

## HomeWork 2 Part 4 Written Assignment

1. In one sentence, explain what the following homogeneous transformation accomplishes when applied to a point  $(x, y, z)$ , in terms of yaw, pitch, roll, and translation.

$$T_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

2. Write out the  $4 \times 4$  homogeneous transformation  $T_2$ , when applied to a point  $(x, y, z)$  in global coordinate frame, translates the point by  $(3, 0, 2)^T$ , then followed by a pitch of 45 degrees. Your answer need not be simplified, and may be represented as a single matrix or the product of two or more matrices.
3. We would like to reverse the transformation applied by  $T_1 T_2$ , that is, write out  $(T_2 T_1)^T$ . Your answer need not be simplified, and may be represented as a single matrix or the product of two or more matrices.
4. Write out the Quaternion equivalent to the rotations in  $T_1$  and  $T_2$  as  $q_1$  and  $q_2$ . Then calculate the product, that is,  $q_1 \circ q_2$  (Hint: Steve's book may be a good source of reference)

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**Solution:** 1. This transformation is rotation by  $R$  and then translation by  $T$ , where

$$R = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R$  implies that the rotation is a roll by  $\frac{\pi}{4}$

2.

$$T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\sqrt{2} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \sqrt{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.

$$T_1^{-1} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & -\frac{3}{\sqrt{2}} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_2^{-1} = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(T_2 T_1)^{-1} = T_1^{-1} T_2^{-2} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} & -2\sqrt{2} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4.

$$q1 = [\cos \frac{\pi}{8} \quad 0 \quad 0 \quad \sin \frac{\pi}{8}] \quad q2 = [\cos \frac{\pi}{8} \quad \sin \frac{\pi}{8} \quad 0 \quad 0]$$

$$q1 \circ q2 = [0.85355 \quad 0.35355 \quad 0.14645 \quad 0.35355]$$

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