CS 498 VR Fall 2017 Cong Shen (cshen19@illinois.edu)
Homework 2 Problem 4 Hongting Liu (hl25@illinis.edu)

HomeWork 2 Part 4 Written Assignment

1. In one sentence, explain what the following homogeneous transformation accomplishes when applied to a point (x, y, z), in terms of yaw, pitch, roll, and translation.
$$T_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$
2. Write out the 4×4 homogeneous transformation T_2 , when applied to a point (x, y, z) in global coordinate frame, translates the point by $(3, 0, 2)^T$, then followed by a pitch of 45 degrees. Your answer need not be simplified, and may be represented as a single matrix or the product of two or more matrices.
3. We would like to reverse the transformation applied by $T_1 T_2$, that is, write out $(T_2 T_1)^T$. Your answer need not be simplified, and may be represented as a single matrix or the product of two or more matrices.
4. Write out the Quaternion equivalent to the rotations in T_1 and T_2 as q_1 and q_2. Then calculate the product, that is, $q_1 \circ q_2$ (Hint: Steve's book may be a good source of reference)

Solution: 1. This transformation is rotatation by R and then translation by T, where

$$R = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R implies that the rotation is a roll by $\frac{\pi}{4}$

2.
$$T_2 = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\sqrt{2} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\sqrt{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.
$$T_1^{-1} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & -\frac{3}{\sqrt{2}} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_2^{-1} = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(T_2T_1)^{-1} = T_1{}^{-1}T_2{}^{-2} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} & -2\sqrt{2} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4.

$$q1 = \begin{bmatrix} cos\frac{\pi}{8} & 0 & 0 & sin\frac{\pi}{8} \end{bmatrix} \quad q2 = \begin{bmatrix} cos\frac{\pi}{8} & sin\frac{\pi}{8} & 0 & 0 \end{bmatrix}$$

$$q1 \circ q2 = \begin{bmatrix} 0.85355 & 0.35355 & 0.14645 & 0.35355 \end{bmatrix}$$

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