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OF THE CRITIQUE:
A Hike with Kant in the Land of
Transcendental Metaphysics

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1 Introduction

How is mathematics possible? How is physics possible? Is metaphysics as a science possible at all? These are some of the questions which Immanuel Kant's *Critique of Pure Reason* attempts to answer, through a systematic, and entirely novel presentation of a *transcendental* metaphysics. Over the course of some eight hundred-odd pages, Kant examines the human faculties of *sensibility*, *understanding*, and *reason*, culminating in a complete metaphysical system with the ambition to serve as a foundation to all physics and a posteriori knowledge. Just exactly how successful is Kant's endeavour, which even he himself likens to a 'Copernican Revolution?' What *exactly* is the 'synthetic unity of apperception?' What do the dialectical 'illusions' of pure reason hold, for the nature of truth in metaphysics? And are the mathematical and physical sciences truly *empirical*, under the terms of Kant's transcendental foundation? This paper seeks to explore these questions, and more, in a critical discourse of Kant's Critique. Thus, one may call this essay a Critique of the Critique of Pure Reason: a paper that seeks not to refute Kant's thesis (at least, not all of it), but rather more of a tourist's diary. A journal that documents our expedition through this strange new country, that asks both the tourist's questions — as well as record the sights and marvels of the trip.

We will begin our metaphysical adventure with the Critique's *Transcendental Doctrine of the Elements*, where we walk along with Kant on his journey to discover the a priori foundations (i.e. the epitomes *elements*) of all human knowledge. Starting at the trailhead of *Transcendental Aesthetics*, we will explore the human faculty of sensibility, to find the a priori forms which underpin all sensate intuition. Moving on to the *Transcendental Logic*, we will attempt the very same campaign upon the faculty of understanding, making the demonstrative leap from *judgements* to *categories*, all in pursuit of the *pure concepts of understanding*. With our elemental prize in hand, we will follow Kant as he unifies the two faculties, and explore *cognition* — the one true heir to *sensibility* and *understanding*'s union. And finally, once we have summited the peaks of this Analytic mountain, we will at last be able to survey the island of Kant's *transcendental metaphysics*. We will see both the beaches of a priori certainty — as well as the farther fog-banks of the stormy, dialectical sea.

2 The Transcendental Aesthetic

Our expedition begins with the *transcendental aesthetics* — that philosophical trailhead, which separates the orderly boulevards of physics from the uncharted wilderness of metaphysics. For what better way there is to explore beyond the physical — μετὰ τὰ φυσικά — than by starting with the very foundation of our means of physical perception? Indeed through Kant's introductory division of knowledge as *a priori* and *a posteriori*, we are already driven to aesthetics, as it is the very means in which we know things *a posteriori*.¹ The faculty of sensibility is shown to us as a means in which we (i.e. the subject, reasoning human), have the power to intuit *sensations*, which are presented to us by objects that are external to us. The study of sensations, sensibility, and the intuitions which we yield is the science of aesthetics. And it is from this springboard of aesthetics, where Kant begins his Copernican leap.

For we ask ourselves the question: “can there be any *a priori* knowledge?” To which Kant responds with the demonstration that: although cognitions are composed of concepts which are in reference to objects², all concepts by necessity do not directly refer to their underlying object, but rather only to a sensible *intuition*.³ However, our faculty of sensibility is never an active one. We never reach out and ‘grasp’ the objects that we wish to sense with some sort of sensory ectoplasm. Rather, the faculty of sensibility is entirely passive — it is solely our capacity for the *receptivity* of sensation, our ability to be *affected* by objects that are external to us:

The capacity (a receptivity) to acquire presentations as a result of the way in which we are affected by objects is called **sensibility**. Hence by means of sensibility objects are *given* to us, and it alone supplies us with *intuitions*. (Kant et al. B33)

The objective, material study of our sensibility (i.e. the means in which we are receptive to sensation) is an empirical science, one that is perhaps closer kin to ophthalmology⁴, than that of any rank befitting a philosopher. Hence, it is no surprise that Kant dusts away the empirical trappings of a mundane aesthetic science, to ask: “when we abstract away all matter of intuition, what is left there to remain?” For the *matter* of intuition is always an object (and hence, objective, and therefore, empirical) — to strip intuitions of their matter is to leave only their *form*. There can be only two possibilities regarding the *formal* nature of intuition. Either we are to deny its existence altogether — as to say that the evaporation of intuitive matter yields only a bare, deposit-less vapour — or

¹A priori knowledge comes *prior* to experience, lit. ‘from earlier.’ A posteriori knowledge comes *posterior* to experience, lit. ‘from later.’

²Cognitions arise from the union of understanding with sensibility. Kant explores this in further detail starting at the transcendental analytic.

³Kant defines intuition in part, as a direct presentation of an object. This is in contrast to concepts, which are only presentations of intuitions.

⁴The medical science of eyeballs.

we are to acknowledge that there is indeed some metaphysical residue, a crystallisation of formal structure that underlies all human intuition.

The first case is an apparent impossibility to Kant, one that even a tourist can understand on grounds both metaphysical and mundane. For to argue that there is no formal nature behind human intuition is to accept that all intuitions are inherently structureless. It is to accept that there is no deep, underlying relationship between sensory intuitions — a nihilistic capitulation so craven that Kant devotes the majority of the Critique's introduction (as well as a good part of his *Prolegomena*) in refuting. He argues that the lack of an a priori, formal structure behind intuition will prevent the apodeictic⁵ certainty of mathematics and geometry. Likewise, to use a more mundane analogy — the material nature of the human sensory organs by necessity create *some* formal structure which will underpin our intuition. It is no great leap to take such mundane, aesthetic a prioriae, and to make a further jump that abstracts away even the material nature of our sensory biology — and ask the *transcendental* question, of what are the a priori forms behind intuition:

There must, therefore, be a science of all principles of a priori sensibility; I call such a science *transcendental aesthetic*. It constitutes the first part of the transcendental doctrine of elements, and stands in contrast to that [part of the] transcendental doctrine of elements which contains the principles of pure thought and is called transcendental logic. (B36)

So what do we find, in this new-found, metaphysical science, of the transcendental aesthetic? What are the underlying a priori forms behind pure intuition? Kant, the ever-obliging tour guide, is happy to answer us, as we begin our hike within the boundaries of the transcendental aesthetic itself. “In the course of that inquiry it will be found that there are two pure forms of sensible intuition, which are principles for a priori cognition: viz., space and time. We now proceed to the task of examining these” (B37). We are guided to a view of the twin peaks of space (which founds our *outer sense*) and time (which founds our *inner sense*).⁶ These mountains serve as the core geology of our transcendental landscape — we are cautioned against thinking that there are any other. The lesser distractions of colour, extension, or motion are but superficial molehills which take no part in the a priori bedrock of transcendental aesthetics:

Transcendental aesthetic cannot contain more than these two elements, i.e., space and time. This is evident from the fact that all other concepts belonging to sensibility presuppose something empirical. (B58)

⁵Necessarily or self-evidently true.

⁶Interestingly, Kant never presents the outer sense nor inner sense in any formal manner. Hence we take these not to be proper faculties of the human cognition, but more as convenient phrases to reference the sensory characteristic of intuition through space and time.

Now to the untrained gaze of the tourist, it is reasonable for us to ask: “just how firm are the grounds upon which such mountains protrude?” Unable to tell treacherous marsh from wholesome grassland, it is easy for us to assume that Kant’s thesis has become an Idealistic one. For surely, should space and time serve as the synthetic a prioriae which underlie all sensibility — then all of our worldly intuitions are founded upon a mirage — some dreamy, subjective precept that is real only within our minds. Troubled by such thoughts, we would have been lost in the bogs of Idealism⁷ forever, should we not be rescued by Kant’s ready alpenstock, ever on the watch against Idealism. Space and time are *empirically real*, but *transcendentally ideal*. As the necessary a priori which prerequisites intuition, space and time are both universal to all objects of intuition, regardless of subject (i.e. not *subjective*). This is what makes space and time empirically real. Likewise, by not being present in the object (i.e. not a property, accident, or attribute) — but only in the formal nature of our intuition, space and time are *transcendentally ideal*. To use Kant’s words in this matter:

Hence we assert that space is empirically real (as regards all possible outer experience), despite asserting that space is transcendentally ideal, i.e., that it is nothing as soon as we omit [that space is] the condition of the possibility of all experience and suppose space to be something underlying things in themselves.
(B45)

Thus concludes the presentation (or to use Euclidean language, the *enunciation*) of the two transcendental elements of space and time, from the metaphysical science of the transcendental aesthetic. The proper, demonstrative proof of these elements are naturally contained within Kant’s *Critique* itself, to which this simple tourist’s journal dares not to re-attempt. However, naturally it is within the mete of every foreign visitor to question their guide. Hence we will ask (with the characteristic brazenness of a tourist): “to what is the industry of such metaphysical mountains?” Are these mountains mere wastes, suitable for no further purpose — or do they yield productive quarries, that lead us to further knowledge in metaphysics? These transcendental elements of space and time are ultimately useful, for in their nature as the a priori forms that necessitate all intuition, they define the boundaries for what intuition can yield.

For one, it is readily apparent that should space and time be the necessary condition for intuition, it is impossible for there to be any intuitive knowledge of objects without the forms of space and time. Such *immediate* knowledge of the *thing-in-itself*⁸, the cognition of an object without the formal conditions of space and time, is

⁷ *Idealism*: A metaphysical school of thought in which all reality is founded inseparably from human intuition. Such a view rejects the possibility of a true empirical science.

⁸ In contrast with the *mediate* knowledge of an object through the intermediary a priori forms of space and time.

impossible. This impossibility of knowing the thing-in-itself serves as a hard boundary for what intuition is capable of, marking all of our intuitive knowledge as knowledge of appearances alone:

being merely conditions of sensibility, these a priori sources of cognition determine their own bounds; viz., they determine that they apply to objects merely insofar as these are regarded as appearances, but do not exhibit things in themselves. Appearances are the sole realm where these a priori sources of cognition are valid; if we go outside that realm, there is no further objective use that can be made of them. space and time (B56)

However, such a border is not a destructive one. These constraints of the sensible condition are not concessions seceded to foreign occupiers, partitioning what is rightfully ours. But much like the demarcations of a city's walls, this limit is a productive one — which allows wealth and industry to flourish within it's bounds. Our empirical world is of appearances — appearances of worldly objects given to the human subject, through the faculty of sensibility. As all of appearance partakes in the shared a prioriae of space and time, there is now a common *synthetic* denominator, universal to all that is which we perceive. Any cognition is always the union of a piece of a posteriori matter, synthetically mated to an a priori form. Now we are able to make broad, general propositions about the empirical objects of sensation, by using the necessary a priori conditions for their appearance as the foundation for our reasoning.

It is this very generality, to which Kant takes as the foundation for mathematical intuition. Does it now follow that the universal applicability of mathematics rest in the foundation of these a priori forms behind intuition? Although some may think that the foundation of mathematics is secure now (to which we may close Kant's *Critique* forever), I caution that the mere existence of a priori forms behind intuition is not enough to sufficiently explain mathematical reasoning. For intuition serves only as the means in which we *perceive* (lit. intuit) empirical objects, not as a means in which we *understand* them. Mathematics deals not just with the intuitive process of appearances (i.e. the appearance of a triangle, that of a derivative), but also a discursive process of seeking the possibility of appearance (i.e. the creation of proofs, application of theorems). Such an act of seeking the possibility of appearance relies not merely on the a priori forms behind possible intuition, but also upon certain acts of understanding, which can only be found in the *transcendental logic*. Hence, the transcendental aesthetic by itself is insufficient to explain the universal applicability of mathematics. In order to truly understand the broad utility of mathematical thought, we must turn to Kant's *transcendental logic*, which we will in due course.

Kant's transcendental treatment of aesthetics is a unique thesis, and indeed a marvellous one. It's metaphysical duality: the synthetic a prioriae both empirically real and transcendently ideal, is a spectacular marriage that bridges the best parts of Materialism⁹ and Idealism, while avoiding the nihilistic excesses of both. Such a treatment on general aesthetic intuition already establishes a foundation to mathematical intuition, which is indeed highly spatial and temporal (think of all the figures of geometry, or the integrals of calculus!). Furthermore, as far as intuition serves for the purpose of cognition (where cognition is the union of sensibility and understanding) — Kant teases us with one final afterthought, which may prove hauntingly prescient in our age of artificial minds:

There is, moreover, no need for us to limit this kind of intuition — intuition in space and time — to the sensibility of man. It may be (though we cannot decide this) that any finite thinking being must necessarily agree with man in this regard. (B72)

Ultimately, as a work of metaphysical knowledge, the transcendental aesthetics is important to us, because it gives us two important transcendental elements that work with the pure elements of understanding as the basis for cognition. Through this process of analysing the material of sensibility (sensation), we are able to find the a priori concepts of space and time. However, we must not rest on our laurels yet — for although we have visited this one landmark, there are further mountains ahead, which we must conquer in order to view the entirety of the Kantian island. For intuition alone does not cognition make — the scion of the union of sensibility and understanding demands equal contribution from both parents. To understand the metaphysical groundwork for cognition, we must next examine the *transcendental logic*. It is only in light of both the *logic* and the *aesthetic*, could we come to a proper assessment of Kant's *Critique*. Our guide hurries us forth — eager to apply his transcendental method of analysis, to the faculty of understanding.

We will not make him wait.

3 The Transcendental Logic

As the faculty of sensibility deals with sensations, the faculty of understanding deals with concepts. Sensibility without understanding is only reaction, while understanding without sensation is but nothing but computation. It is the union of both sensibility and understanding which yields cognition, that fruit of the mind so characteristic of the human animal. As the science of understanding is named logic, Kant brings us to this aptly named summit of the *transcendental logic*. The ascent takes place in two steps.

⁹A philosophical school of thought which rejects metaphysics altogether, basing all knowledge on empirical intuition.

We first acknowledge that the characteristic nature of any concept is that of *function*: “the unity of the act of arranging various presentations under one common presentation” (B93). Given a raw and unorganised manifold of intuitive presentations, the functional process is one in which such a manifold is unified under a singular concept. That is how a whiff of asphalt, the roar of an engine, and the sparkle of sun-shined chrome all combine to form the singular concept of ‘race car.’

But yet all such unity is also difference — for every way in which various presentations are combined, we also perform the negative act of excluding other, unrelated presentations. Only in such a manner do our concepts have any order. This means that concepts are also *judgements* — acts of discernment and differentiation. Thus lies the basis step of Kant’s inductive proof.¹⁰ As all acts of understanding are concepts, and all concepts judgement... therefore all understanding is judgement. “Now since all acts of the understanding can be reduced to judgments, the understanding as such can be presented as a power of judgment” (B94). This reductive method in which we reduce the problem of finding the a prioriae of concepts to that of judgements is an indispensable one — as a complete, transcendental analysis of the different modes of judgement is a far easier task, than the analysis of infinite unbounded concepts. We are able to start our analysis by drawing upon the prior scholarship of mundane logic, much like beginning a difficult ascent from a pre-established base camp.

Judgement of Quantity

- Universal
- Particular
- Singular

Judgement of Quality

- Affirmative
- Negative
- Infinite

Judgement of Relation

- Categorical
- Hypothetical
- Disjunctive

Judgement of Modality

- Problematic
- Assertoric
- Apodeictic

Figure 1: Kant’s Table of Judgements

However, upon what grounds do we have to reduce the power of understanding, into a power of judgement? Unlike a mathematical proof that may demonstrate equivalency through formal means (e.g. using the side-angle-side postulate to prove the similarity of two different triangles), this leap from concepts to judgements is an unintuitive¹¹ one. As tourists unfamiliar to the customs of this land, we are forgiven to

¹⁰A mathematical analogy. Inductive proofs are completed in two steps, a *basis* step, and an *inductive* step.

¹¹This is a colloquial usage of the word, no relation to Kantian intuition.

voice our suspicions to our guide. There are two problems in Kant's approach. On one hand, the given space of all possible concepts (a concept-space, to borrow a topological vocabulary) may be larger than judgement-space. On the other hand, the four logical judgements that Kant provides may be incomplete. The first question he answers on the basis that the functional nature of concepts makes them formally analogous to judgement (the enunciation which we have walked through in the above paragraph). It is the latter question — that of whether or not the judgements provided are complete — which serves as the essential linchpin that holds the Kantian wagon together. Kant himself recognises the dependency of his transcendental logic on furnishing a complete set of judgements:

These concepts [judgements] can be collected in an essay that will be more or less comprehensive ... but by this — as it were, mechanical — procedure, we can never reliably determine at what point that inquiry will be completed. (B92)

Alas, Kant himself never clearly elucidates the reasons why his table of judgements is apodeictically complete. Although it is clear that he borrows the technical apparatus of classical logic, he modifies said apparatus to suit his transcendental use, justifying only his modifications, and not the choice of the apparatus itself. Whether this is because the justification was already apparent to him, or simply an oversight is unknown to us. Kant's judgements hold a certain resemblance to modern fields of mathematical logic. However, there is no single axiomatic formalisation that neatly corresponds to all four judgements which he lists. The judgements of quality are analogous to propositional calculus, while the judgements of relation are valid forms of logical inference within propositional logic¹². Judgements of quantity are analogous to predicate logic, while the judgements of modality bear the closest resemblance to modal logic. The fact that these Kantian forms of judgement do not correspond neatly to any axiomatically-complete system of logic (for indeed, modal logic as a field has only had axiomatic formalisations in this century) makes it difficult to demonstrate apodeictically that such a table of judgement is complete. Hence, the transcendental analysis of these judgements may not rest upon as certain of a foundation as Kant implies. However, assuming that these judgements are indeed complete, we may move on forth with our journey and not embarrass our guide any further. Let us delve into the pure elements of understanding which we seek.

Using the four logical judgements (or the 'logical function in judgements', as per Kant's typical verbosity), we next perform the inductive or transcendental step — of abstracting away from the content of judgement, to look at its form. Mired in the difficulty of a rocky ascent, we are forced to take a brief detour to examine the byway of *synthesis*, before the path forward becomes clear. Recall that the characteristic nature

¹²We will use this as an analogy in a later section.

of concepts is their unifying function — that act of uniting presentations. However the very act of judgement (i.e. unifying act) presupposes a priori the existence of a further concept, which gives unity to the synthesis of said disparate presentations. Hence, for all four formal logical judgement, there exists an associated a priori concept which gives synthetic unity to the presentations which underlie the judgement. These transcendental concepts — now fully abstracted away from any material content of judgement, are the *categories*: the *pure elements of understanding* of the transcendental logic.

Category of Quantity

- Unity
- Plurality
- Allness

Category of Quality

- Reality
- Negation
- Limitation

Category of Relation

- Inherence & Subsistence
- Casuality & Dependence
- Community & Interaction

Category of Modality

- Possibility/Impossibility
- Existence/Nonexistence
- Necessity/Contingency

Figure 2: Kant's Table of Categories

These twelve categories which are given to us from the faculty of understanding serve as the same transcendental foundation for conception, as the a priori forms of space and time relate to intuition. Having met these elements which compose the second part of the transcendental doctrine of elements, we are now once again left to ask of their utility. "What do these categories tell us about metaphysics? Are space, time, and the categories sufficient grounds for a complete metaphysical foundation?" The answer to this question is a nuanced one. For the transcendental elements of space, time, and category, are still insufficient in isolation. As with chemical elements, they are inert in their pure, transcendently isolated state. They are the mere feedstock of reactions, which yield the dizzying chemistry that is cognition. For as we have hinted at throughout our journey, it is the union of intuition and concept which leads to cognition.

4 The Union of Sensibility and Understanding

This union is conceived of originally as a mundane¹³ one. For it seems that the imposition of a sensible intuition upon understanding leads by its very nature to an *empirical concept*. That is, a cognition which refers to an object of intuition. This would make this marriage a metaphysically barren one, for such empirical cognitions are fundamentally *objective*, and hence contingent — there is no space for a priori knowledge here. There can be no morganatic marriage¹⁴ in which we can salvage the union — for the nature of a category is to bring differing contents into synthesis, and the pure element of intuition are by their definition, contentless. Our exploration of the transcendental landscape would end by necessity, at this insurmountable cliff. This appears to be the case, at least to the untrained eyes of a visitor. However, Kant is able to solve this seemingly impossible unity, through an act of transcendental clarity.

Recall that empirical concepts do not refer to singular, individual intuitions — but rather, they refer to manifolds of intuition, as exemplified by the earlier ‘race car’ example. These manifolds of intuition, which we talk about in the transcendental sense (i.e. abstracted away from their content) still contain the a priori forms of space and time, as these are the necessary conditions to intuition. The existence of these a priori manifolds, which are not experience, but only the possibility of experience¹⁵ (as they contain the formal a prioriae of space and time) — yields a transcendental manifold which is capable of union with the categories. But what manner of a manifold is this? An abstract and unconcerned manifold¹⁶ of possible experience can never be united in synthesis with the categories of pure understanding. Categories are just the a priori forms of judgements, e.g. the form of $((P \vee Q) \wedge \neg P) \rightarrow Q$ ¹⁷. Even though the intuitive manifold provides us with the propositional terms P, Q (to extend the propositional logic metaphor), and the categories provide us with all the brackets and squiggly signs; the ultimate logical implication (denoted by the symbol “ \rightarrow ”) is found in neither. Hence, with a simply unconcerned manifold, there is can be no union, and hence no cognition.

4.1 The Synthetic Unity of Apperception

This leads us to what is perhaps *the* most thought-provoking idea in Kant’s entire *Critique of Pure Reason*, which is the *synthetic unity of apperception*. This synthetic unity of apperception is the unifying act which is capable of bringing intuition together with concepts, yielding cognition. It is the “ \rightarrow ” symbol of every judgement. “This is what

¹³As opposed to transcendental.

¹⁴A marriage between persons of unequal social rank, e.g. Archduke Franz Ferdinand and Sophie Chotek.

¹⁵Kant sometimes uses the word presentation, which is a more general form that also encompasses non-intuitive sources.

¹⁶A manifold that does not contain an experiential subject. I have not explained this distinction yet, but it will become clear in due course.

¹⁷Propositional notation for a disjunctive judgement.

the little relational word *is* in judgements intends [to indicate], in order to distinguish the objective unity of given presentations [the manifold] from the subjective one.”

(B142) Kant leads us on, explaining:

The *I think* must be *capable* of accompanying all my presentations. For otherwise something would be presented to me that could not be thought at all ... presentation that can be given prior to all thought is called *intuition*.

Hence everything manifold in intuition has a necessary reference to the *I think* in the same subject in whom this manifold is found.¹⁸ But this presentation [i.e., the *I think*] is an act of spontaneity; i.e., it cannot be regarded as belonging to sensibility. I call it *pure apperception*. (B132)

Our guide goes on to elaborate on the offspring of such a union, which draw their resemblance from space (corresponding to the outer sense) and time (corresponding to the inner sense), in sections entitled *On the Synthesis of Apprehension in Intuition*, and the *On the Synthesis of Reproduction in Imagination*. However these discussions are merely the procedural lemmata of his transcendental analytic, contingent on the synthetic unity of apperception, and we will not elaborate upon them here. Instead we will spend the rest of our time examining the synthetic unity of apperception, this unifying force which marries sensibility and understanding.

5 The Element of Cognition

What makes the synthetic unity of apperception so essential to the process of cognition? Kant canonicalises it as the basis for the possibility of understanding in respect to intuition. His presentation is as follows:

1. The supreme principle for the possibility of all intuition in reference to sensibility was, according to the transcendental aesthetic, that everything manifold in intuition is subject to the formal conditions of space and time.
2. The supreme principle for the possibility of all intuition in reference to understanding is that everything manifold in intuition is subject to the conditions of the original synthetic unity of apperception.

(B137)

On a surface level, it is clear that all possible cognition (i.e. ‘intuition in reference to understanding’) requires the synthetic unity of apperception. However the more interesting realisation is that this also gives all of our intuitions a fundamentally subjective nature — there will always have to be an ‘*I think*’ particle attached to our cognitions, a self-referential burdock that ties these cognitions to the cognitive subject.

¹⁸This subject is the thinking self, i.e. the human being.

It is never possible to *cognise* in a manner where the subject is removed from the object. Hence, *all objective cognition is dependent on the cognising subject*.

This is truly a audacious conclusion, and it is important for the traveller to not completely misunderstand it. For we are *not* saying that all cognition is subjective, in the sense of wayward Idealism. Rather, the presence of the subject in cognition is an a priori necessity for objective cognition in the first place:

The reference to this necessary unity is there even if the judgment itself is empirical and hence contingent – e.g., Bodies are heavy. By this I do not mean that these presentations belong necessarily to one another in the empirical intuition. Rather, I mean that they belong to one another by virtue of the necessary unity of apperception in the synthesis of intuitions (B142)

It is through the subject, that different intuitions are brought into unity. Without this synthetic unity of apperception, different terms cannot be brought into commensuration by a common measure, allowing no objective determination, only associative ones. Kant elaborates:

Only through this [reference to the original apperception and it's necessary unity] does this relation [among presentations] become a *judgement*, i.e. a relation that is *valid objectively* and can be distinguished adequately from a relation of the same presentations that would only have a subjective validity – e.g., a relation according to the laws of association. According to these laws, all I could say is: When I support a body, then I feel a pressure of heaviness. I could not say: It, the body, is heavy – which amounts to saying that these two presentations are not merely together in perception ... but are combined in the object, i.e., combined independently of what the subject's state is. (B143)

This transcendental process, in which the presence of the subject (in the form of the synthetic unity of apperception) is contained within the cognitive process, but yet somehow 'cancels out' the subjectivity of presentations – allowing true objective cognition – is a gymnastic leap of thought of Comănescian¹⁹ proportions. It is important that we understand this synthetic unity of apperception, for it speaks to the deep nature of how cognition works. A simplified demonstration can be given in the following manner:

1. Through the faculty of sensibility, we yield the manifolds of intuition.
2. Through the faculty of understanding, we yield conception.
3. We begin the synthetic process of merging sensibility and understanding:

¹⁹Nadia Comănesci, a Romanian gymnast who achieved the first perfect score at the Olympics.

- (a) The manifolds of intuition, which are *subjective to the sensible self*, contain the a prioriae of space, time, and the synthetic unity of apperception.
- (b) The a priori categories behind concepts are by necessity, *subjective to the judgemental self*.
- (c) The two subjective elements are combined synthetically through their common denominator.

4. Hence, the resulting union of intuition and concept yields cognition.

Thus concludes Kant's treatment synthetic unity of apperception, the only means in which cognition (including objective cognition) can arise. It is the synthesising process of the transcendental elements of sensibility (aesthetics) and understanding (logic), that is only possible "through the necessary reference of the manifold of intuition to the one [self]" (B140).

6 The Kantian Theory of Mind

We are now at the summit. The rarefied atmosphere of these transcendental heights have melted away all contingency. These transcendental elements, like the rays of an ascendant sun, shine brilliantly upon our visage, unobstructed by the smog and haze of a posteriori pollution. Our journey was a difficult ascent, one which leaves us with a breathless countenance — weary from the exertions of our guide's transcendental analytic. And speaking of which ... where is our guide? Look up and turn around! — but alas, he's nowhere to be seen. We're left alone, with these precepts, and only the ghost of a smile as our transcendental Virgil leaves us. There are further peaks, and farther ranges²⁰ — but none matter to us now, other than the view which is ahead. It is now time to contemplate.

What is the nature of the synthetic unity of apperception, this ghost which fuels the engine of cognition? We are shown that all cognition requires it, as an a priori essence of the self. *Does this make the synthetic unity of apperception the faculty (or nature) of self-awareness?* Self awareness is one of the indisputable hallmarks of truly sapient beings — it is a faculty which only until recently, we ascribed only to humans.²¹ For neither sensibility nor understanding are sufficient for intelligence. The former is within the domain of all sensate creatures — from plants, to insects, to clockwork automata. Even the latter is not enough. Computer programs can be highly conceptual, capable of discretionary judgements, but yet they lack the general-purpose learning that is so characteristic of intelligence. The nature of this inadequacy seems to be

²⁰The transcendental dialectic in particular, forms the bulk of Kant's remaining treatise — but it is only an exposition of the consequences of the analytic. We will only reference it's findings.

²¹Empirical experiments with higher-order primates and dolphins are highly suggestive of self-awareness in other creatures.

plausible: while an artificial neural network is able to execute judgements on input, all such judgement takes the unwieldy form of contingent association. “When I support a body, then I feel a pressure of heaviness” (B143), v.s. “given input x , return output y .” There is never that spark of true cognition, where “the nature of x is y .” On a formal level, the very foundations of computational machine learning is *entirely* based on what Kant so disparagingly calls “the laws of association,” with linear algebra serving as the building blocks of such associative neural networks. While it is beyond the scope of this paper to examine the architectural and technical foundations of artificial intelligence²², the necessity of the synthetic unity of apperception for self-awareness is a thesis as intriguing indeed.

Going back to the treatise of our erstwhile guide, Kant himself seems to offer an indirect, almost coquettish answer. He never definitively state whether or not the synthetic unity of apperception is the wellspring of self-awareness. However, amidst a brief aside on empirical psychology in the *Paralogisms of Pure Reason*, he comments:

Apperception is itself the basis of the possibility of the categories, which in turn present nothing but the synthesis of the manifold of intuition insofar as this manifold has unity in apperception. Hence self-consciousness as such is the presentation of what is the condition of all unity and is yet itself unconditioned. Hence we can say about the thinking I (the soul) — which thinks itself as substance, as simple, as numerically identical in all time, and as the correlate of all existence from which all other existence must be inferred — that it cognises *not so much itself through the categories*, but cognises the categories, and through them all objects, in the absolute unity of apperception and hence *through itself*.
(A399)

Verbose, comprehensive, and tedious to a fault — nonetheless Kant gives us a hint in the affirmative. Most tellingly, he is careful to make the distinction that our apperception is not the simple faculty of *intuiting oneself* (i.e. self-reflection).²³ But rather, that all of our cognition (“the categories, and through them all objects”) is done through this synthetic unity of apperception. It is definitely *much* too early of us to crown such a faculty with the gaudy chalice of the term ‘soul’ (and indeed, Kant himself answers in the negative — at least, to the existence of an *immortal* soul) — but to take such a synthetic unity as the source for self-awareness is a plausible thesis indeed.

Subscribing to such a gentle ‘plausible thesis’ (even though such concessions to common likelihood would cause Kant to shudder) yields us another interesting conclusion. That this self-perceptive act — call it self-awareness or the synthetic unity of apperception — *is a necessary condition for all cognitive beings*. Without the

²²Advances by Goodfellow et al. in generative adversarial neural networks (GANNs) may serve as an interesting comparative thesis to explore.

²³Historically, apperception is the empirical, psychological term of a kind of self-consciousness.

imposition of the a priori form of the self into the very nature of intuition, the synthetic process of merging sensibility and understanding can never happen (as per Kant's demonstration in the *Analytic*). This conclusion provides us with a possible answer to Kant's prescient question, that which so haunts the author of this essay: "It may be (though we cannot decide this) that any finite thinking being must necessarily agree with man in this regard" (B72). Perhaps it has indeed been decided ... and the lack of artificial, cognitive agents is the proof. There is room for further inquiry, however. Kant's casual qualification of his question — "that of any finite thinking being", supports our examination, in the same circumspect way that a bloodstain may support a detective on a case.

For consider the following hypothesis (this paragraph takes the form of a proof via *reductio*). Let there be given a *finite*, thinking being — those a priori forms of intuition consist not of space nor time. Instead, it is capable of fulfilling all the sensory precepts of an objective intuition without said limitation (i.e. it is not simply insensate). Let it be further granted that such a creature is fully capable of the synthetic unity of apperception, viz. it is fully self-aware. What manner of a creature is this? It is a Flatlandian creature, the solipsistic Monarch of Pointland, a denizen of Edwin Abbott's novella.²⁴ Sensate, but yet neither in space nor time, such a creature perceives all sensation as thoughts originating in his own mind. The lack of a spatial intuition by necessity abolishes the *outer sense*, which makes all objective perception a part of the self. Likewise, the lack of a temporal intuition by necessity abolishes the inner sense — as well as to make all occurrences simultaneous, and concurrent. Such a creature — who exists as it's existence has been granted — is by it's very nature is not a *finite* thinking being. To perceive all as a part of the self, separated neither by distance, nor by time — is to be an infinite, eternal being. Thus completes the arc of this proof via *reductio*, and the demonstration is complete. The proposition is that all finite thinking beings must cognise through the a priori forms of space, and time.

7 The Limitations of Rational Metaphysics

It is clear now that the cognitive act defines and constrains by necessity, the very boundaries of what the cognitive agent (i.e. the knowing being) can know. While we are unable to generate any meaningful knowledge about the '*thing-in-itself*' (the phrase which Kant uses to refer to objects outside of the intuitive forms), Kant demonstrates quite consistently throughout the *Critique* that our knowledge is still empirical. Indeed, it is his very thesis that the broad applicability of mathematics to the natural world is due to his very transcendental construction. The gist of his argument can be found summarised in his *Prolegomena to Any Future Metaphysics*:

²⁴In the novel *Flatland: A Romance of Many Dimensions*, the protagonist (a Square) lives in a 2-dimensional world. During the course of their adventures, they ultimately visit 'Pointland' — inhabited by a single Point, who perceives all others as thoughts originating in his own mind.

Nature considered *materialiter* is the *sum of all objects of experience*. We are here only concerned with this, since otherwise things that could never become objects of an experience²⁵ if they had to be cognised according to their nature would force us to accept concepts whose significance could never be given *in concreto*. ... cognition of that which cannot be an object of experience would be hyperphysical. (Kant and Hatfield 4:296)

Therefore, it is in the necessary nature of empirical cognition to allow us to apply mathematical intuition to it. The presence of Kant's transcendental elements allow the possibility of such mathematical intuition. And it is a third, derivative faculty of reason (which Kant properly introduces in the Transcendental Dialectic) — that anticipates the preconditions of said cognitions, always searching for its antecedent cause. It is this faculty of human reason which crowns the *Critique* with its infamous title. Yet, this is also the name to which Kant then proceeds to censure, on the basis of reason's metaphysical limitations. For despite reason's empirical industry (which we do not doubt), it is lost outside the confines of possible experience. The ultimate brand of banishment is cast, when Kant demonstrates a series of dialectical conflicts of pure reason, termed the paralogisms, antinomies, and the theological ideal.

These dialectical illusions all share a common form, which is an eventual proof that for such a given syllogism, pure reason can be coerced into proving the falsity of both possible options. Given the syllogism of "Either A is True, or B is True" (i.e. $A \vee B$), should both options be shown to be false, then neither option can be true as the result of rejecting its antithesis. This results not in a contradiction, but rather yields alternate pairs of self-consistent theses and antitheses. This *dialectic* (defined in the truest etymology of that word, *διαλεκτική*) is taken by Kant to be the final nail in the logic of pure reason, rendering it forever banished from the kingdom of metaphysics. But yet, is such a sentence truly just, for pure reason?

The Kantian argument lies in the seemingly unreconcilable infinitude of arguments, supporting each thesis and antithesis — always non-contradictory, but yet never true. The application of reason to metaphysics seems to open an infinite maw of endless dialectic, which leaves us never able to apodeictically solve the questions relating to experience, being, soul-hood, and God. It is this infinitude which denies certainty, and the denial of certainty which is the root of reason's 'crime.' Here for once, I *categorically reject Kant's thesis*. I argue for an alternative perspective, one which vindicates reason in its metaphysical utility. I accept the self-consistent infinitude of pure reason in metaphysics, and deign to restore it back to its place of honour.

²⁵ e.g. things that are not subject to the formal a prioriae of space, time, and unified with the categories.

7.1 The Parable of the Suicidal Mathematician

I present the body of my counterargument in the form of a parable²⁶, a tale with the name of *The Suicidal Mathematician*.

Once upon a time there was a young MATHEMATICIAN, whose eyes were bright with spark and promise. One day, they sits in the class of a visiting PROFESSOR, a foreigner who presents a lecture on *Gödel's Theorems of Incompleteness*.²⁷ In the lecture hall, the PROFESSOR explains:

"Using mathematics of the greatest rigour and exactitude, we have apodeictically proved that: *there is no formal axiomatic system that can prove it's own consistency*. Any such proof will depend on *axioms outside of the system itself*."

Upon hearing this, the MATHEMATICIAN pales with horror, trembling all over with fear. For surely, if no axiomatic system can prove itself, then all of mathematics is baseless and false! Mathematics will always yield consistent, non-contradictory theorems, that yet can never be proven.

Haunted by this knowledge, the young MATHEMATICIAN flees the lecture hall, and loses all hope and confidence. After a month of abject despair, the young MATHEMATICIAN resolves to end their life.

Perched atop the parapets of their mathematics department, the MATHEMATICIAN is just about to jump, when they are swiftly plucked off the ledge by a stranger. It is the PROFESSOR!

"What is the meaning of this?" asks the PROFESSOR. The young MATHEMATICIAN blushes, flustered at their most unlikely rescue. They explains their thoughts and contemplations from before. The PROFESSOR listens kindly, and chuckles, before sitting down to explain:

"*Gödel's Theorems of Incompleteness* is not a proof of mathematics's falsehood," the PROFESSOR says. "But rather, it is a proof of *mathematics infinitude*."

"For while you can never prove the truth-hood of a system entirely from within, there will always be an outside axiom that when granted, yields apodeictic certainty for the system."

"Such 'outside axioms' are not arbitrary choices, but are rather the self-consistent parts of even greater axiomatic systems which encompasses the former."

"But surely these greater systems are still themselves unprovable, and hence incomplete?!" sobs the young MATHEMATICIAN in reply.

"Ah, but you see — that's the beauty of it. There will always be greater systems, stretching on in an infinite series. And each system yields a whole universe of math, each as self-consistent and true as the other."

"Gödel's Theorems show us that mathematics is not a closed loop, a shallow pond, finite in it's shores. But rather, it is an endless ocean, infinite in scope —

²⁶Unfortunately, an Aesopian fable with animal characters was rejected upon editorial feedback.

²⁷Two theorems published by Kurt Gödel in 1931, which proves the incompleteness of all formal systems, including that of mathematics.

bounded only by the heights of human potential.”

With that parting thought, the young MATHEMATICIAN is enlightened, and the PROFESSOR leaves them to their thoughts.

Hence my perspective is the same. Although Kant is correct that the apodeictic certainty of reason will never yield an absolute metaphysics, these dialectical illusions are not mere sophistries. But rather, they are dual pairs of axiomatic systems, which depend on each other. Self-consistent within themselves, but yet unable to demonstrate their own certainty from within them. Hence the refutation of any thesis with its twin antithesis requires ‘outside axioms’ from its twin. Thus, this makes the application of reason to metaphysics not *false*, as is the Kantian argument. But rather, it makes metaphysics *infinite*. The exploration of the very nature of *existence*, *being*, and *God* is not a closed loop or shallow pond. But rather, “it is an endless ocean, infinite in scope — bounded only by the heights of human potential.”

8 Conclusion

The sun is setting now, and the thin, transcendental atmosphere of these peaks howl with a cold wind. As the last rays of the sun give away to the glow of an ascendant moon, the full scope of this Kantian island comes into view. It's rather befitting of a *Spirit-Seer*,²⁸ for the contours of their island to resolve best only through moonlight. We have travelled far in this journey, and although our guide is gone — the way back home is as clear and as 'apodeictically' certain, as a Kantian demonstration. In our journey in this land of metaphysics, we traversed the slopes of the transcendental aesthetic, finding out the a priori forms behind all sensible intuition. These forms serve as the necessary prerequisite for mathematical intuition, even if they do not extend as much to mathematical cognition. We are then lead to the even higher mountain, of a transcendental logic, in which we examine the origin of categories, that a priori element of pure understanding. The ascent is a difficult one, an act of technical mountaineering assisted by a leap from concepts to judgement, and from judgement to category. Even so, it is not a leap unscathed, for the completeness of Kant's table of judgements is still yet perfect — even though it suffices for our path. Well above the snow line now, we finally prepare to summit — reaching the ultimate pinnacle of the synthetic unity of apperception. It is here in which we finally catch our breath, and delve into the nature of self-awareness, finite beings, and the nature of cognitive agents.

As the moonlight illuminates the on the island's shore, we are reminded once more of the words from our departed guide: "It is the land of truth, and is surrounded by a vast and stormy ocean", he echoes. "Where illusion properly resides and many fog banks and much fast-melting ice feign new-found lands." (Kant et al. B295) But we only smile at Kant in return. For this island of truth — so charming in it's completeness, is none other than a precious jewel, which crowns a yet greater treasure. The ocean which surrounds it, glittering so brightly by the light of the stars, whispers us to come ever so forward. For while Kant sees it's boundless horizons and endless expanse as a folly, we are not so begrudged ourselves. The dialectical application of pure reason to metaphysics yields infinitude, yes — but it is the welcome infinitude of a vast and challenging ocean. Look now, the storms are calming. There is a gap in the tempest, the fog-banks have cleared. The ocean beckons us forwards.

It's time to set sail, into infinity.

²⁸*Dreams of a Spirit-Seer*, an early work by Kant which predates the *Critique* by several decades.

9 Bibliography

All quotations from Immanuel Kant's *Critique of Pure Reason* are sourced from the 1996 Werner S. Pluhar translation issued by Hackett Publishing. Citation line numbers refer to the Second (i.e. 'B') edition of the *Critique*, as per the *Akademie* manuscript.

Kant, Immanuel and Gary Hatfield. *Prolegomena to Any Future Metaphysics: That Will Be Able to Come Forward as Science: With Selections from the Critique of Pure Reason*. Cambridge Texts in the History of Philosophy, Cambridge UP, 2004.

Kant, Immanuel, et al. *Critique of Pure Reason*. Paperback, Hackett Publishing Company, 1996.

Kant, Immanuel, et al. *Critique of pure reason*. Penguin classics, Penguin Books, 2007.

Kant, Immanuel, et al. *Kritik Der Reinen Vernunft*. Nach der 1. und 2. Originalausg. Philosophische Bibliothek, Koch, Neff & Oetinger & Co, 1956.

Technical Notes

This essay is typeset using \LaTeX , an Open Source document typesetting language by Donald Knuth, and version-controlled via Git. The git repository containing notes, source code, and revision history is available at the following link.

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An additional section on the structure of the *Critique* and its organisation has been omitted from this document for the sake of conciseness. It is available as an optional appendix in the git repository.

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