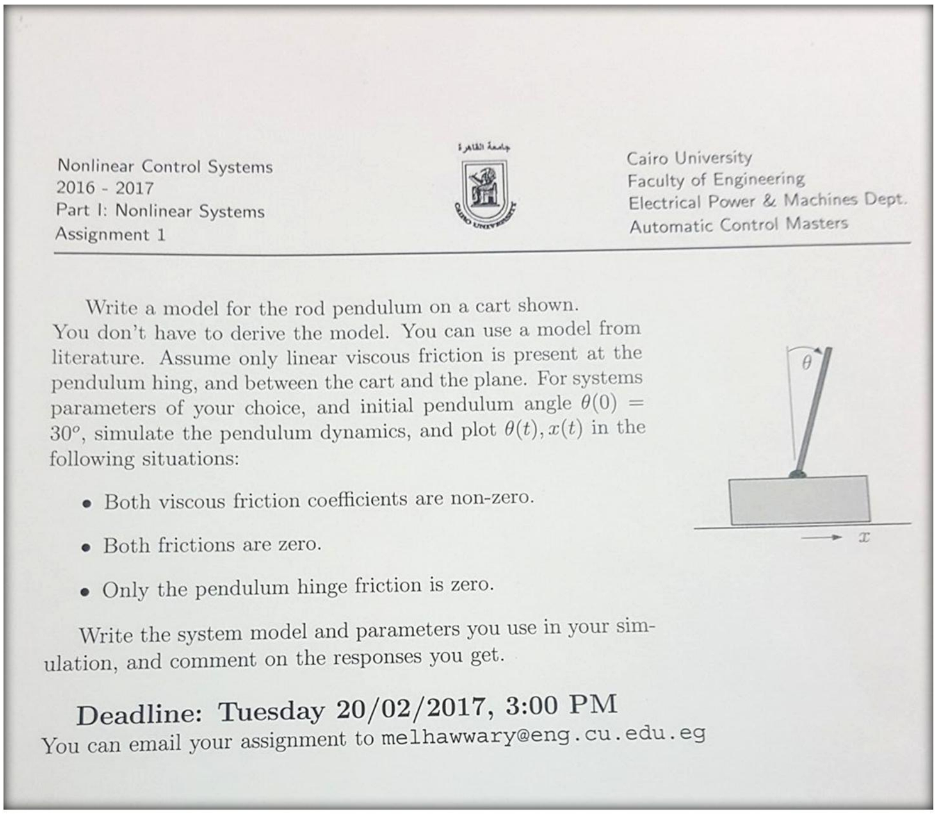
Assignment 1

Inverted pendulum on a cart



Nonlinear Control Systems
2016 - 2017
Part I: Nonlinear Systems
Assignment 1

Cairo University
Faculty of Engineering
Electrical Power & Machines Dept.
Automatic Control Masters

Write a model for the rod pendulum on a cart shown. You don't have to derive the model. You can use a model from literature. Assume only linear viscous friction is present at the pendulum hing, and between the cart and the plane. For systems parameters of your choice, and initial pendulum angle $\theta(0) = 30^o$, simulate the pendulum dynamics, and plot $\theta(t), x(t)$ in the following situations:

- Both viscous friction coefficients are non-zero.
- Both frictions are zero.
- Only the pendulum hinge friction is zero.

Write the system model and parameters you use in your simulation, and comment on the responses you get.

Deadline: Tuesday 20/02/2017, 3:00 PM
You can email your assignment to `melhawwary@eng.cu.edu.eg`

The model:

For cart with friction:

$$\ddot{x} = \frac{-b}{M}\,\dot{x} - \frac{ml}{M}\,\ddot{\theta}\,\cos\theta + \frac{ml}{M}\,\dot{\theta}^2\,\sin\theta + F$$

For cart without friction:

$$\ddot{x} = -\frac{ml}{M}\,\ddot{\theta}\,\cos\theta + \frac{ml}{M}\,\dot{\theta}^2\,\sin\theta + F$$

For a cart with friction and pendulum without friction:

$$\ddot{x} = -\frac{ml}{M}\,\ddot{\theta}\,\cos\theta + F$$

For pendulum with friction:

$$\ddot{\theta} = \frac{-1}{l\,\cos\theta}\,\ddot{x} + \frac{1}{\cos\theta}\,\dot{\theta}^2\,\sin\theta$$

For pendulum without friction:

$$\ddot{\theta} = \frac{-1}{l\,\cos\theta}\,\ddot{x}$$

Where:

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b = 0.2 (the coefficient of friction between cart and plane).
l = 0.6 (the length of the pendulum).
M = 4 (the mass of cart).
m = 2 (the mass of pendulum).
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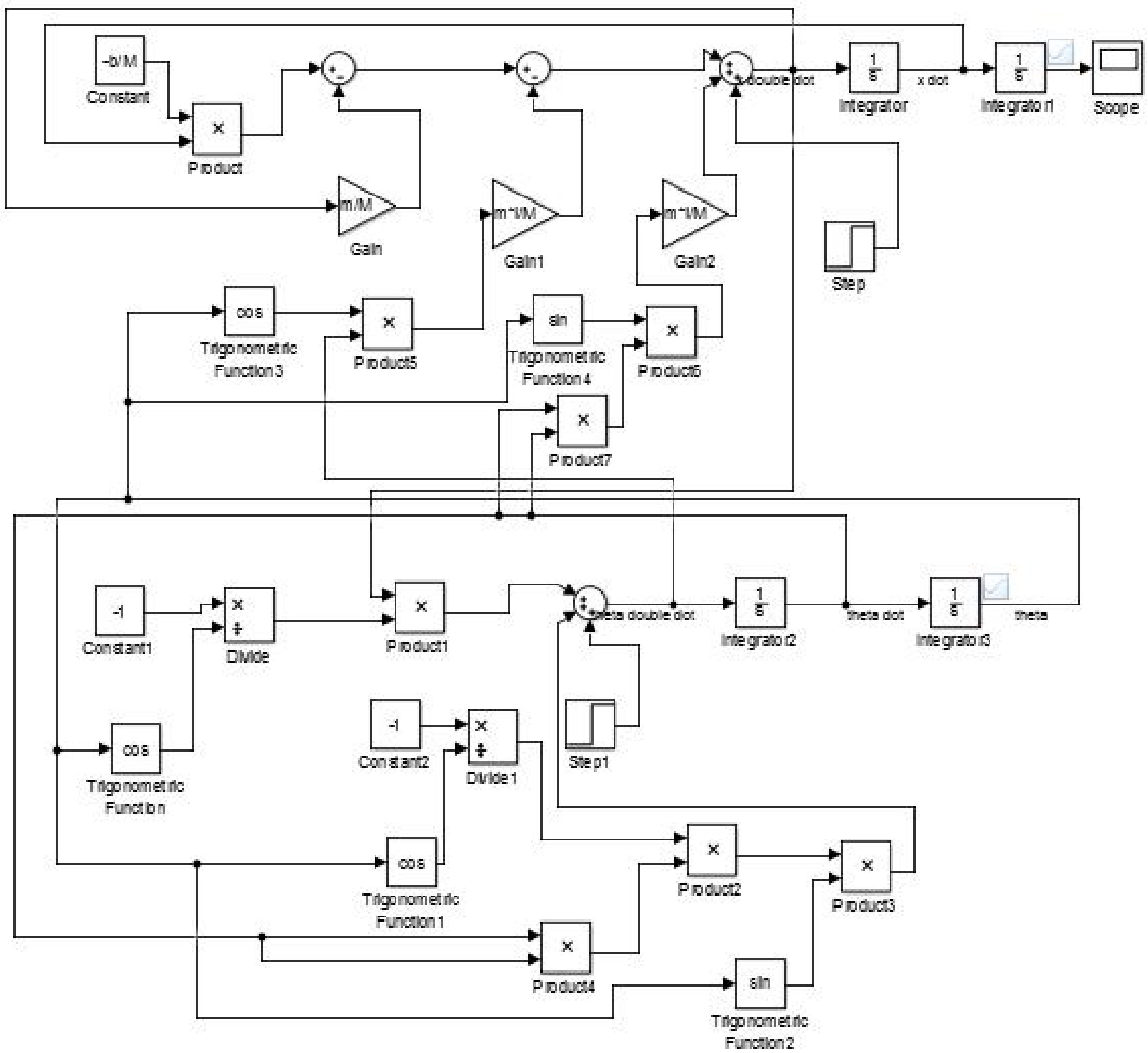
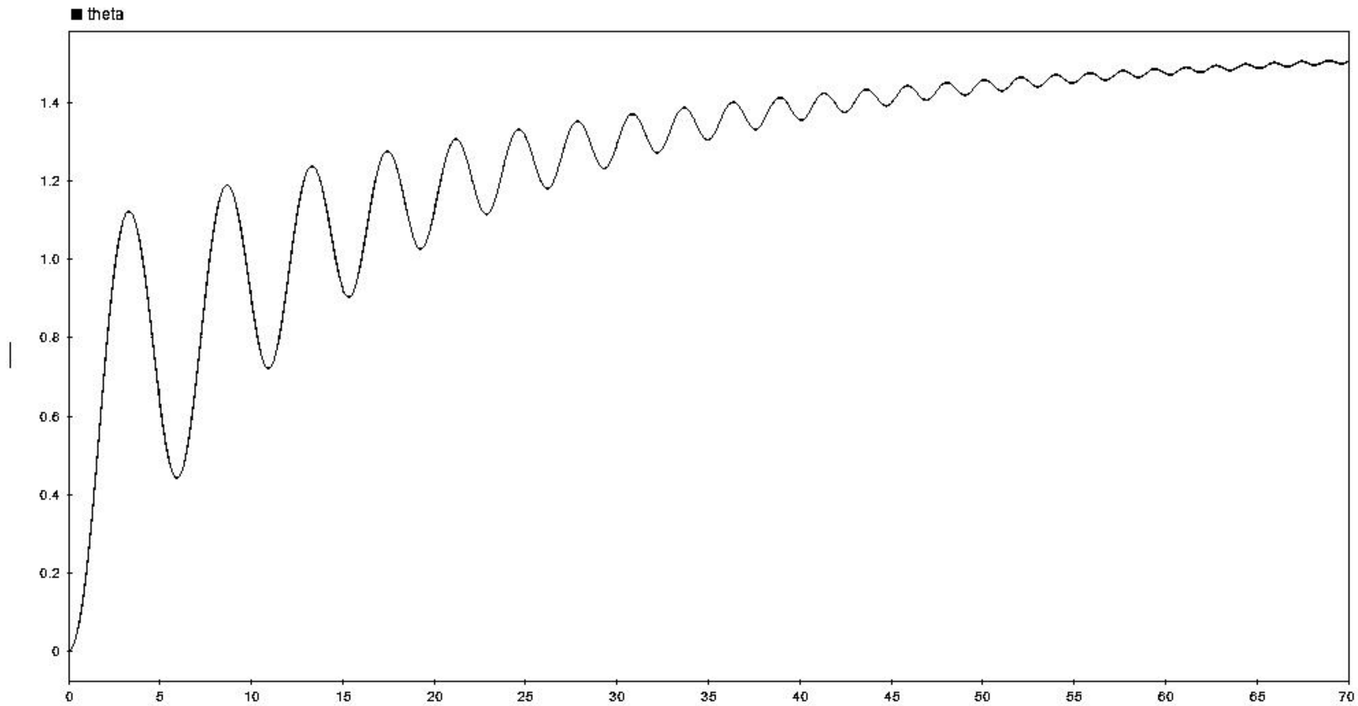
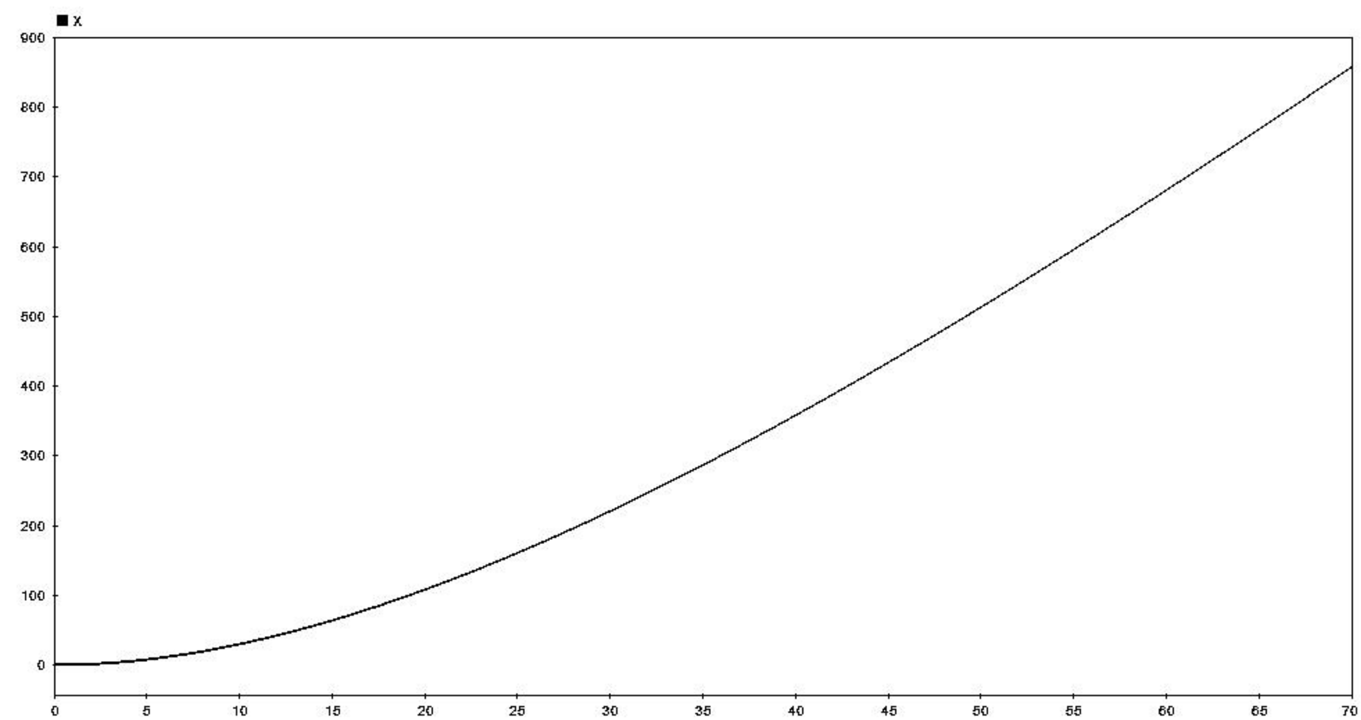
Model implementation on Simulink:

Here is the trial for solving the second order nonlinear differential equations governing the linear motion of the cart and the angular motion of the pendulum:

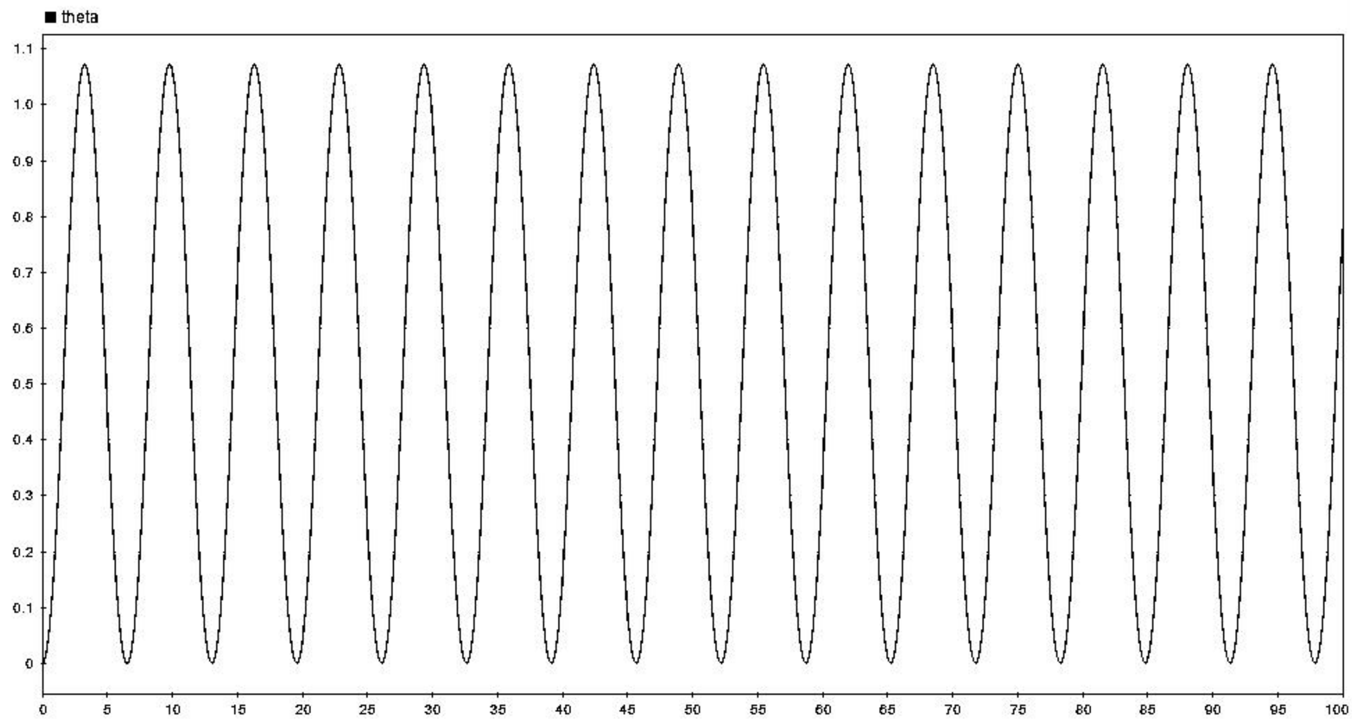


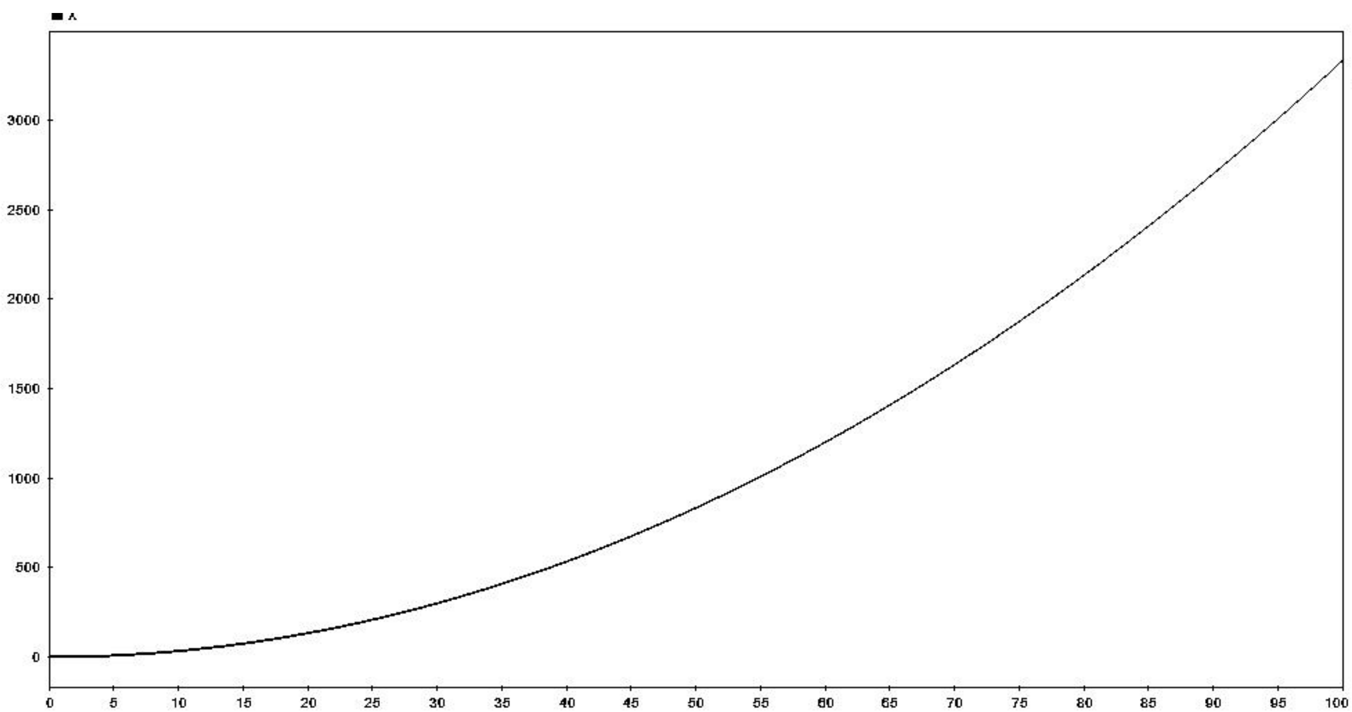
Both viscous frictions are non-zero



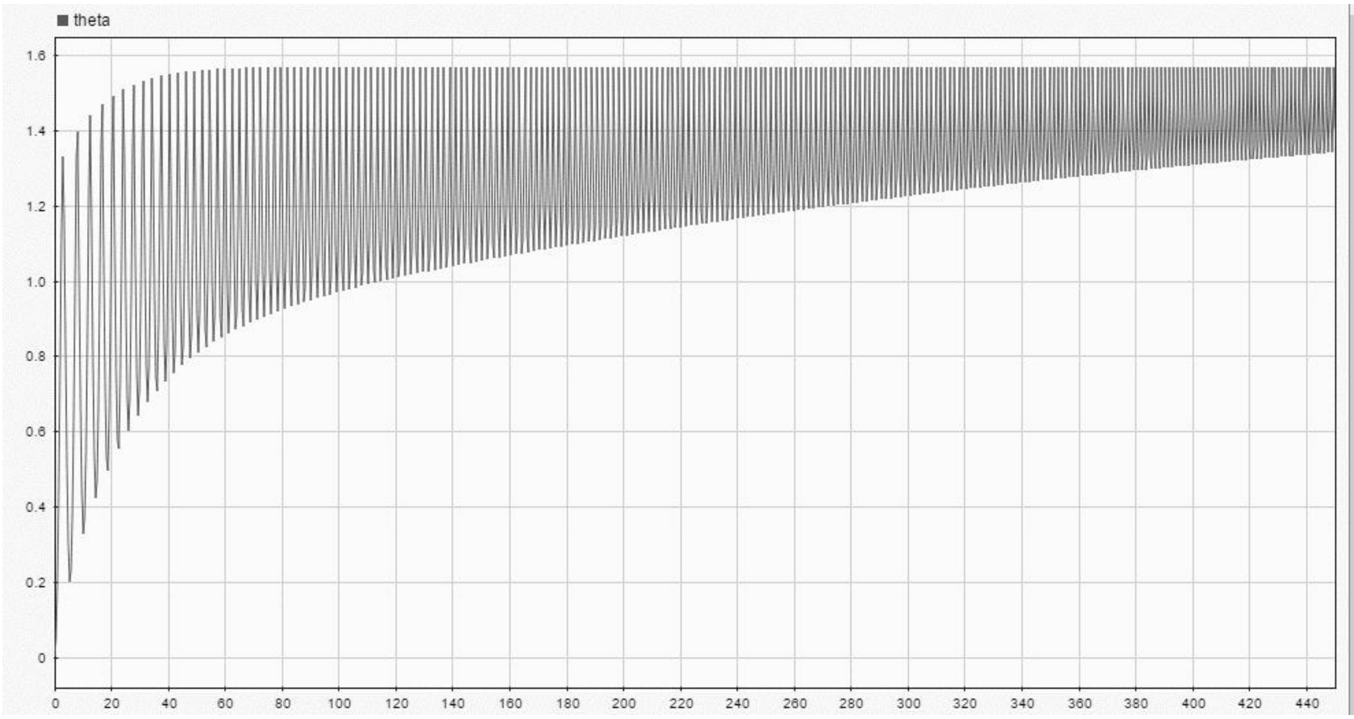


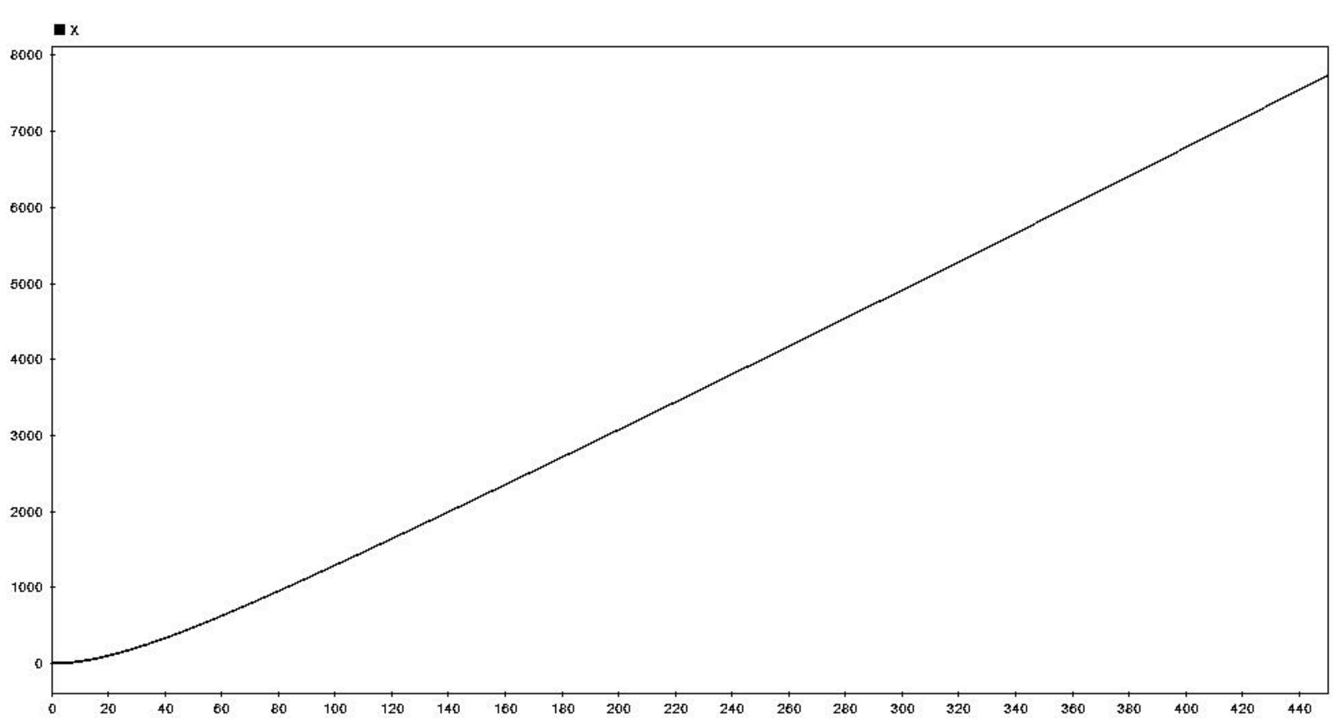
Both frictions are zero





Only the pendulum hinge friction is zero:





Both viscous frictions are non-zero:

The oscillations of the pendulum's angle will eventually die due to the frictional force at the hinge.

The cart will continue to move with acceleration until eventually the change in velocity will tend to zero and the cart will be moving with constant velocity.

Both frictions are zero:

The oscillations of the pendulum's angle will never die and the pendulum will continue to move as time tends to infinity

The cart's velocity will continue to increase as time tends to infinity.

Only the pendulum hinge is zero:

The pendulum's angular movement will oscillate for a very long time but decreasing and will continue to decrease until the change in velocity of the cart stops and the cart's velocity becomes constant, then the oscillations will die