Robotics: Fundamentals

Prof. Mark Yim
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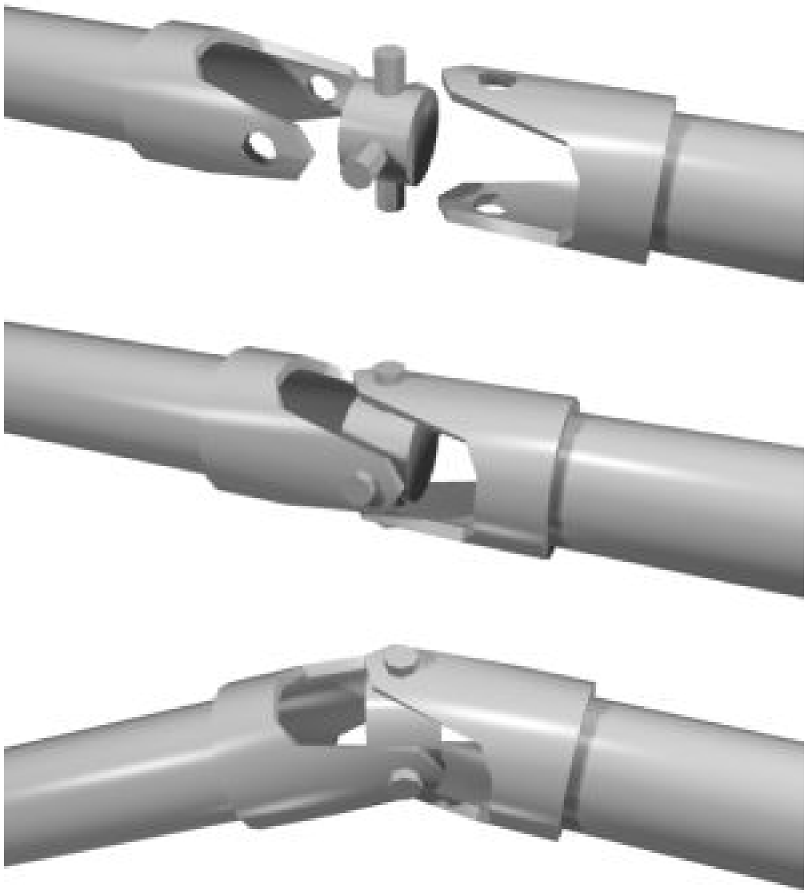
Week 5: Degrees of Freedom

The Goal

- Understanding the position and orientation of robot links.
- Computing end-effector positions from joint angles. Computing joint angles from end effector positions.

Universal Joint Example



Degrees of Freedom

Grublers Criterion

$$F = \lambda(n - j - 1) + \sum_{i=1}^{j} f_i$$

Where F = number of DOF

n = number of links

j = number of joints

λ = number of DOF in the space

f_i= number of DOF permitted by joint j_i

Degrees of Freedom

Constraint formulation

$$F = \lambda(n-1) - \sum_{i=1}^{j} C_i$$

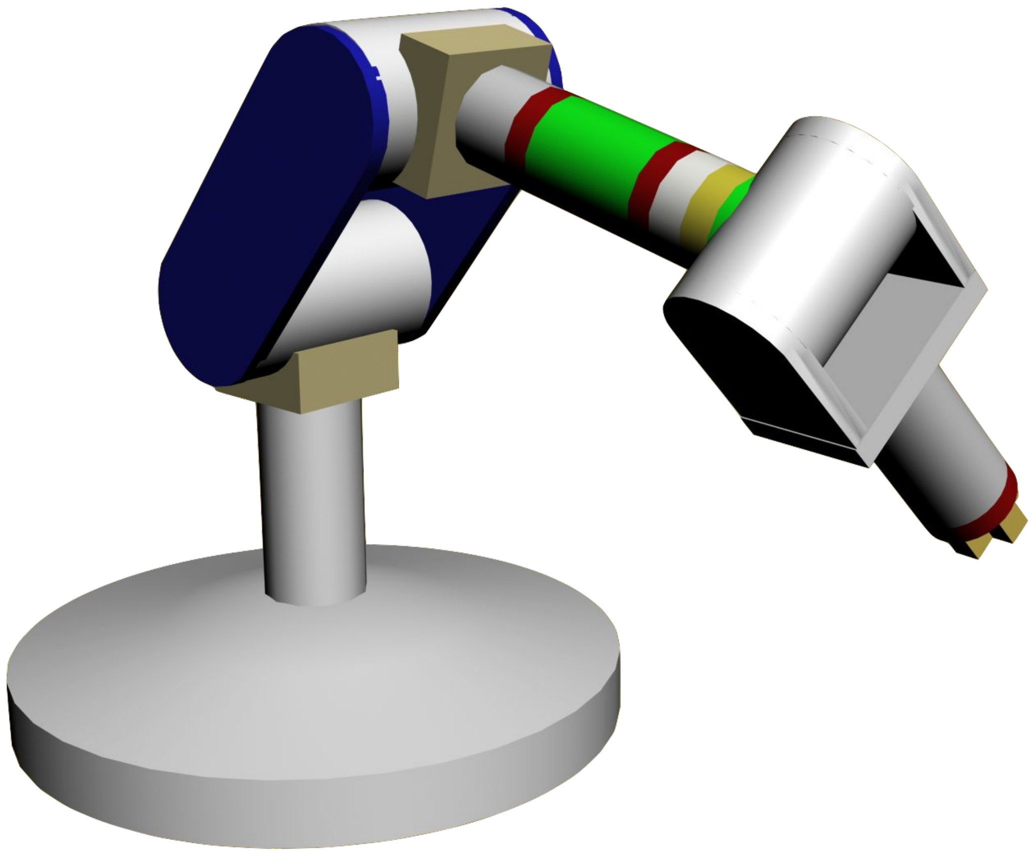
Where F = number of DOF

n = number of links

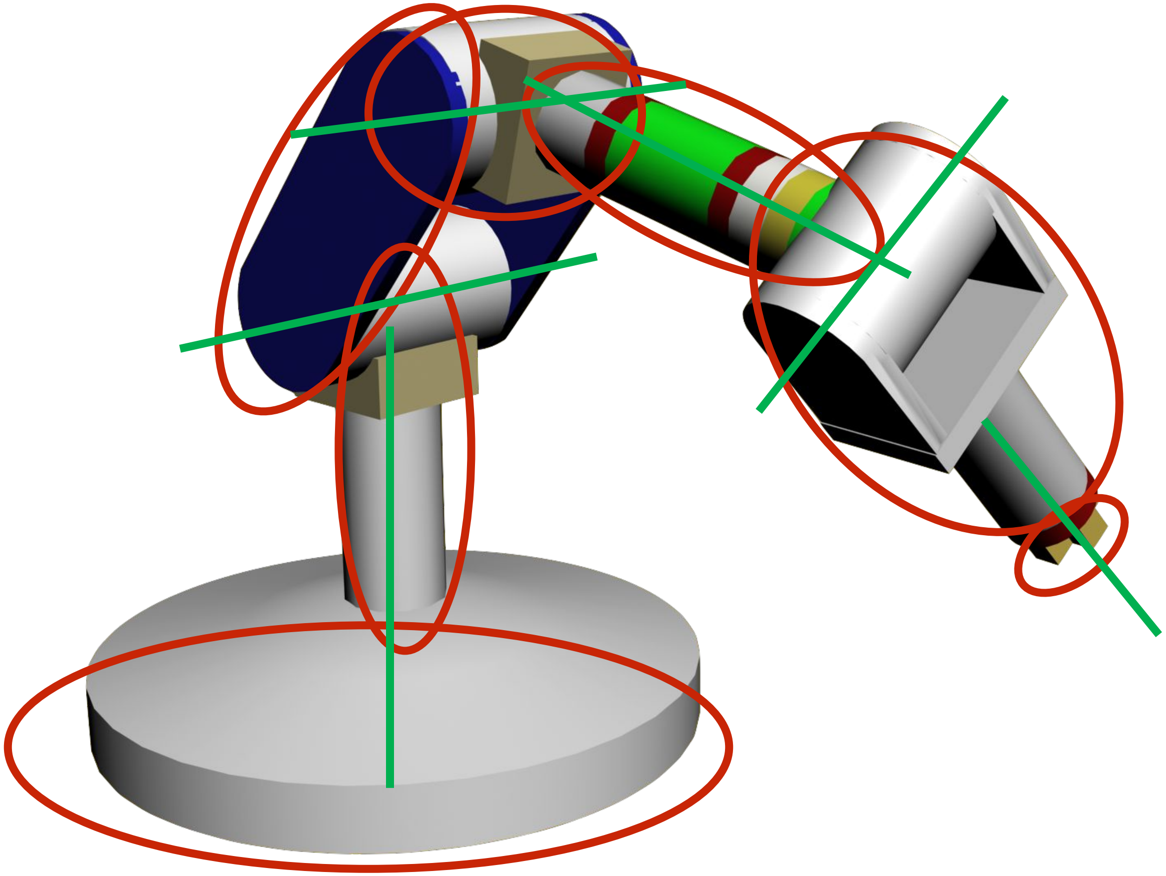
j = number of joints

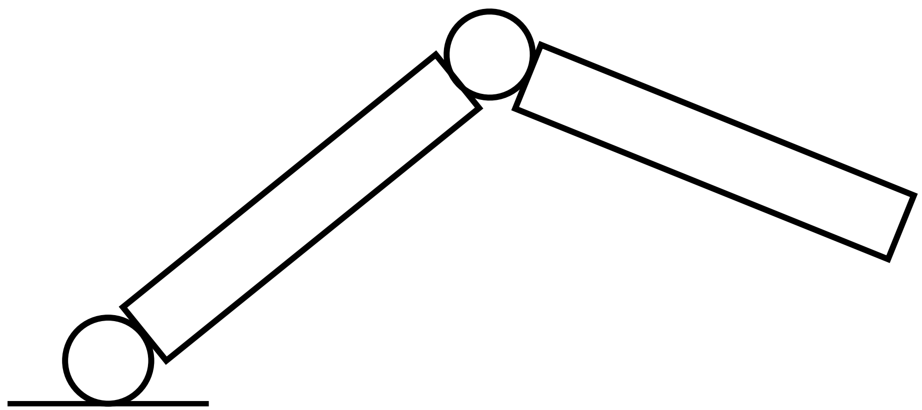
λ = number of DOF in the space

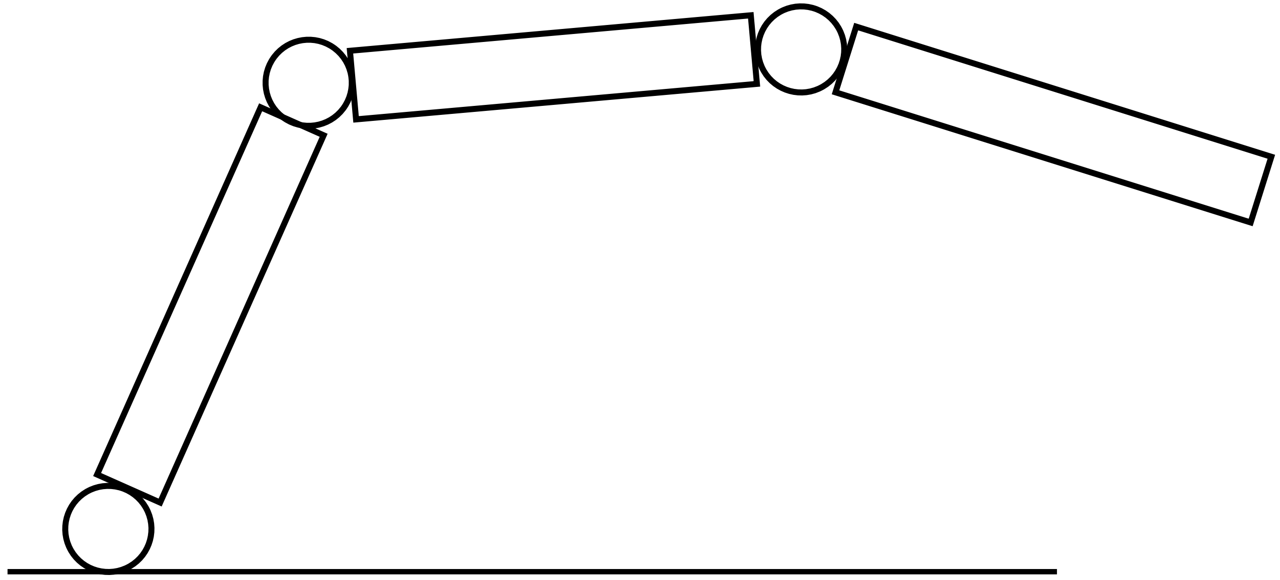
C_i= number of DOF constrained by joint j_i



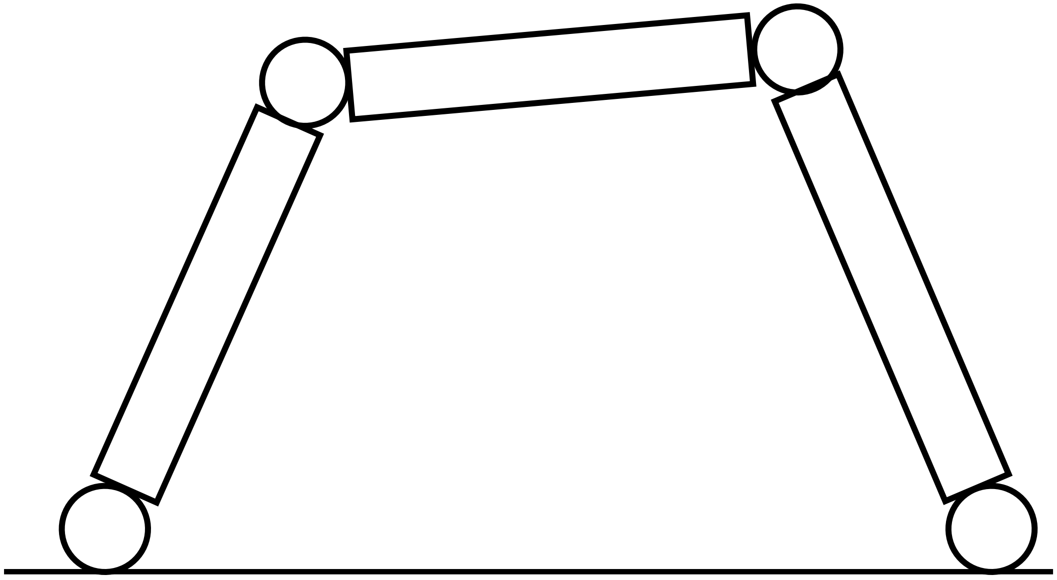
7 links: Base, spine, shoulder, shoulder twist, forearm, gripper
6 joints between the 7 links, all revolute
6(7-1) – 6(5) = 6 degrees of freedom



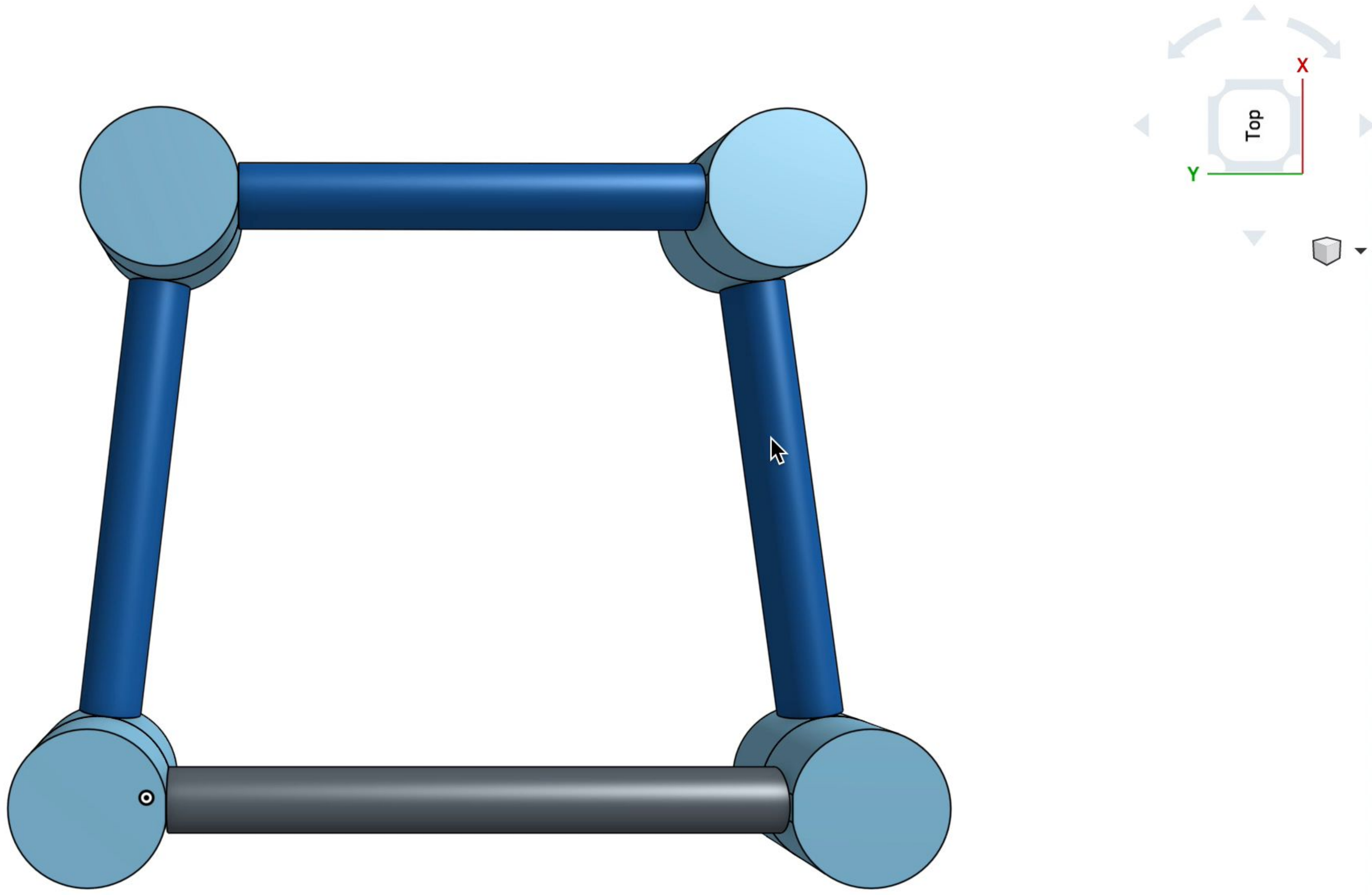




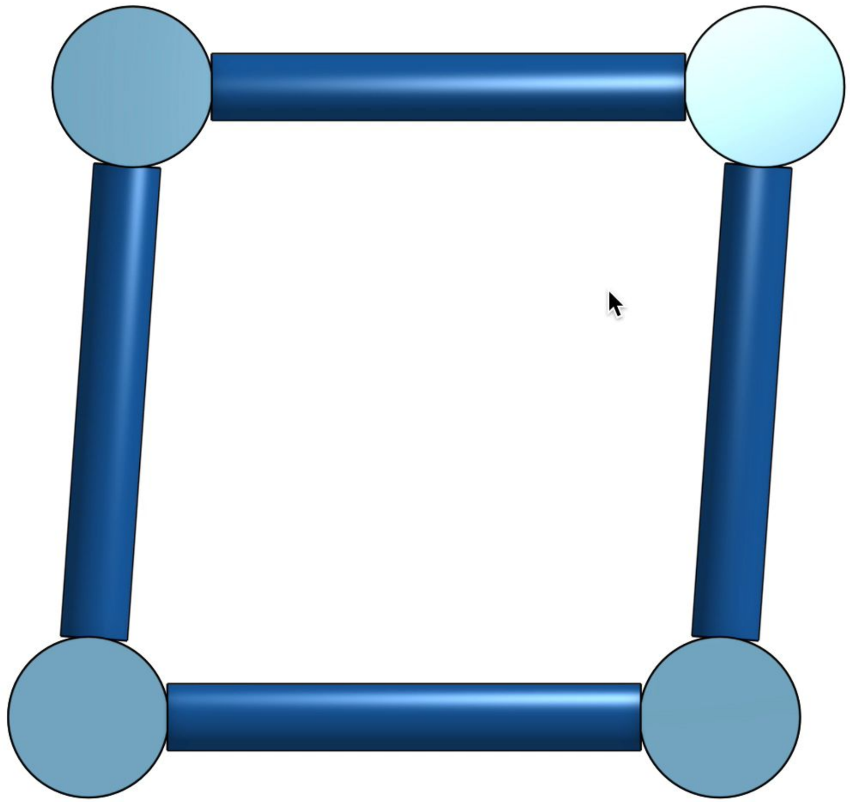
4-bar linkage



Four-bar linkage

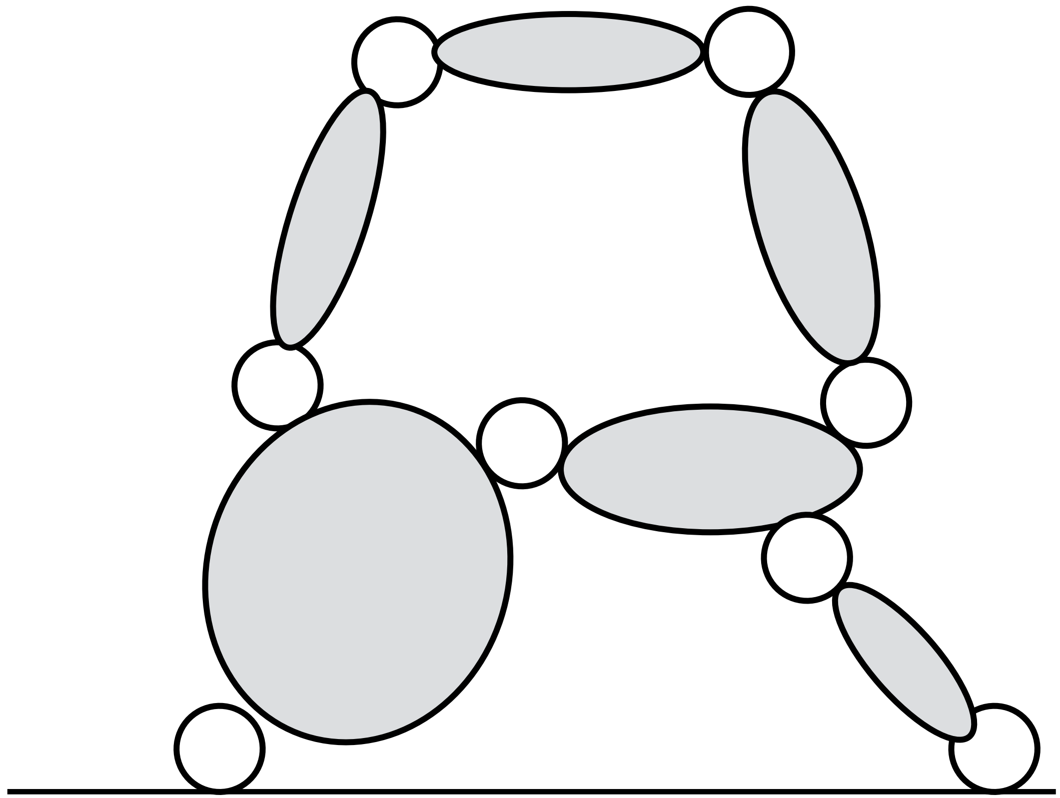


Parallel Four-bar

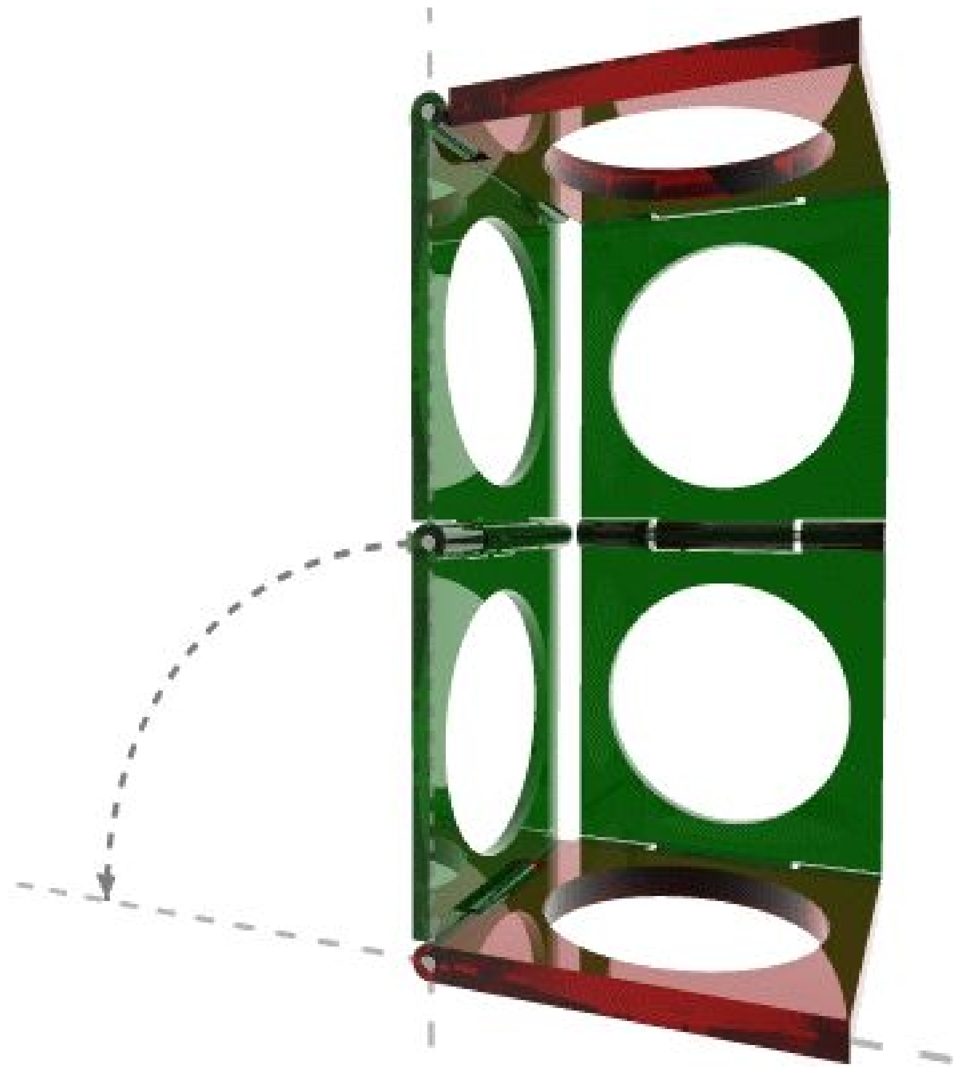


Penn Engineering

More complex linkage



Pathological Exceptions



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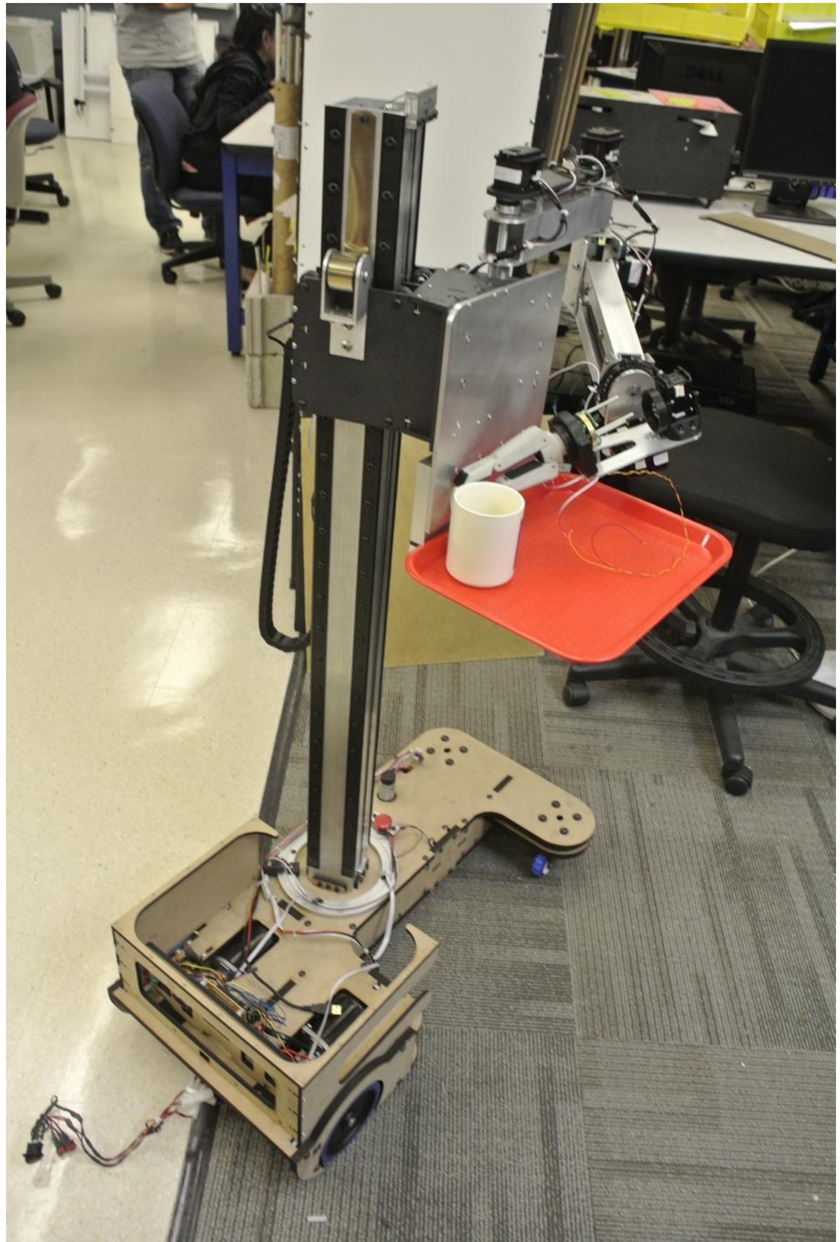
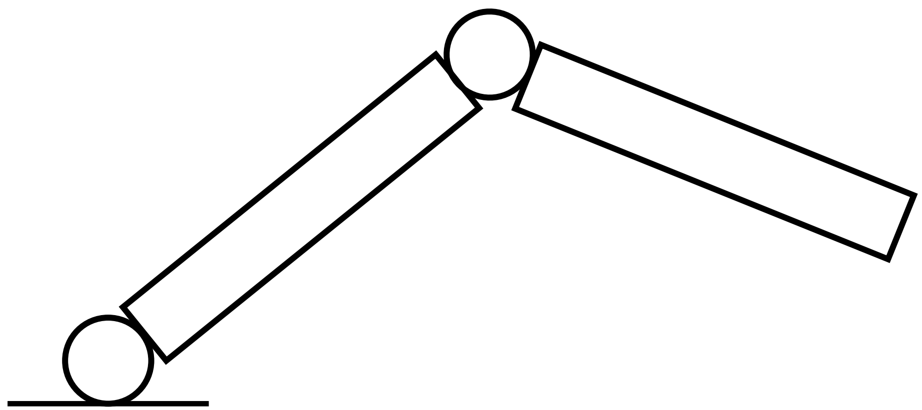
Week 5: Forward Kinematics and DH Parameters

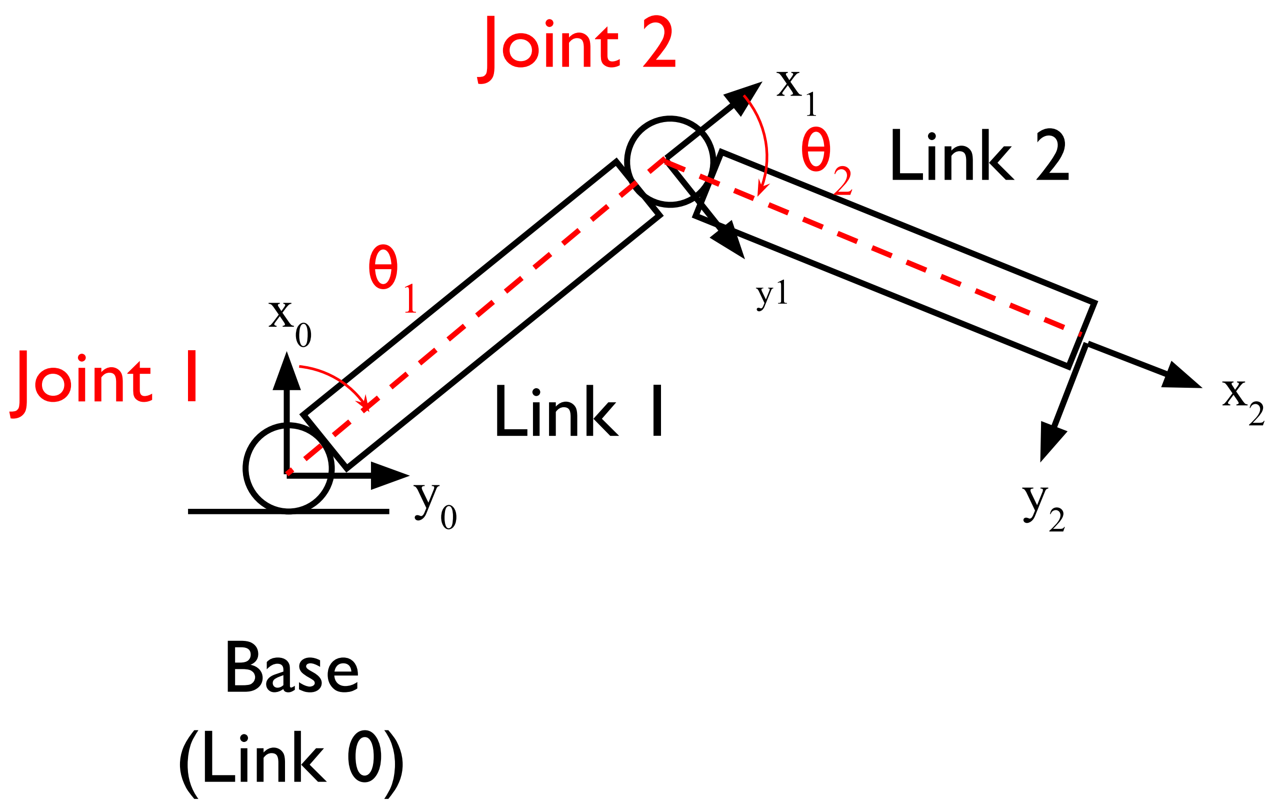
Forward Kinematics



Labeling Conventions

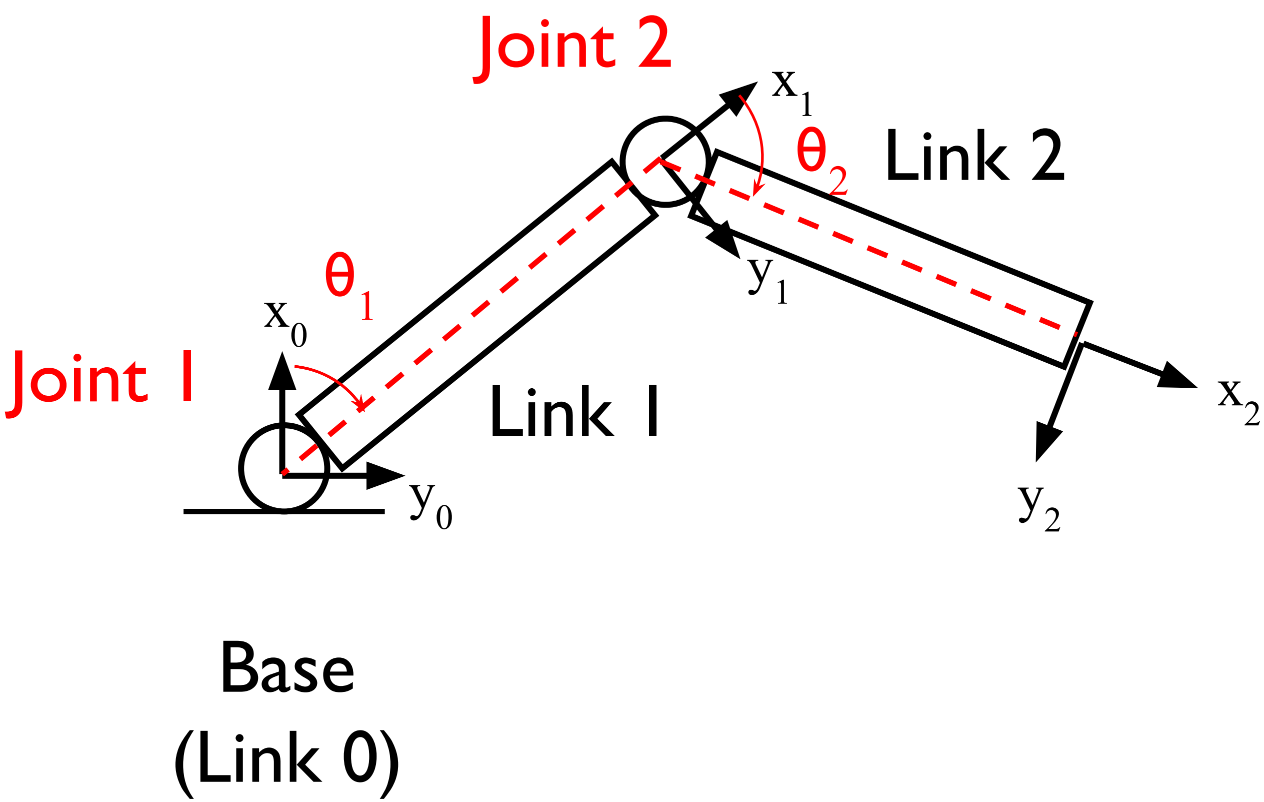


Labeling Conventions



Planar Forward Kinematics

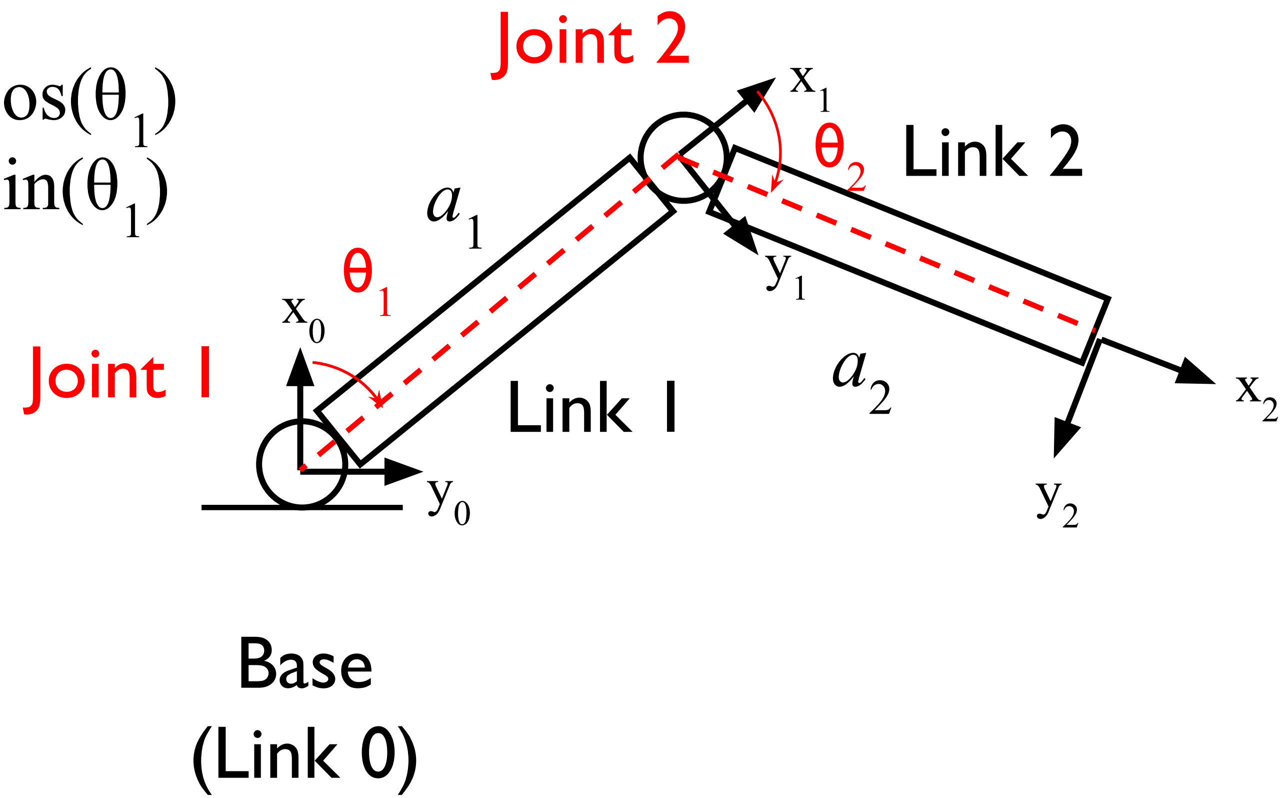
$$A_1 = \begin{bmatrix} R_0^1 & \boldsymbol{d}_0^1 \\ 0 & 1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} R_1^2 & \boldsymbol{d}_1^2 \\ 0 & 1 \end{bmatrix}$$



Planar Forward Kinematics

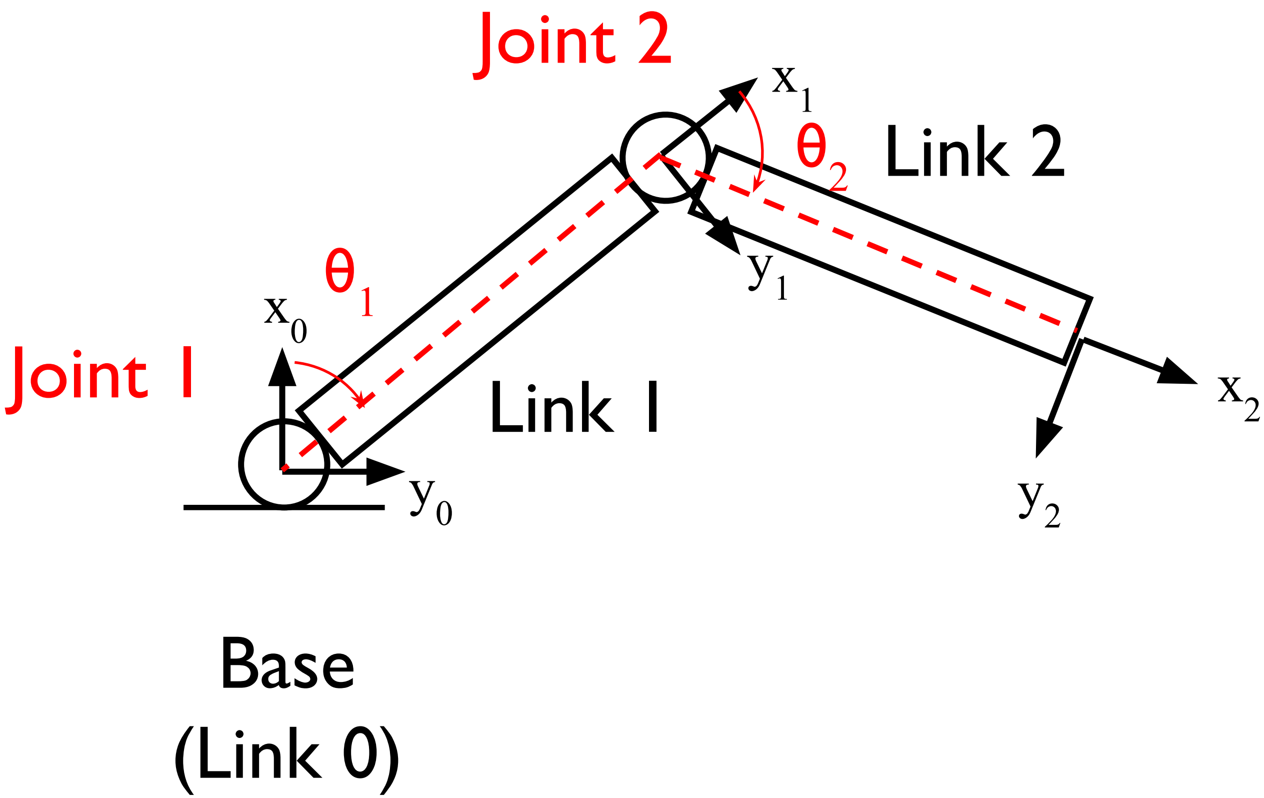
$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 c_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2 c_2 \\ s_2 & c_2 & 0 & a_2 c_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$c_1 = \cos(\theta_1)$$
$$s_1 = \sin(\theta_1)$$

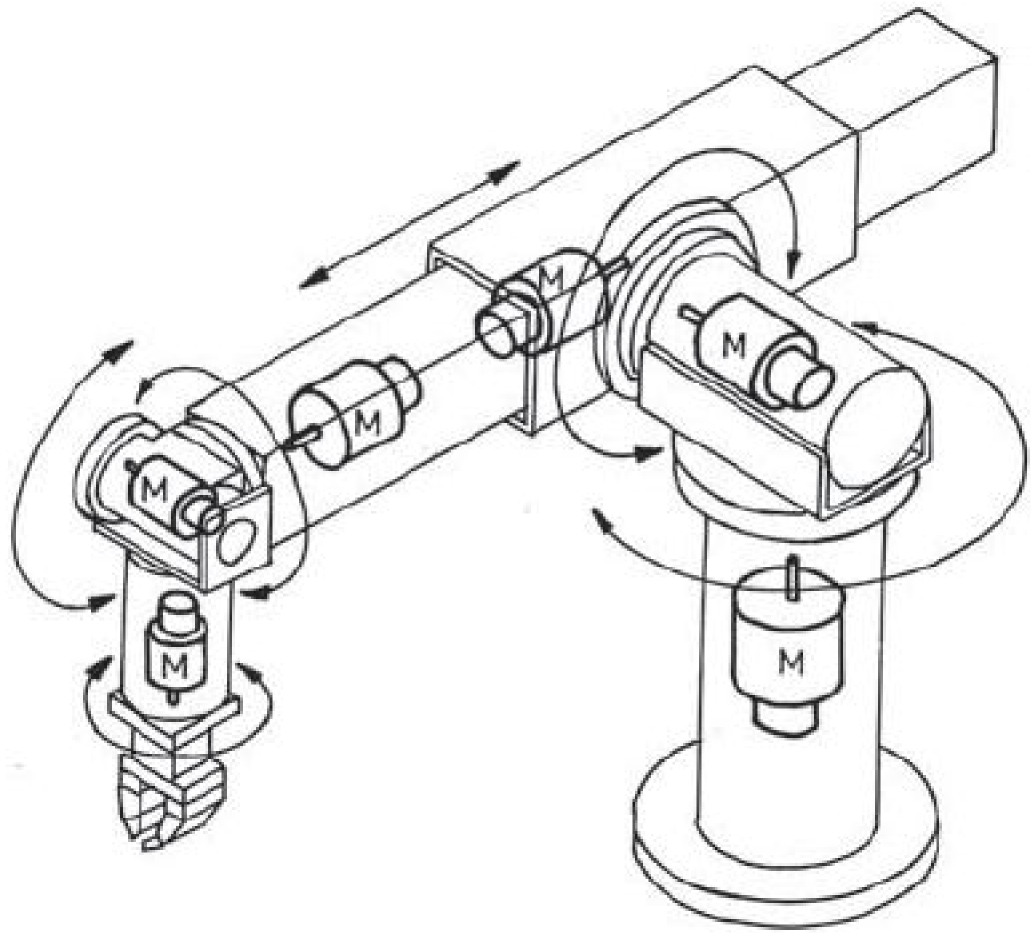


Planar Forward Kinematics

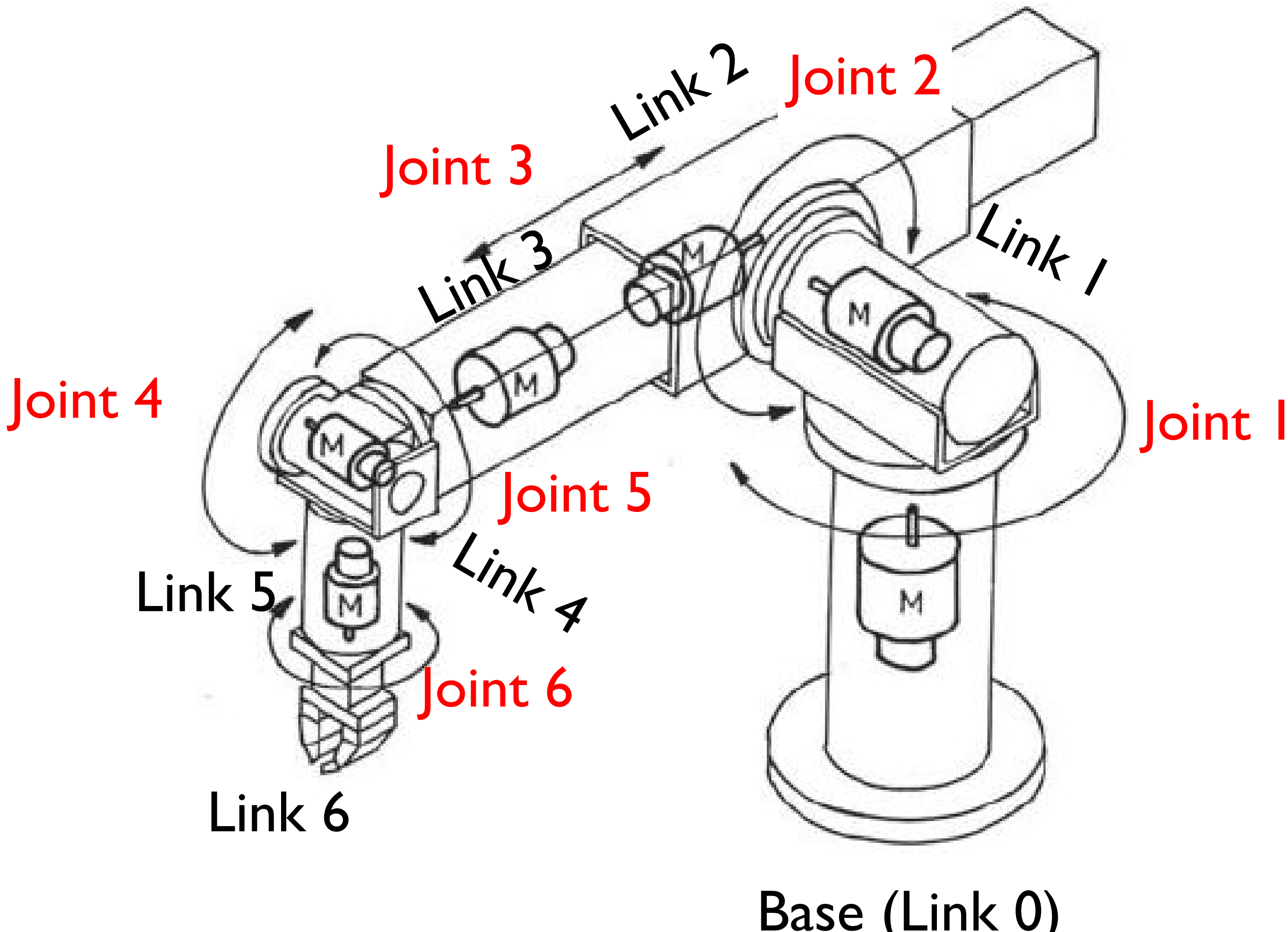
$$T_{02} = A_1 A_2 = \begin{bmatrix} R_{02} & \boldsymbol{d}_{02} \\ 0 & 1 \end{bmatrix}$$



3D 6DOF Links/Joints



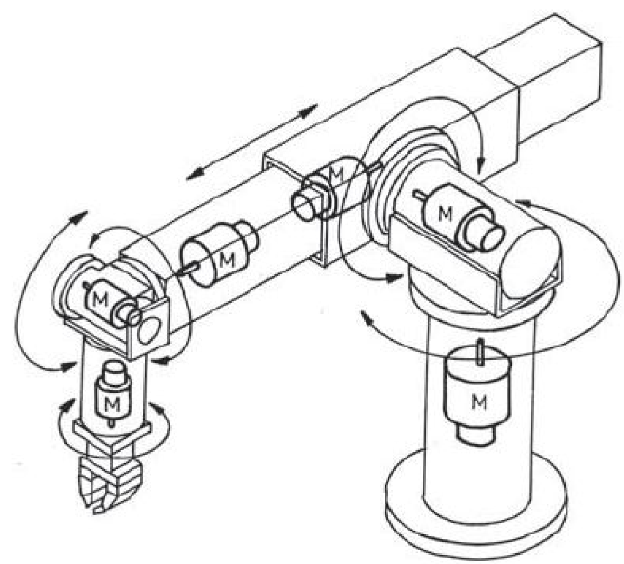
3D 6DOF Links/Joints



RRPRRR

3D 6DOF Transformation

$$T_{06} = A_1 A_2 A_3 A_4 A_5 A_6 = \begin{bmatrix} R_{06} & \boldsymbol{d}_{06} \\ 0 & 1 \end{bmatrix}$$

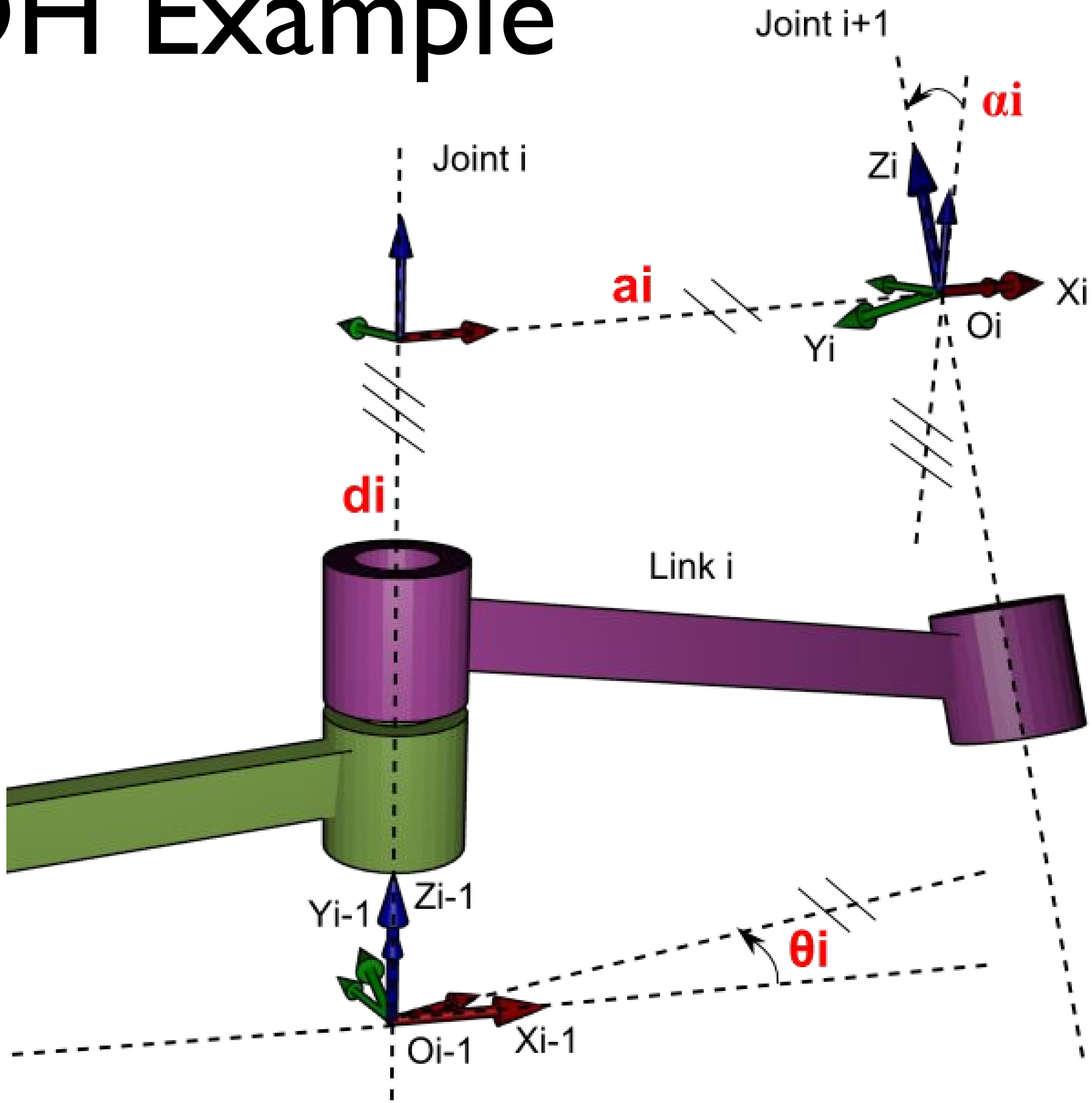


Denavit-Hartenberg Convention

4 parameters for each link/joint i

- a_i is **link length** of link i
- $\boldsymbol{a}_i$ is **link twist** of link i
- d_i is the **link/joint offset** of link/joint i
- θ_i is the **joint angle** of joint i

DH Example



DH Link Transformation

$$A_i = Rot_{z,\theta_i} Trans_{z,d_i} Trans_{x,a_i} Rot_{x,\alpha}$$

$$A_i = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_\alpha & -s_\alpha & 0 \\ 0 & s_\alpha & c_\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_i = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

DH Frame Placement Rules

[DH1] The axis $Z_{i\text{-}1}$ is the joint axis for joint i

- Axis of revolution for revolute joint

- Axis of translation for prismatic

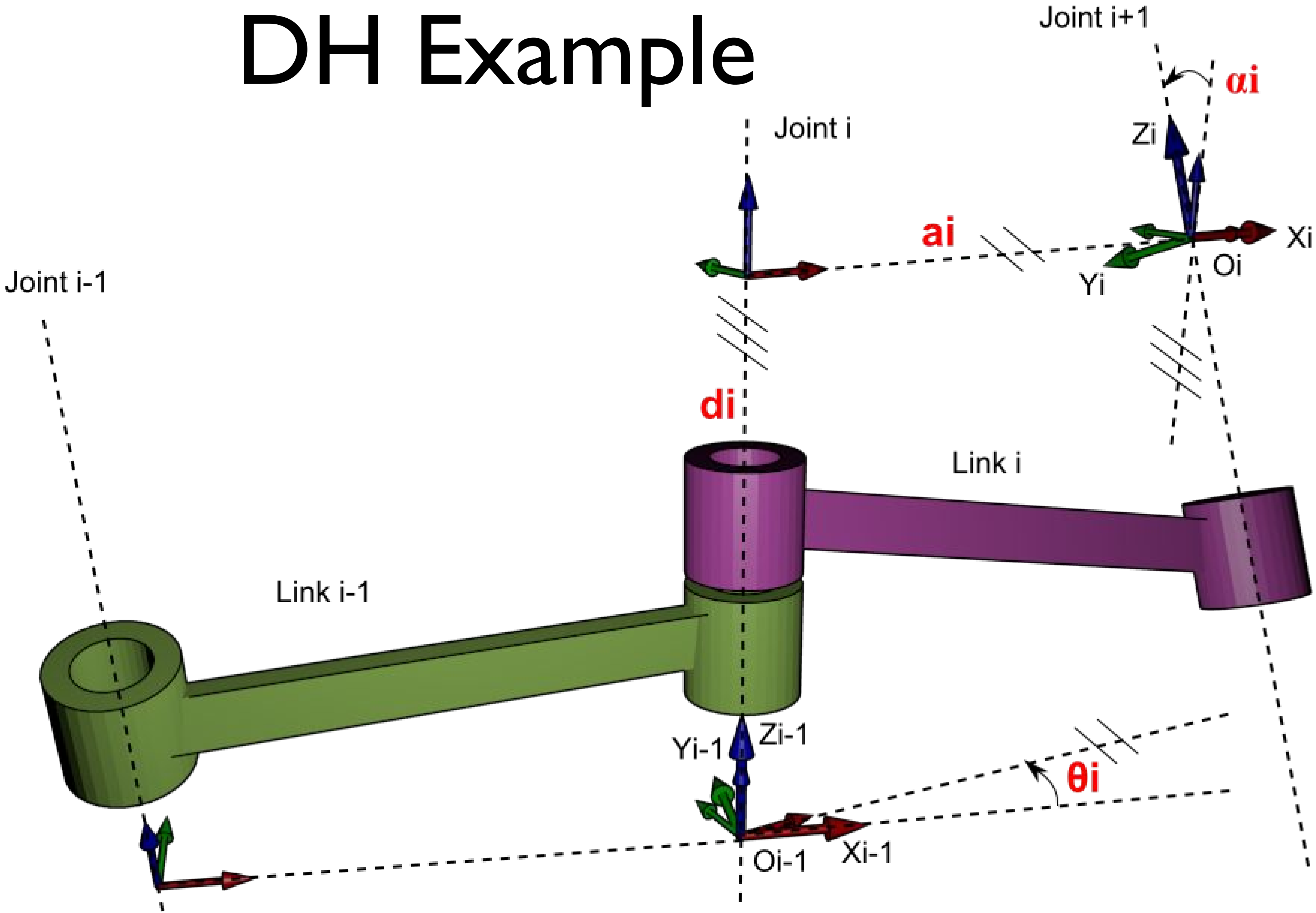
[DH2] The axis X_i is perpendicular to the axis $Z_{i\text{-}1}$

[DH3] The axis X_i intersects the axis $Z_{i\text{-}1}$

DH Parameters

- a_i is distance between Z_i and $Z_{i\text{-}1}$ along X_i
- α_i is the angle between Z_i and $Z_{i\text{-}1}$ about X_i
- d_i is distance between X_i and $X_{i\text{-}1}$ along $Z_{i\text{-}1}$
- θ_i is the angle between X_i and $X_{i\text{-}1}$ about $Z_{i\text{-}1}$

DH Example

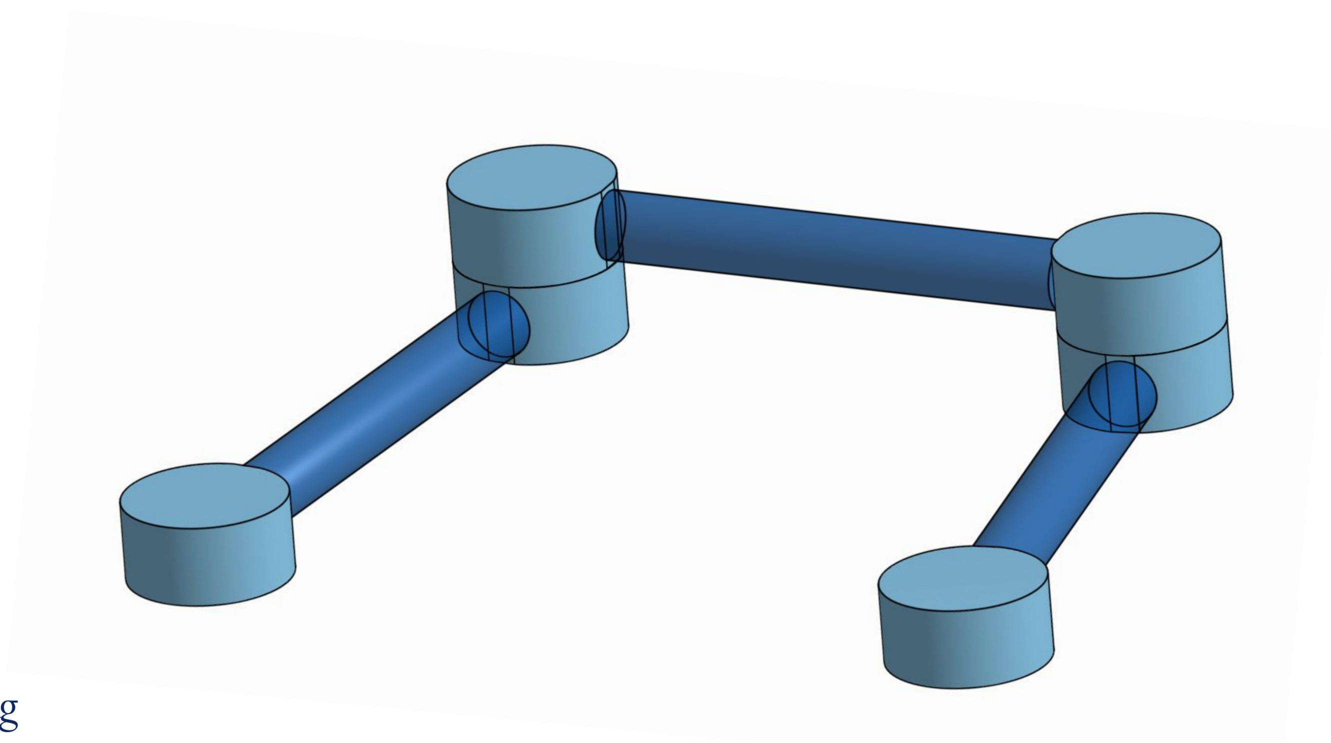

Joint i+1
αi
Joint i
Zi
ai
Xi
Oi
Yi
Joint i-1
di
Link i
Link i-1
Yi-1
Zi-1
θi
Oi-1
Xi-1

DH Process

1. Label z_i axes
2. Set base frame and end effector frame x_0 and y_0 as arbitrary
3. For $i=1, \ldots n-1$,
   A. Find common normal between z_i and z_{i-1} (z_i and z_{i-1} parallel is a special case)
   B. Establish x_i on this normal
   C. Establish y_i perpendicular to x_i and z_i to form a right handed coordinate frame
4. Create a table of all link parameters $a_i\ d_i\ \alpha_i\ \theta_i$
5. Form homogeneous transformation A_i for each link
6. Form $T_0^n = A_1 \cdots A_n$

Special Case

- If z_i and z_{i-1} are parallel:
- Choose any d. Other parameters are the same as before



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Week 5: Examples of Forward Kinematics

3link Cylindrical Robot DH Parameters

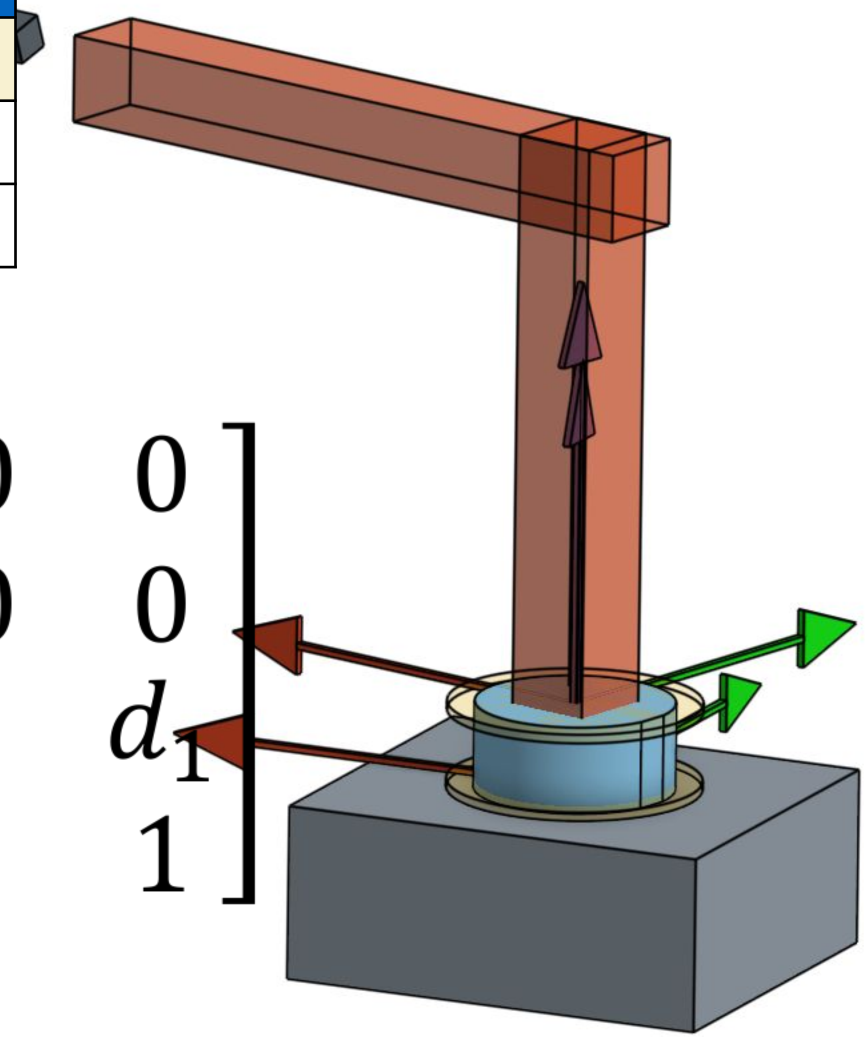
Link	a_i	a_i	d_i	θ_i
1	0	0	d_1	$\underline{\boldsymbol{\theta}_{\mathbf{1}}}$
2	0	-90	$\underline{\mathbf{d}_{\mathbf{2}}}$	0
3	0	0	$\underline{\mathbf{d}_{\mathbf{3}}}$	0

<u>Bolded</u> are joint variables

Link 1: revolute joint

Link	a_i	a_i	d_i	θ_i
1	0	0	d_1	$\underline{\boldsymbol{\theta_1}}$
2	0	-90	$\underline{\mathbf{d_2}}$	0
3	0	0	$\underline{\mathbf{d_3}}$	0

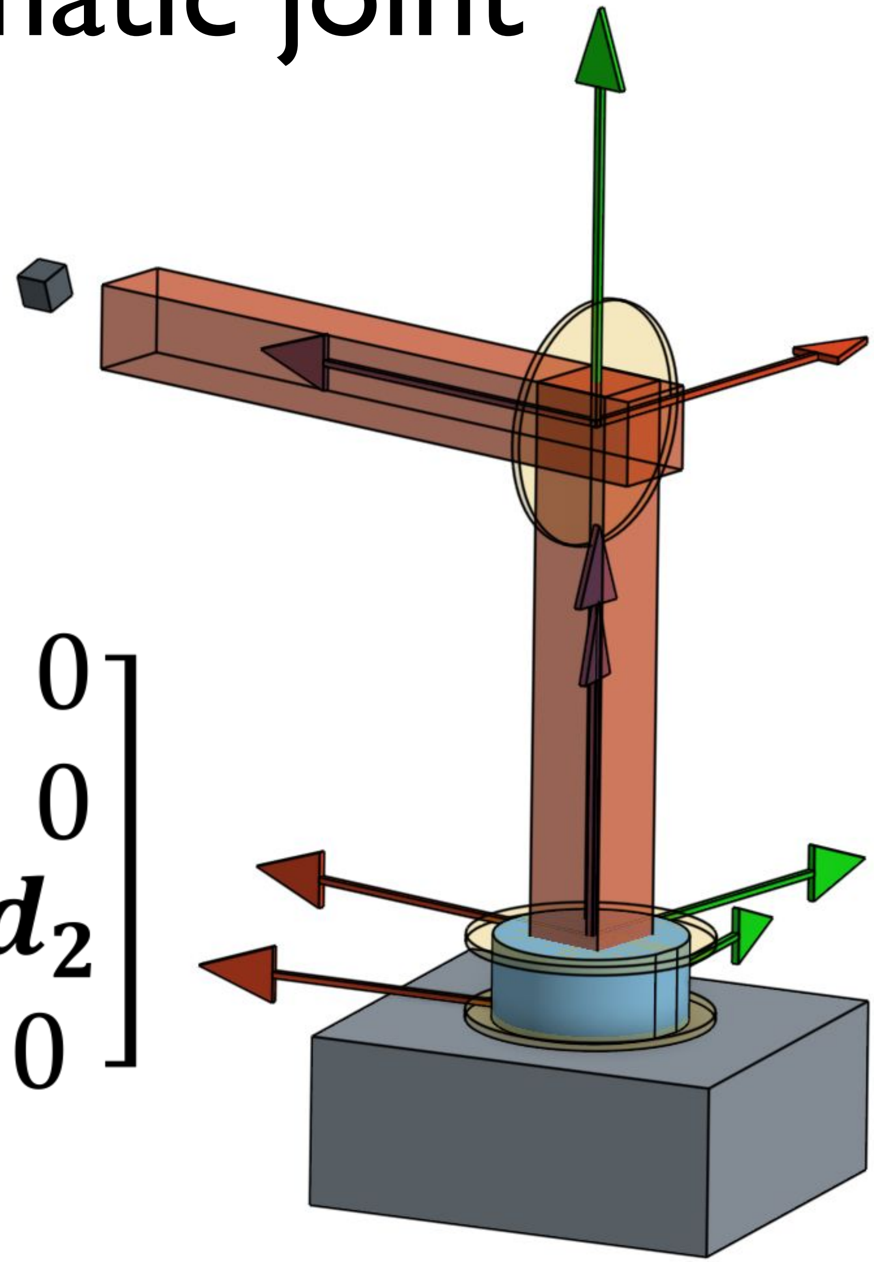
$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Link 2: prismatic joint

Link	a_i	α_i	d_i	θ_i
1	0	0	d_1	$\underline{\boldsymbol{\theta}_1}$
2	0	-90	$\underline{\mathbf{d}_2}$	0
3	0	0	$\underline{\mathbf{d}_3}$	0

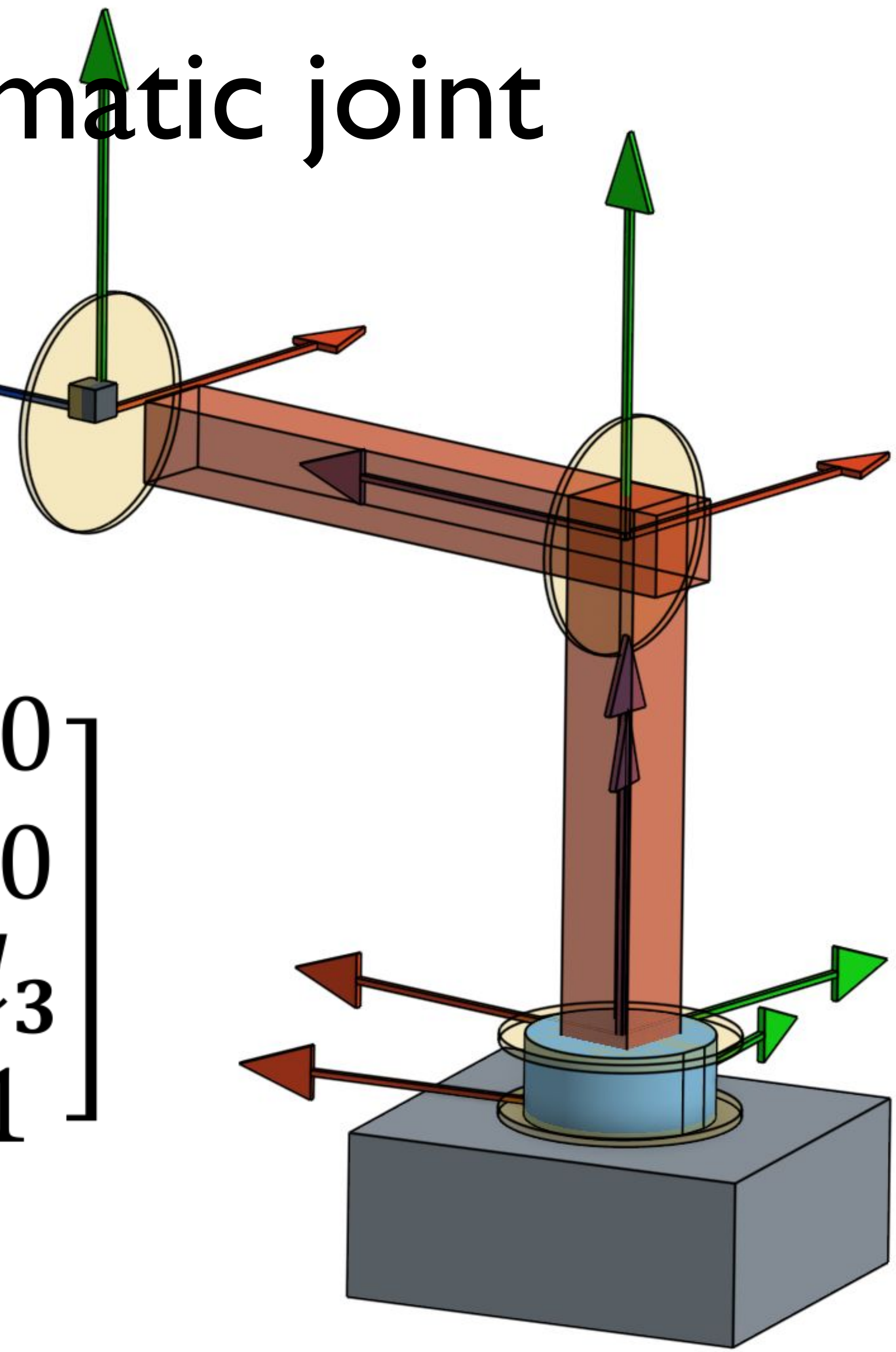
$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & \boldsymbol{d_2} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



Link 3: prismatic joint

Link	a_i	α_i	d_i	θ_i
1	0	0	d_1	$\underline{\boldsymbol{\theta_1}}$
2	0	-90	$\underline{\mathbf{d_2}}$	0
3	0	0	$\underline{\mathbf{d_3}}$	0

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \boldsymbol{d_3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Penn Engineering

End-effector Transform

$$T_{03} = A_1 A_2 A_3$$

$$T_{03} = \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

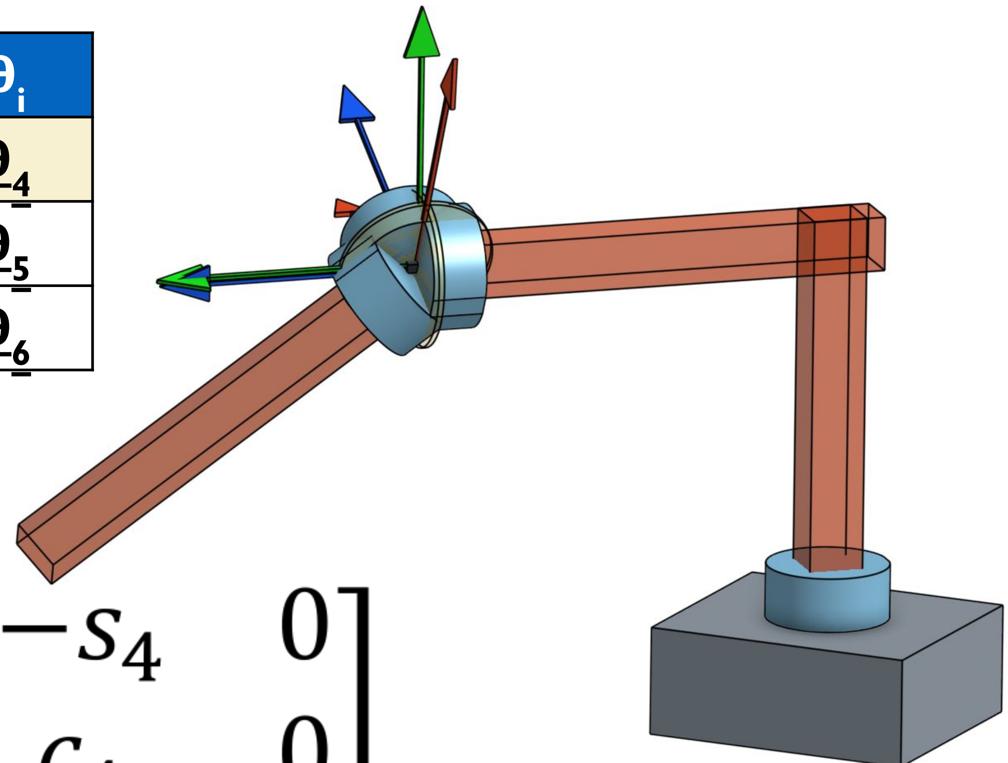
Spherical Wrist DH Parameters

Link	a_i	a_i	d_i	θ_i
4	0	-90	0	$\underline{\boldsymbol{\theta_4}}$
5	0	90	0	$\underline{\boldsymbol{\theta_5}}$
6	0	0	d_6	$\underline{\boldsymbol{\theta_6}}$

<u>Bolded</u> are joint variables

Link 4: revolute joint

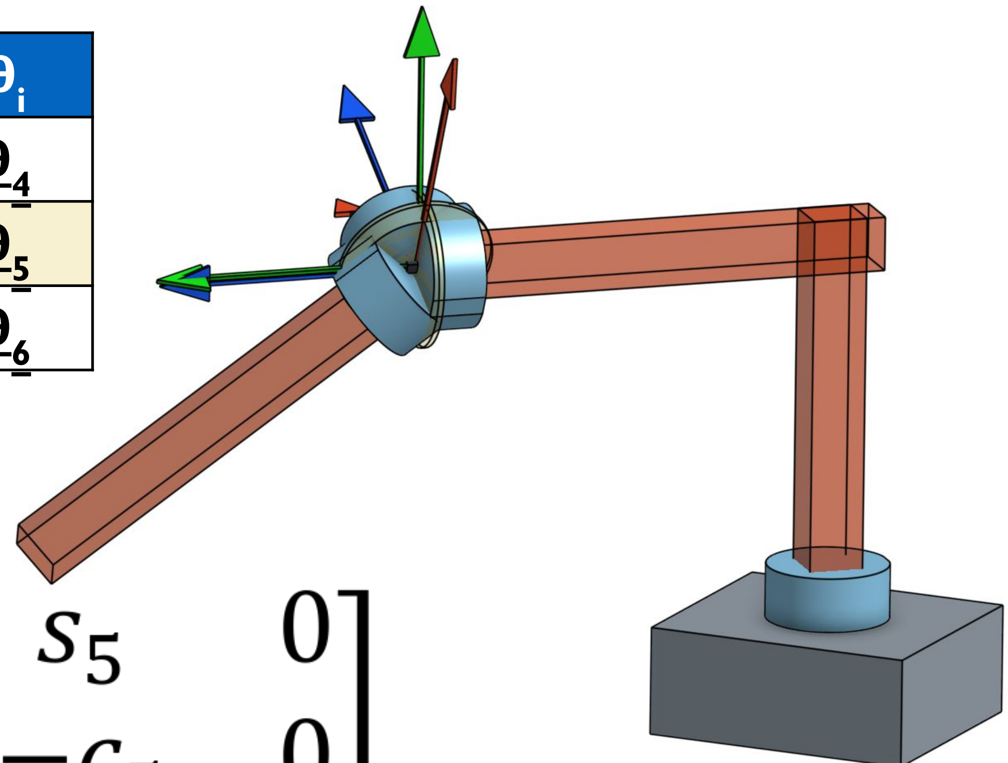
Link	a_i	α_i	d_i	θ_i
4	0	-90	0	$\underline{\theta_4}$
5	0	90	0	$\underline{\theta_5}$
6	0	0	d_6	$\underline{\theta_6}$



$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link 5: revolute joint

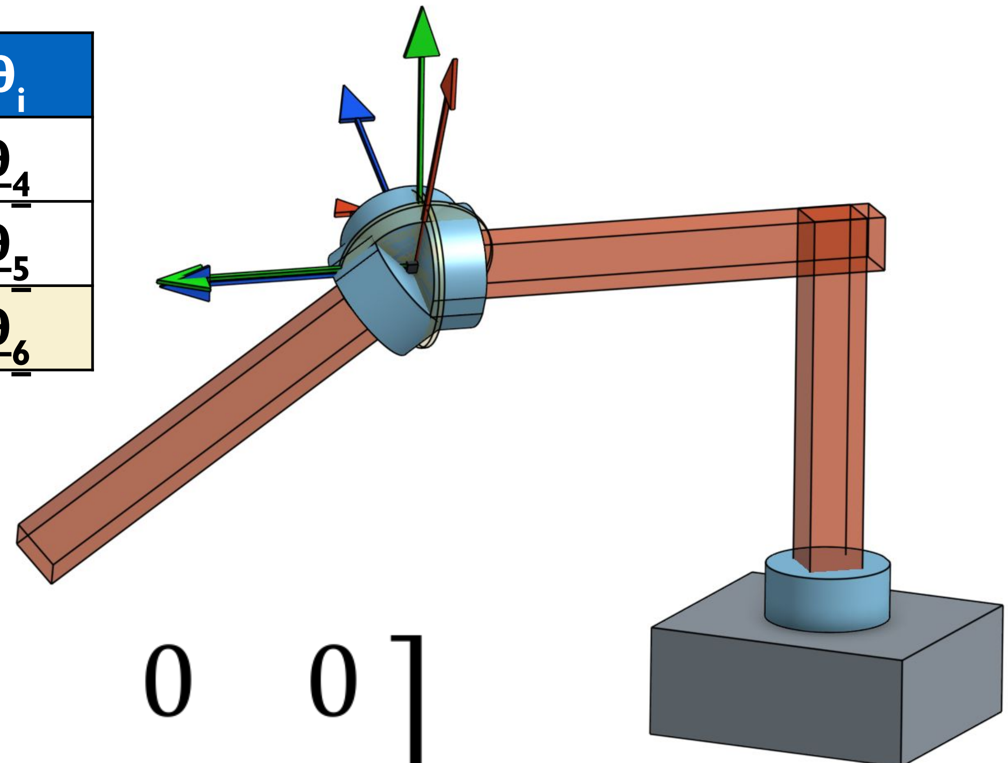
Link	a_i	α_i	d_i	θ_i
4	0	-90	0	$\underline{\boldsymbol{\theta_4}}$
5	0	90	0	$\underline{\boldsymbol{\theta_5}}$
6	0	0	d_6	$\underline{\boldsymbol{\theta_6}}$



$$A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link 6: revolute joint

Link	a_i	α_i	d_i	θ_i
4	0	-90	0	$\underline{\boldsymbol{\theta}_4}$
5	0	90	0	$\underline{\boldsymbol{\theta}_5}$
6	0	0	d_6	$\underline{\boldsymbol{\theta}_6}$



$$A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

End-effector Transform

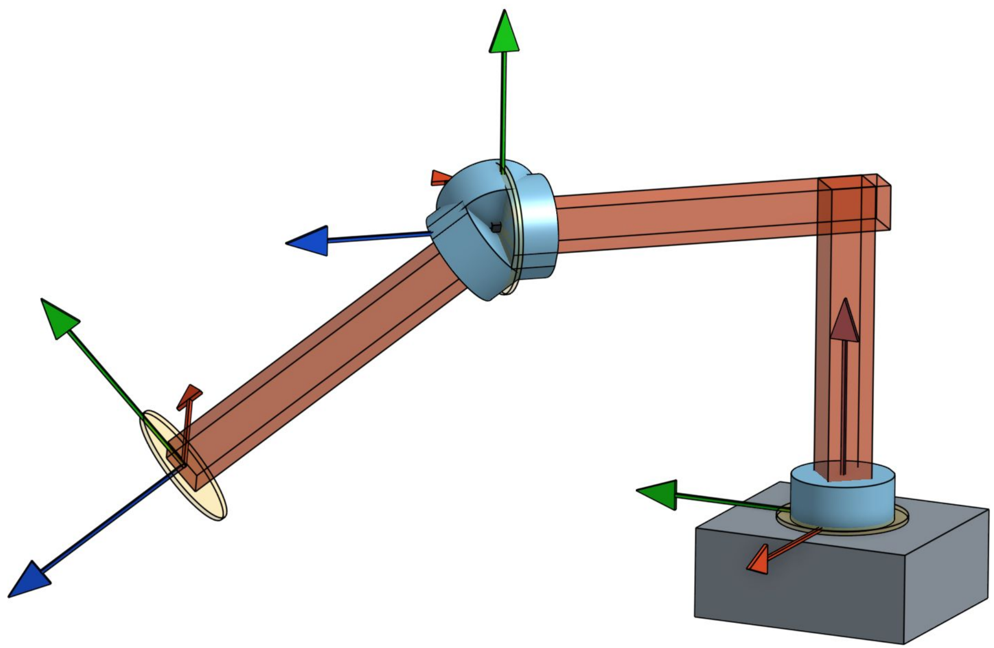
$$T_{36} = A_4 A_5 A_6$$

$$T_{36} = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_5c_6 & c_4s_5 & c_4s_5d_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & s_4s_5d_6 \\ -s_5c_6 & s_5s_6 & c_5 & c_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

End-effector Transform

$$T_{06} = T_{03}T_{36}$$

$$= \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_4 c_5 c_6 - s_4 s_6 & -c_4 c_5 s_6 - s_5 c_6 & c_4 s_5 & c_4 s_5 d_6 \\ s_4 c_5 c_6 + c_4 s_6 & -s_4 c_5 s_6 + c_4 c_6 & s_4 s_5 & s_4 s_5 d_6 \\ -s_5 c_6 & s_5 s_6 & c_5 & c_5 d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Final Transform

$$T_{06} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$r_{11} = c_1c_4c_5c_6 - c_1s_4s_6 + s_1\, s_5c_6$$
$$r_{21} = s_1c_4c_5c_6 - s_1s_4s_6 - c_1s_5c_6$$
$$r_{31} = -s_4c_5c_6 - c_4s_6$$
$$r_{12} = -c_1c_4c_5s_6 - c_1s_4s_6 - s_1s_5s_6$$
$$r_{22} = -s_1c_4c_5s_6 - s_1s_4c_6 + c_1s_5s_6$$
$$r_{32} = s_4c_5s_6 - c_4c_6$$
$$r_{13} = c_1c_4s_5 + s_1c_5$$
$$r_{23} = s_1c_4s_5 + c_1c_5$$
$$r_{33} = -s_4s_5$$
$$d_x = c_1c_4s_5d_6 - s_1c_5d_6 - s_1d_3$$
$$d_y = s_1c_4s_5d_6 + c_1c_5d_6 + c_1d_3$$
$$d_z = -s_4s_5d_6 + d_1 + d_2$$

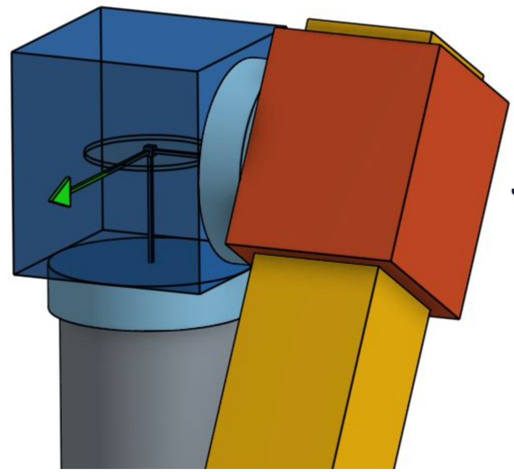
Stanford Arm (RRP) DH Parameters

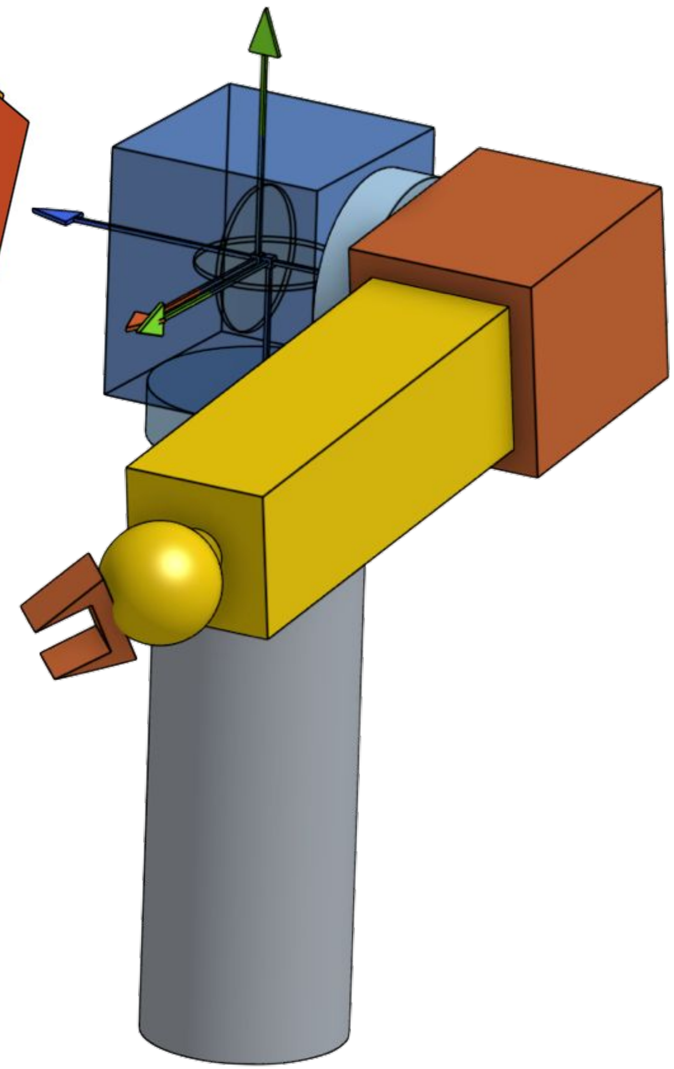
Link	a_i	a_i	d_i	θ_i
1	0	-90	0	$\underline{\boldsymbol{\theta_1}}$
2	0	90	d_2	$\underline{\boldsymbol{\theta_2}}$
3	0	0	$\underline{\boldsymbol{d_3}}$	0
4	0	-90	0	$\underline{\boldsymbol{\theta_4}}$
5	0	90	0	$\underline{\boldsymbol{\theta_5}}$
6	0	0	d_6	$\underline{\boldsymbol{\theta_6}}$

<u>Bolded</u> are joint variables

Link 1: revolute joint

Link	a_i	α_i	d_i	θ_i
1	0	-90	0	$\underline{\boldsymbol{\theta_1}}$
2	0	90	d_2	$\underline{\boldsymbol{\theta_2}}$
3	0	0	$\underline{\boldsymbol{d_3}}$	0
4	0	-90	0	$\underline{\boldsymbol{\theta_4}}$
5	0	90	0	$\underline{\boldsymbol{\theta_5}}$
6	0	0	d_6	$\underline{\boldsymbol{\theta_6}}$

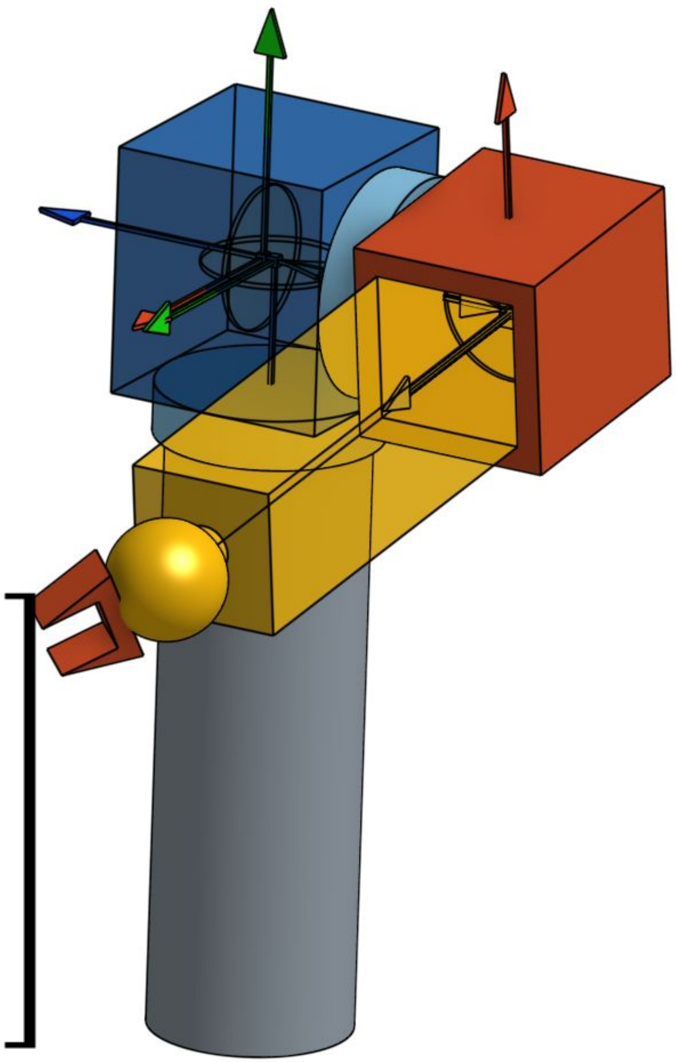




$$A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link 2: revolute joint

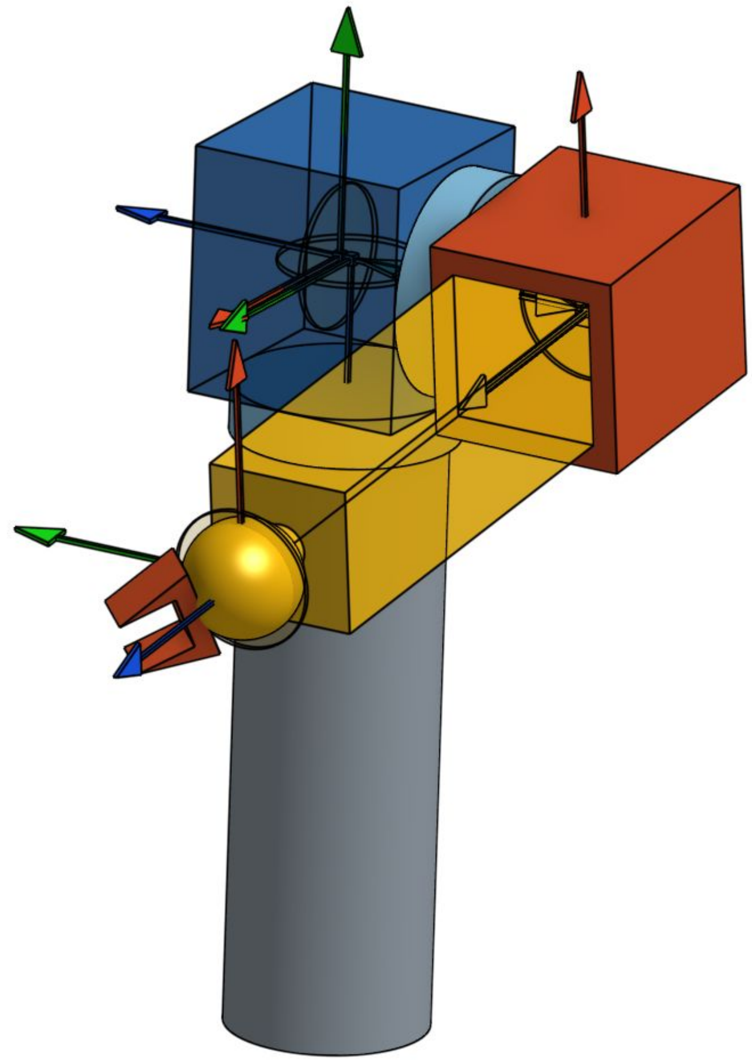
Link	a_i	a_i	d_i	θ_i
1	0	-90	0	$\underline{\boldsymbol{\theta}_1}$
2	0	90	d_2	$\underline{\boldsymbol{\theta}_2}$
3	0	0	$\underline{d_3}$	0
4	0	-90	0	$\underline{\boldsymbol{\theta}_4}$
5	0	90	0	$\underline{\boldsymbol{\theta}_5}$
6	0	0	d_6	$\underline{\boldsymbol{\theta}_6}$



$$A_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link 3: prismatic joint

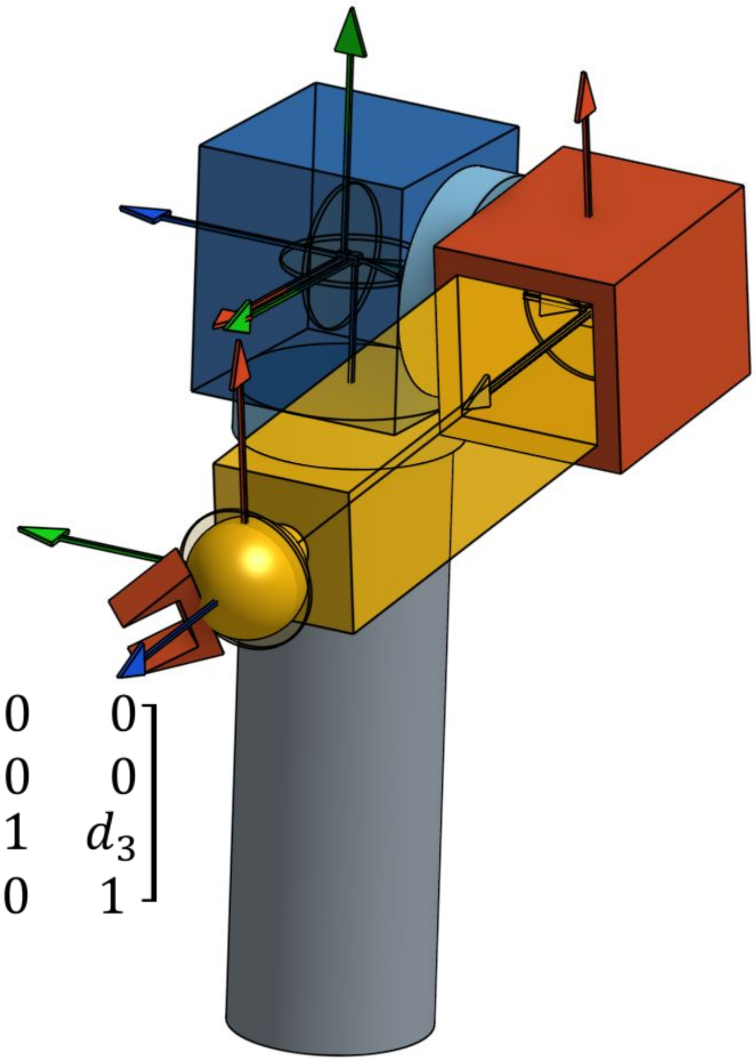
Link	a_i	α_i	d_i	θ_i
1	0	-90	0	$\underline{\boldsymbol{\theta}_1}$
2	0	90	d_2	$\underline{\boldsymbol{\theta}_2}$
3	0	0	$\underline{\boldsymbol{d}_3}$	0
4	0	-90	0	$\underline{\boldsymbol{\theta}_4}$
5	0	90	0	$\underline{\boldsymbol{\theta}_5}$
6	0	0	d_6	$\underline{\boldsymbol{\theta}_6}$



$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link 1-3: prismatic joint

Link	a_i	a_i	d_i	θ_i
1	0	-90	0	$\underline{\boldsymbol{\theta_1}}$
2	0	90	d_2	$\underline{\boldsymbol{\theta_2}}$
3	0	0	$\underline{\boldsymbol{d_3}}$	0
4	0	-90	0	$\underline{\boldsymbol{\theta_4}}$
5	0	90	0	$\underline{\boldsymbol{\theta_5}}$
6	0	0	d_6	$\underline{\boldsymbol{\theta_6}}$



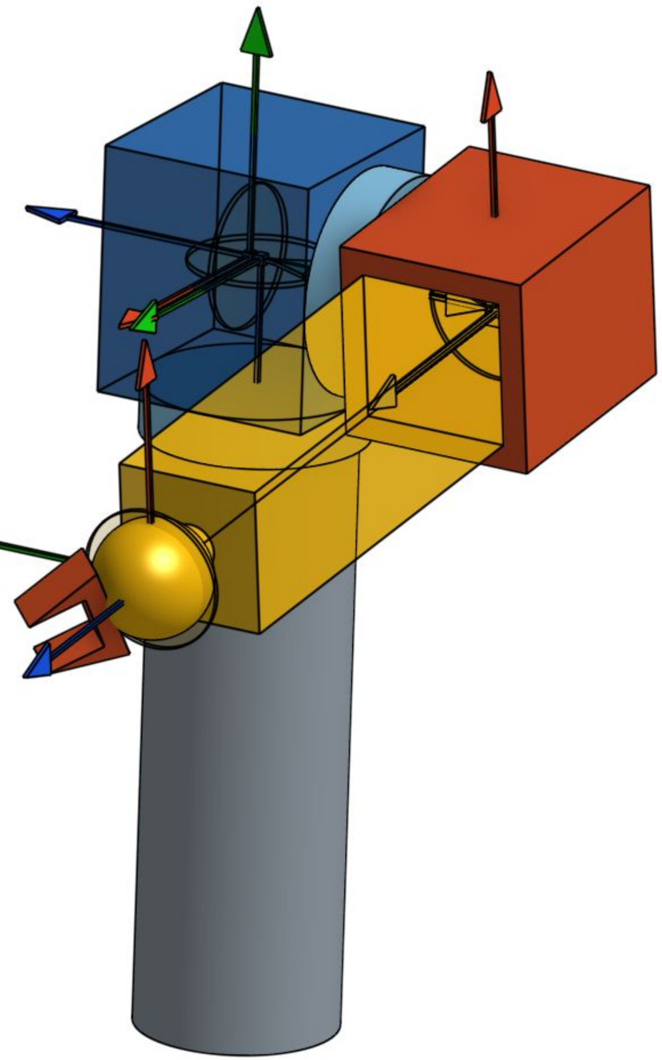
$$T_{03} = A_1 A_2 A_3$$

$$\begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Penn Engineering

Link 1-3: prismatic joint

Link	a_i	a_i	d_i	θ_i
1	0	-90	0	$\underline{\boldsymbol{\theta_1}}$
2	0	90	d_2	$\underline{\boldsymbol{\theta_2}}$
3	0	0	$\underline{\boldsymbol{d_3}}$	0
4	0	-90	0	$\underline{\boldsymbol{\theta_4}}$
5	0	90	0	$\underline{\boldsymbol{\theta_5}}$
6	0	0	d_6	$\underline{\boldsymbol{\theta_6}}$



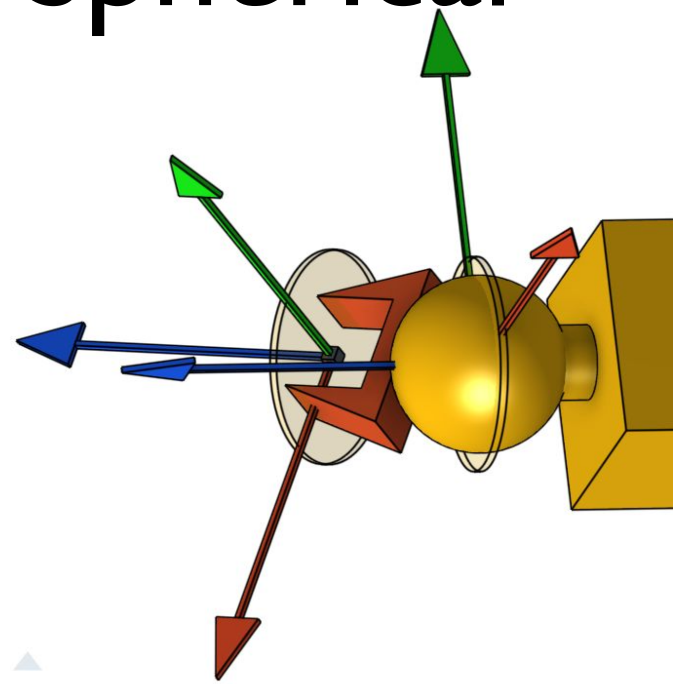
$$T_{03} = A_1 A_2 A_3$$

$$\begin{bmatrix} c_1c_2 & -s_1 & c_1s_2 & c_1d_3s_2 - d_2s_1 \\ c_2s_1 & c_1 & s_1s_2 & c_1d_2 + d_3s_1s_2 \\ -s_2 & 0 & c_2 & c_2d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Penn Engineering

Links 4-6: 3-axis Spherical Joint

Link	a_i	α_i	d_i	θ_i
1	0	-90	0	$\underline{\boldsymbol{\theta}_1}$
2	0	90	d_2	$\underline{\boldsymbol{\theta}_2}$
3	0	0	$\underline{\boldsymbol{d_3}}$	0
4	0	-90	0	$\underline{\boldsymbol{\theta}_4}$
5	0	90	0	$\underline{\boldsymbol{\theta}_5}$
6	0	0	d_6	$\underline{\boldsymbol{\theta}_6}$



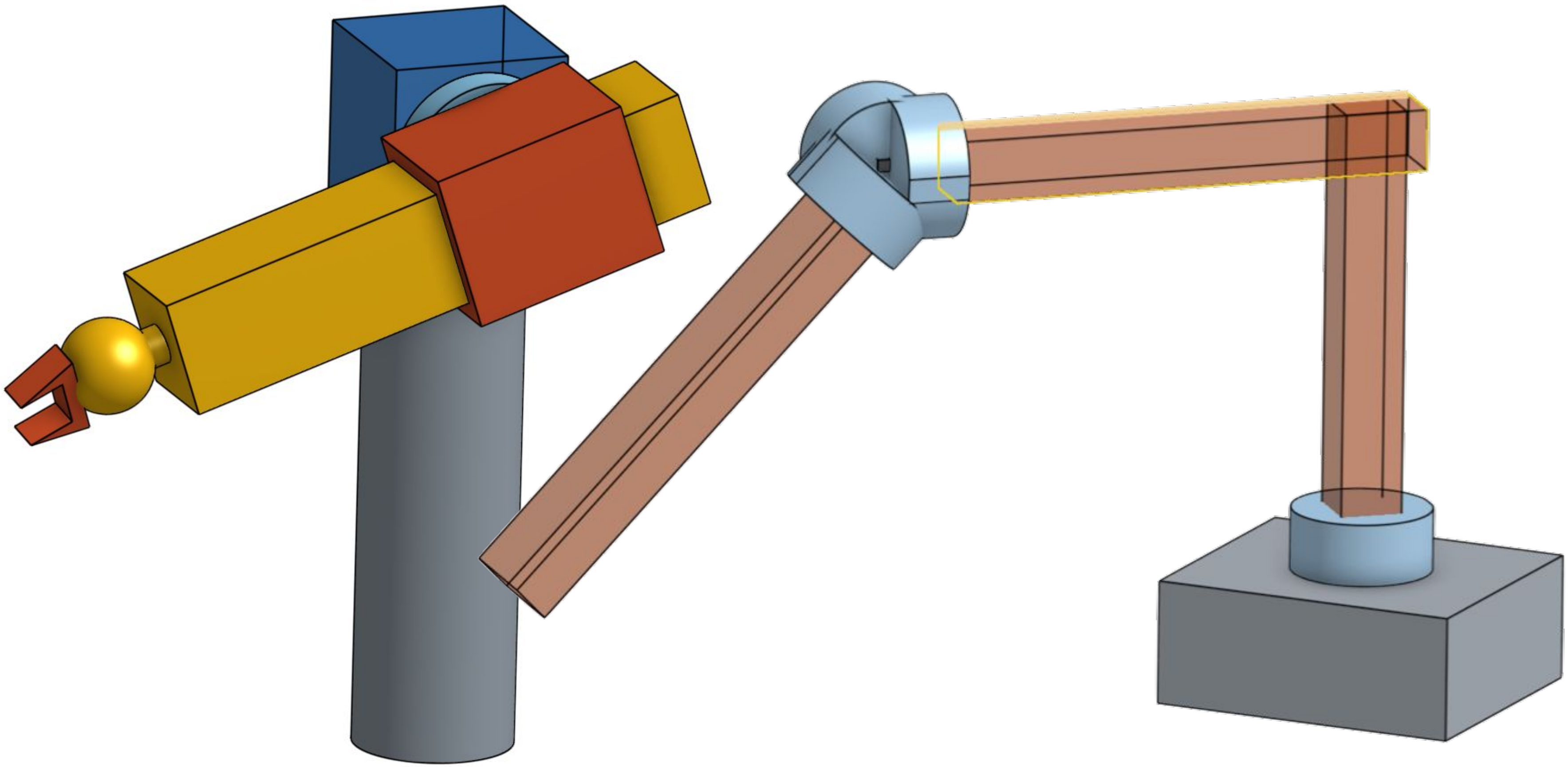
$$T_{36} = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_5c_6 & c_4s_5 & c_4s_5d_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & s_4s_5d_6 \\ -s_5c_6 & s_5s_6 & c_5 & c_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Stanford Arm Transform

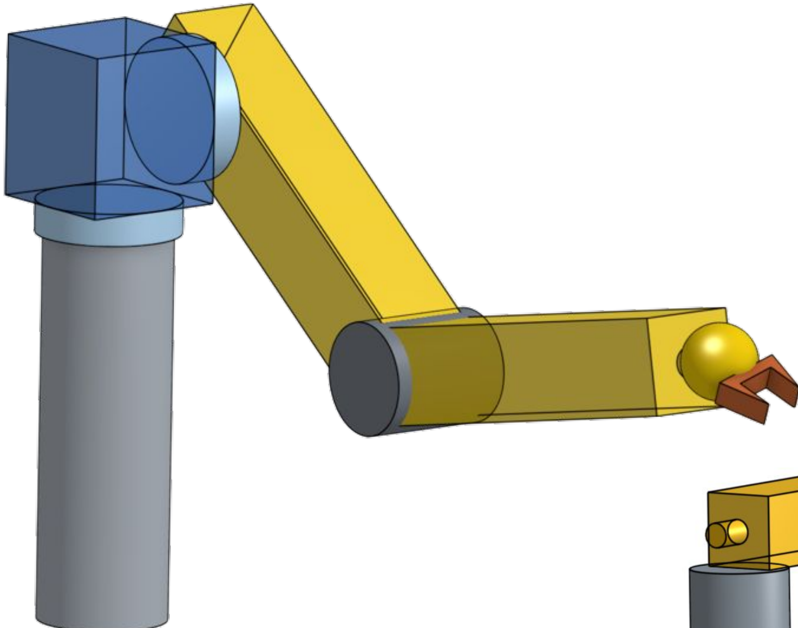
$$T_{06} = T_{03}T_{36} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & P_x \\ r_{21} & r_{22} & r_{23} & P_y \\ r_{31} & r_{32} & r_{33} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

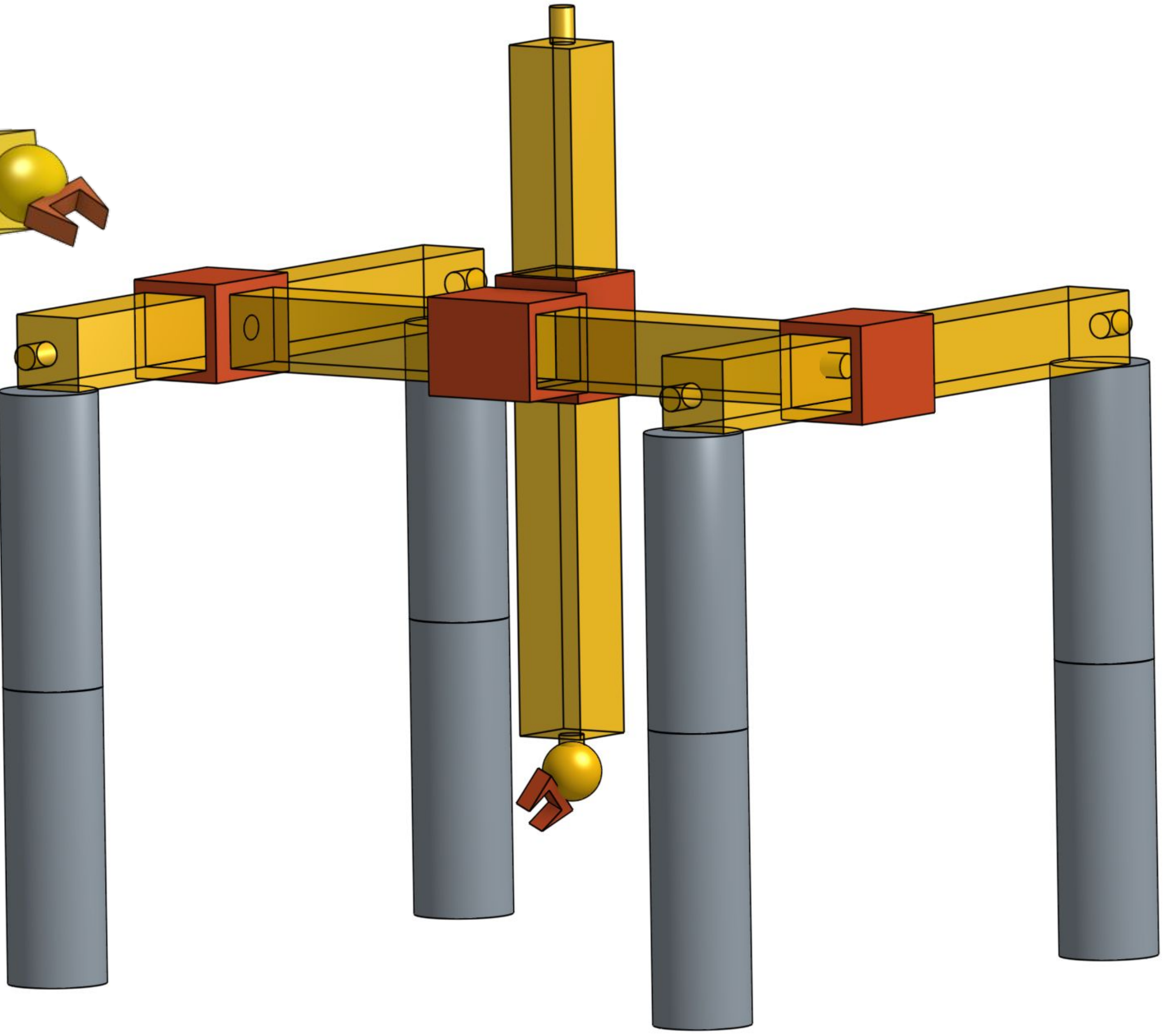
$$r_{11} = c_1[c_2(c_4c_5c_6 - s_4c_6) - s_2s_5c_6] - s_1(s_4c_5c_6 + c_4s_6)$$
$$r_{21} = s_1[c_2(c_4c_5c_6 - s_4c_6) - s_2s_5c_6] + c_1(s_4c_5c_6 + c_4s_6)$$
$$r_{31} = -s_2(c_4c_5c_6 - s_4s_6) - c_2s_5c_6$$
$$r_{12} = c_1[-c_2(c_4c_5s_6 + s_4c_6) + s_2s_5s_6] - s_1(-s_4c_5c_6 + c_4s_6)$$
$$r_{22} = s_1[-c_2(c_4c_5s_6 + s_4c_6) + s_2s_5s_6] + c_1(-s_4c_5c_6 + c_4s_6)$$
$$r_{32} = s_2(c_4c_5c_6 + s_4s_6) + c_2s_5c_6$$
$$r_{13} = c_1(c_2c_4s_5 + s_2c_5) - s_1s_4s_5$$
$$r_{23} = s_1(c_2c_4s_5 + s_2c_5) + c_1s_4s_5$$
$$r_{33} = -s_2c_4s_5 + c_2c_5$$
$$P_x = c_1s_2d_3 - s_1d_2 + d_6(c_1c_2c_4s_5 + c_1s_2c_5 - s_1s_4s_5)$$
$$P_y = s_1s_2d_3 + c_1d_2 + d_6(s_1c_2c_4s_5 + s_1s_2c_5 + c_1s_4s_5)$$
$$P_z = c_2d_3 + d_6(c_2c_5 - s_2c_4s_5)$$

Standard configurations



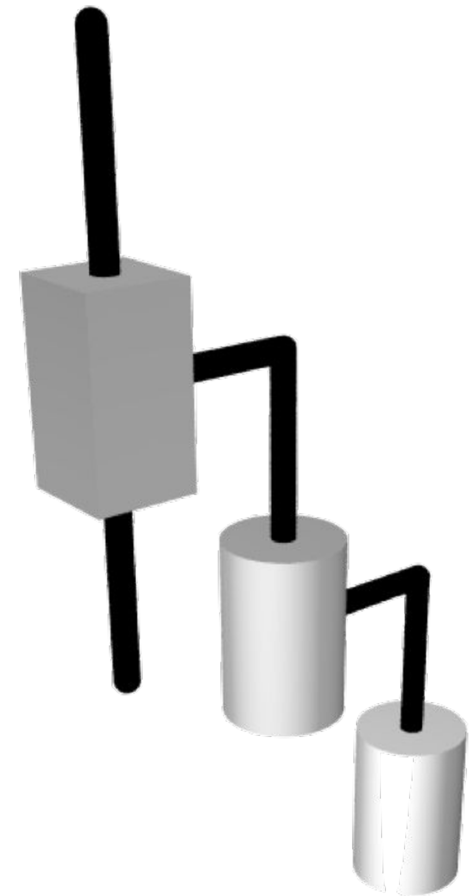
Standard configurations





SCARA robot arm

Selective Compliance Articulated Robot Arm.



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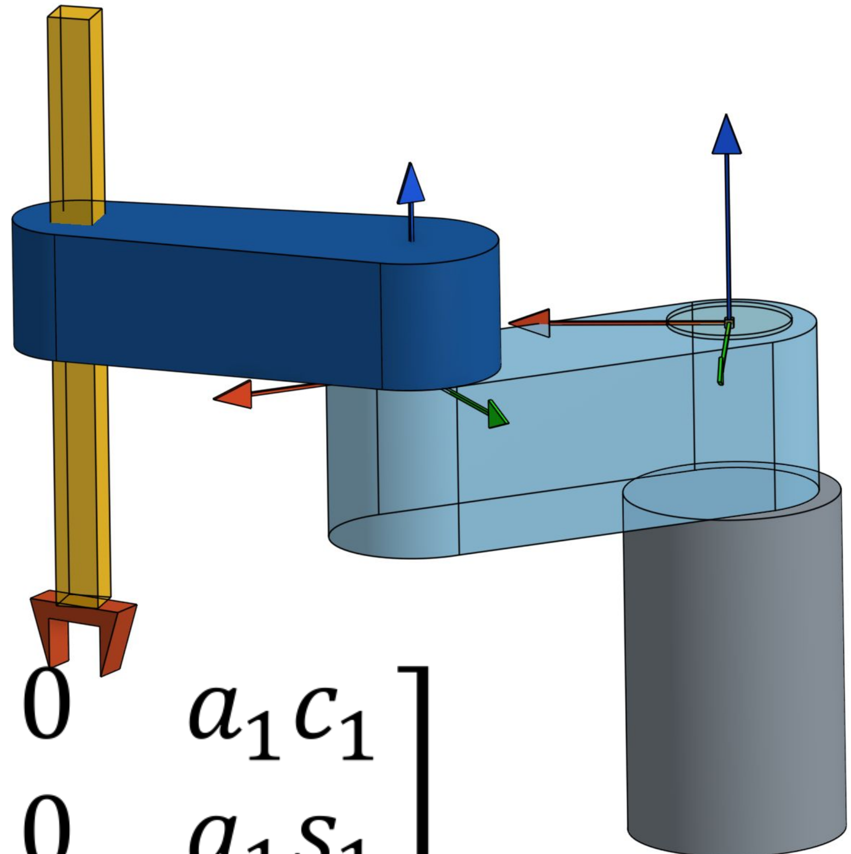
SCARA Arm DH Parameters

Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	$\underline{\boldsymbol{\theta}_1}$
2	a_2	180	*0*	$\underline{\boldsymbol{\theta}_2}$
3	0	0	$\underline{\boldsymbol{d}_3}$	0
4	0	0	d_4	$\underline{\boldsymbol{\theta}_4}$

<u>Bolded</u> are joint variables

Link 1: z-axis revolute joint

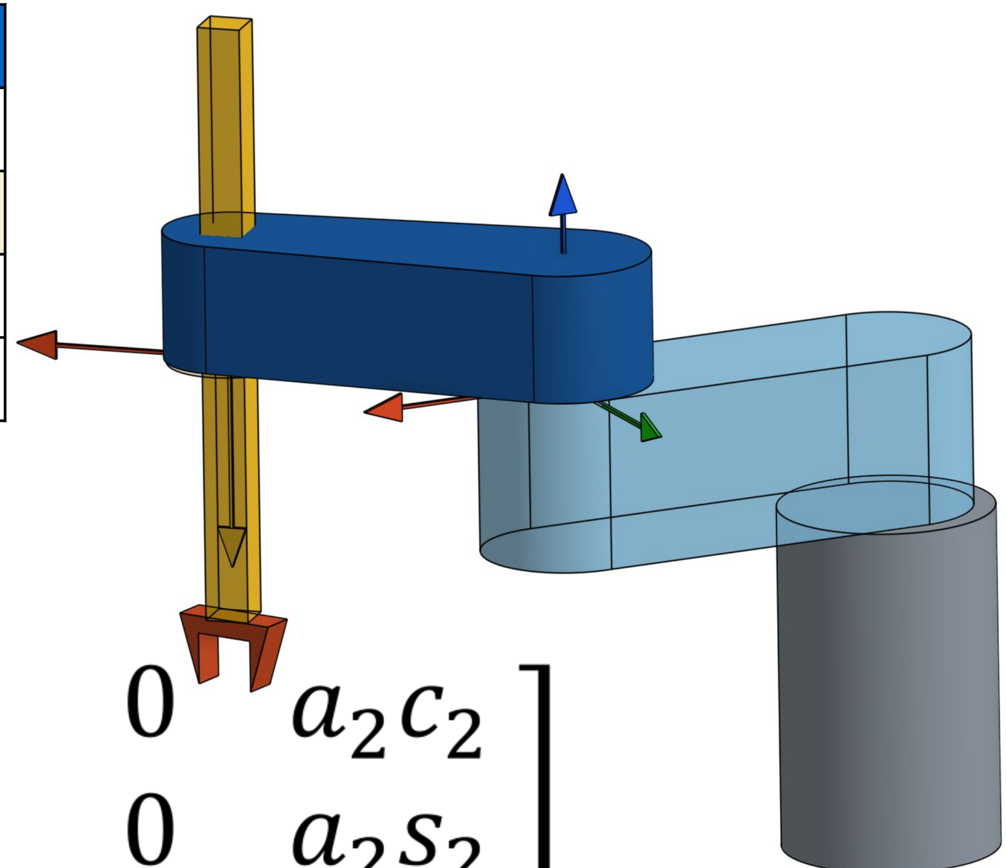
Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	$\underline{\boldsymbol{\theta_1}}$
2	a_2	180	*0*	$\underline{\boldsymbol{\theta_2}}$
3	0	0	$\underline{\boldsymbol{d_3}}$	0
4	0	0	d_4	$\underline{\boldsymbol{\theta_4}}$



$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link 2: z-axis revolute joint

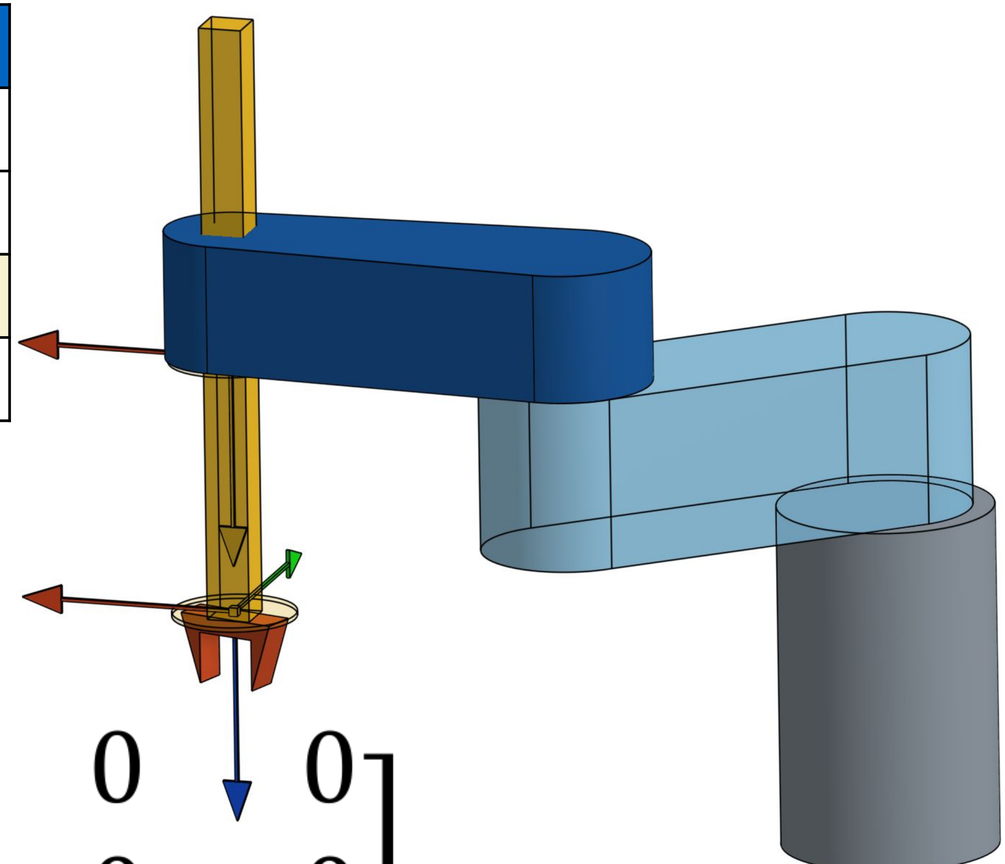
Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	$\underline{\theta_1}$
2	a_2	180	0	$\underline{\theta_2}$
3	0	0	$\underline{d_3}$	0
4	0	0	d_4	$\underline{\theta_4}$



$$A_2 = \begin{bmatrix} c_2 & s_2 & 0 & a_2 c_2 \\ s_2 & -c_2 & 0 & a_2 s_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link 3: prismatic joint

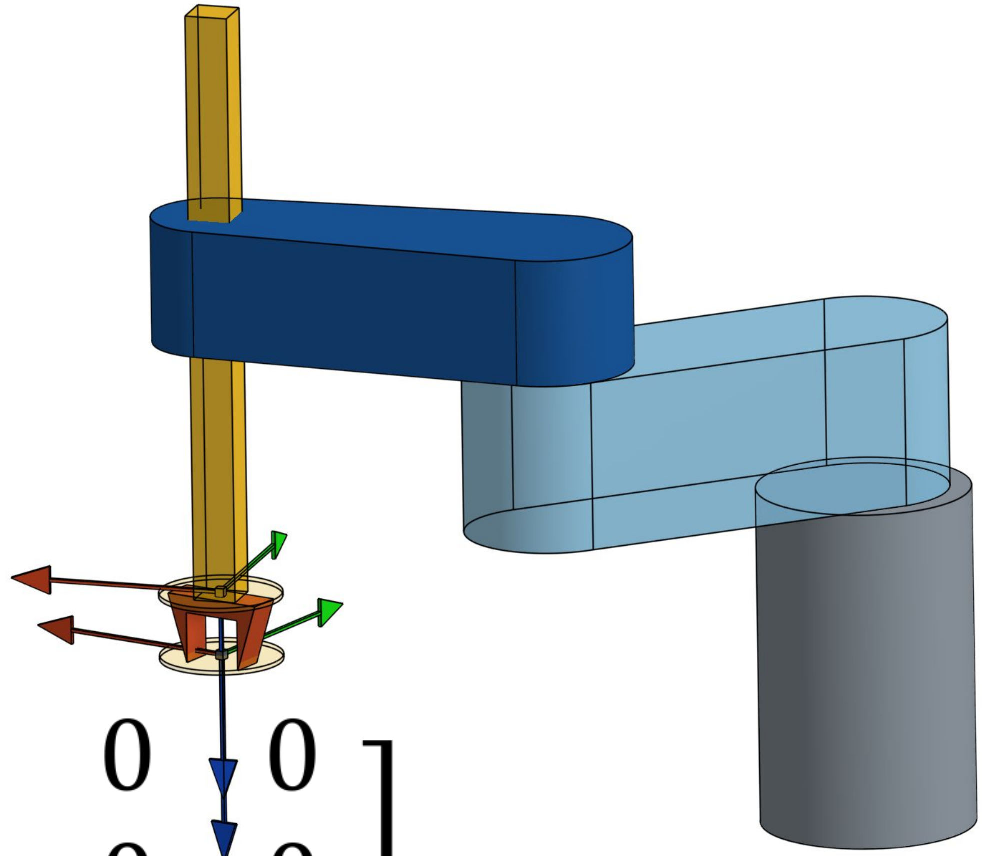
Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	$\underline{\theta_1}$
2	a_2	180	0	$\underline{\theta_2}$
3	0	0	$\underline{d_3}$	0
4	0	0	d_4	$\underline{\theta_4}$



$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link 4: *z*-axis revolute joint

Link	a_i	a_i	d_i	θ_i
1	a_1	0	0	$\underline{\theta_1}$
2	a_2	180	*0*	$\underline{\theta_2}$
3	0	0	$\underline{d_3}$	0
4	0	0	d_4	$\underline{\theta_4}$



$$A_4 = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

End-effector Transform

$$T_{04} = A_1 A_2 A_3 A_4$$

$$T_{04} = \begin{bmatrix} c_{12}c_4 + s_{12}s_4 & -c_{12}s_4 + s_{12}c_4 & 0 & a_1c_1 + a_2c_{12} \\ s_{12}c_4 - c_{12}s_4 & -s_{12}s_4 - c_{12}c_4 & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & -1 & -d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$