> • Possibility/Impossibility • Existence/Nonexistence • Necessity/Contingency

Figure 2: Kant's Table of Categories

These twelve categories which are given to us from the faculty of understanding serve as the same transcendental foundation for conception, as the a priori forms of space and time relate to intuition. Having met these elements which compose the second part of the transcendental doctrine of elements, we are now once again left to ask of their utility. "What do these categories tell us about metaphysics? Are space, time, and the categories sufficient grounds for a complete metaphysical foundation?" The answer to this question is a nuanced one. For the transcendental elements of space, time, and category, are still insufficient in isolation. As with chemical elements, they are inert in their pure, transcendently isolated state. They are the mere feedstock of reactions, which yield the dizzying chemistry that is cognition. For as we have hinted at throughout our journey, it is the union of intuition and concept which leads to cognition.

This union is conceived of originally as a mundane⁸ one. For it seems that the imposition of a sensible intuition upon understanding leads by it's very nature to an *empirical concept*. That is, a cognition which refers to an object of intuition. This would make this union a metaphysically barren one, for such empirical cognitions are fundamentally *objective*, and hence contingent – there is no space for a priori knowledge here. There can be no morganatic marriage⁹ in which we can salvage the union – for the nature of a category is to bring differing contents into synthesis, and the pure element of intuition are by their definition, contentless. Our exploration of the transcendental landscape end by necessity at this insurmountable cliff. This appears to be the

⁸As opposed to transcendental.

⁹A marriage between persons of unequal social rank, e.g. Archduke Franz Ferdinand and Sophie Chotek.

case, at least to the untrained eyes of a visitor. However, Kant is able to solve this seemingly impossible unity, through an act of transcendental clarity.

Recall that empirical concepts do not refer to singular, individual intuitions – but rather, they refer to manifolds of intuition, as exemplified by the earlier ‘race car’ example. These manifolds of intuition, which we talk about in the transcendental sense (i.e. abstracted away from their content) still contain the a priori forms of space and time, as these are the necessary conditions to intuition. The existence of these a priori manifolds, which are not experience, but only the possibility of experience¹⁰ (as they contain the formal a prioriae of space and time) – yields a transcendental manifold which is capable of union with the categories. But what manner of a manifold is this? An abstract and unconcerned manifold¹¹ of possible experience cannot ever be united in synthesis with the categories of pure understanding. Categories are just the a priori forms of judgements, e.g. the form of $((P \vee Q) \wedge \neg P) \rightarrow Q$ ¹². Even though the intuitive manifold provides us with the propositional terms P, Q (to extend the propositional logic metaphor), and the categories provide us with all the brackets and squiggly signs; the ultimate logical implication (denoted by the symbol ‘ \rightarrow ’) is found in neither. Hence, with a simply unconcerned manifold, there is can be no union, and hence no cognition.

This leads us to what is perhaps *the* most thought-provoking idea in Kant’s entire *Critique of Pure Reason*, which is the *synthetic unity of apperception*. This synthetic unity of apperception is the unifying act which is capable of bringing intuition together with concepts, yielding cognition. It is the ‘ \rightarrow ’ symbol of every judgement. Kant leads us on, explaining:

The *I think* must be *capable* of accompanying all my presentations. For otherwise something would be presented to me that could not be thought at all ... presentation that can be given prior to all thought is called *intuition*.

Hence everything manifold in intuition has a necessary reference to the *I think* in the same subject in whom this manifold is found.¹³ But this presentation [i.e., the *I think*] is an act of spontaneity; i.e., it cannot be regarded as belonging to sensibility. I call it *pure apperception*.

¹⁰Kant sometimes uses the word presentation, which is a more general form that also encompasses non-intuitive sources.

¹¹A manifold that does not contain an experiential subject. I have not explained this distinction yet, but it will become clear in due course.

¹²propositional notation for a disjunctive judgement

¹³This subject is the thinking self, i.e. the human being.

(B132)

Kant goes on to further elaborate regarding the offspring of such a union, which draw their resemblance from space (corresponding to the outer sense) and time (corresponding to the inner sense), in sections entitled *On the Synthesis of Apprehension in Intuition*, and the *On the Synthesis of Reproduction in Imagination*. However these discussions are merely the procedural lemmata of his transcendental analytic, contingent on the synthetic unity of apperception, and we will not elaborate upon them here. Instead we will spend the rest of our time examining the synthetic unity of apperception, this unifying force which marries sensibility and understanding.

The Synthetic Unity of Apperception

Conclusion

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All quotations from Immanuel Kant's *Critique of Pure Reason* are sourced from the 1996 Werner S. Pluhar translation issued by Hackett Publishing. Citation line numbers refer to the Second (i.e. 'B') edition of the *Critique*, as per the *Akademie* manuscript.

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Technical Notes

This essay is typeset using \LaTeX , an Open Source document typesetting language by Donald Knuth, and version-controlled via Git. The git repository containing notes, source code, and revision history is available at the following link.

<https://github.com/ShenZhouHong/kant-aufsatz>

This essay is written using the EssayTemplate, an open source \LaTeX essay template designed for the Humanities by Shen Zhou Hong. It is available at:

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An additional section on the structure of the *Critique* and its organisation has been omitted from this document for the sake of conciseness. It is available as an optional appendix in the git repository.

This \LaTeX essay is also available in Microsoft Word, OpenOffice, HTML, and plain text upon request.