统计咨询与实践第一次作业

张申铎 2176112379, 祖劭康 2173412124

目录

1 问题 1

我们对 MNIST 数据集进行了主成分分析，对结果进行了分析，并对数据进行了降维可视化。

1.1 MNIST 数据集准备

```
## Warning: package 'keras' was built under R version 4.0.3
```

MNIST 手写数字数据集是机器学习中最经典的数据集之一，其包含 60000 个 28 乘 28 的手写数字样本。R 语言没有内置 MNIST 数据集，我们使用 Keras 包内的 MNIST 数据集。

我们观测一下 MNIST 数据的形态。

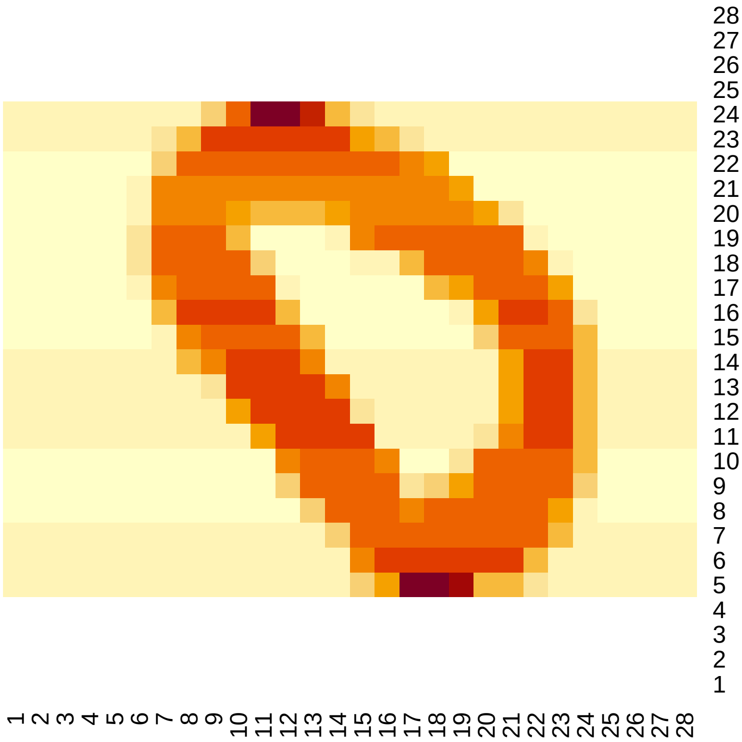
```
mnist$train$x[100,1:6,1:6]
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    0    0    0    0    0    0
## [2,]    0    0    0    0    0    0
```

```
## [3,]    0    0    0    0    0    0
## [4,]    0    0    0    0    0    0
## [5,]    0    0    0    0    0    0
## [6,]    0    0    0    0    0    0
```

对其抽取样本进行初步可视化。

```
heatmap(mnist$train$x[400,,],Colv = NA, Rowv = NA, scale = "row", )
```



我们将 MNIST 矩阵形式的数据拉直成数据框。

```
pcadata<-data.frame(t(as.matrix(c(1:28^2))))
for (i in 1:10000) {
  pcadata[nrow(pcadata)+1,] <- as.vector(mnist$train$x[nrow(pcadata)+1,,])
}
```

计算其数字 1 与数字 8 的不同像素位置的几个基本统计量

数字 1

```
summary(pcadata[mnist$train$y==1,400])
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##     0.0     0.0    48.0   103.1   251.0   255.0    5615
```

```
summary(pcadata[mnist$train$y==1,700])
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##       0       0       0       0       0       0    5615
```

数字 8

```
summary(pcadata[mnist$train$y==8,400])
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##     0.0    56.0   209.0   159.9   253.0   255.0    4907
```

```
summary(pcadata[mnist$train$y==8,700])
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##       0       0       0       0       0       0    4907
```

1.2 分析过程与结果

由于 MNIST 数据集过大，数据清洗与 PCA 分析时间过长。我们只采用了其中训练集的 1000 个样本进行降维。由于有显著信息的变量过多，报告打不下，为了保证 80% 的信息，我们需要 38 个主成分。

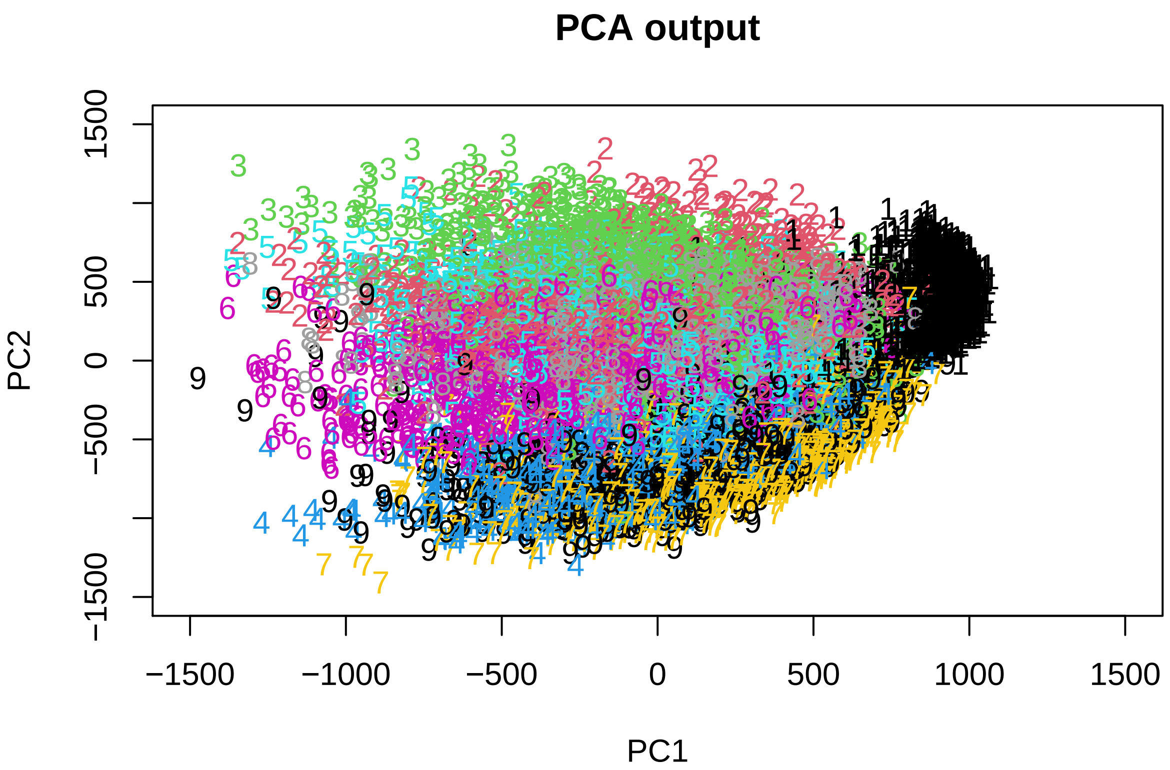
```
pcamnist <- prcomp(pcadata,rank=38)
summary(pcamnist)
```

```
## Importance of first k=38 (out of 784) components:
##                             PC1       PC2       PC3       PC4       PC5
## Standard deviation     593.0788 497.56143 457.91305 431.02790 405.30373
## Proportion of Variance   0.1021   0.07187   0.06088   0.05394   0.04769
## Cumulative Proportion    0.1021   0.17399   0.23487   0.28881   0.33650
##                              PC6       PC7       PC8       PC9      PC10
## Standard deviation     389.36623 336.09412 316.92757 309.71704 281.36123
## Proportion of Variance   0.04401   0.03279   0.02916   0.02785   0.02298
## Cumulative Proportion    0.38051   0.41331   0.44247   0.47032   0.49330
```

```
##                            PC11      PC12      PC13      PC14      PC15
## Standard deviation     270.93242 265.57429 240.61012 240.10678 235.62637
## Proportion of Variance   0.02131   0.02048   0.01681   0.01674   0.01612
## Cumulative Proportion    0.51461   0.53509   0.55190   0.56863   0.58475
##                            PC16      PC17      PC18      PC19      PC20
## Standard deviation     229.73770 210.32808 208.3031 199.80815 196.73280
## Proportion of Variance   0.01532   0.01284   0.0126   0.01159   0.01124
## Cumulative Proportion    0.60008   0.61292   0.6255   0.63711   0.64834
##                           PC21      PC22      PC23      PC24      PC25
## Standard deviation     188.3304 186.01165 180.03788 177.9954 171.46227
## Proportion of Variance   0.0103   0.01005   0.00941   0.0092   0.00854
## Cumulative Proportion    0.6586   0.66869   0.67810   0.6873   0.69583
##                            PC26      PC27      PC28      PC29      PC30
## Standard deviation     170.23286 166.0351 163.12154 157.22572 153.71945
## Proportion of Variance   0.00841   0.0080   0.00773   0.00718   0.00686
## Cumulative Proportion    0.70424   0.7123   0.71997   0.72715   0.73401
##                            PC31      PC32      PC33      PC34      PC35
## Standard deviation     149.75903 146.99962 142.99832 142.13940 137.6093
## Proportion of Variance   0.00651   0.00627   0.00594   0.00587   0.0055
## Cumulative Proportion    0.74052   0.74679   0.75273   0.75860   0.7641
##                            PC36      PC37      PC38
## Standard deviation     134.76719 132.73443 128.28624
## Proportion of Variance   0.00527   0.00512   0.00478
## Cumulative Proportion    0.76937   0.77448   0.77926
```

我们首先选择最大的两个主成分进行可视化，结果如下

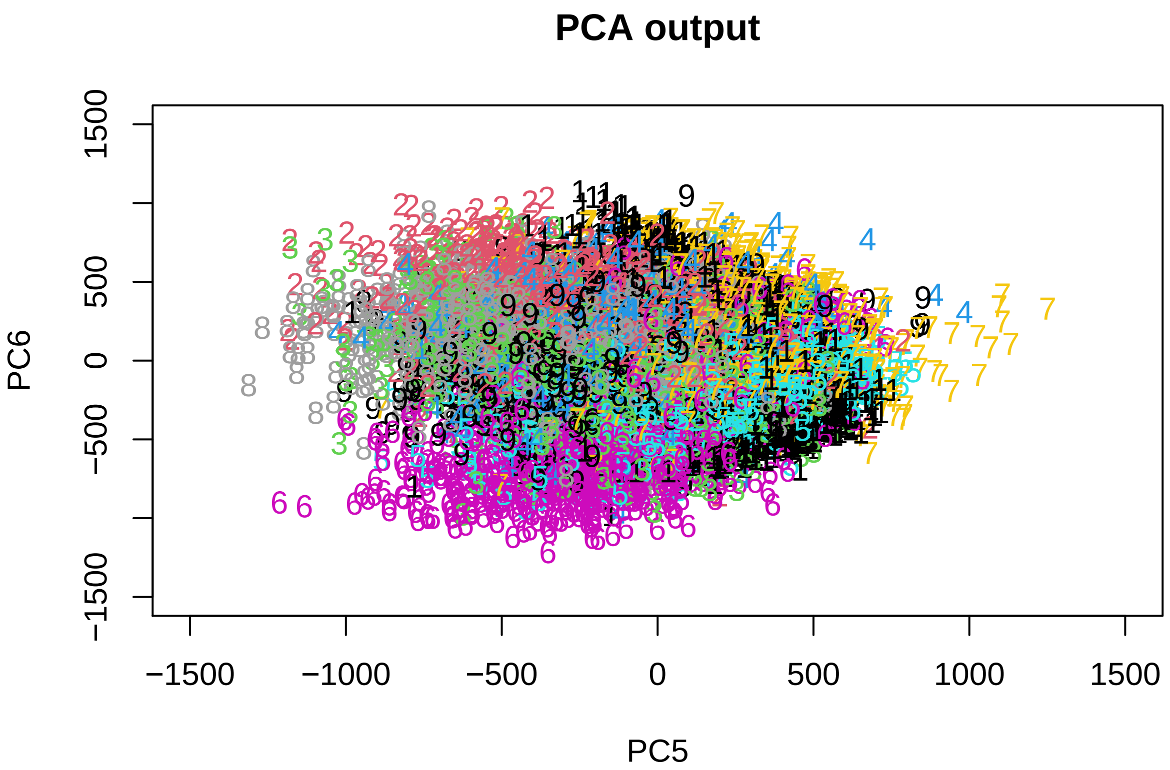
```
plot(pcamnist$x[,1:2], pch = as.character(mnist$train$y), col = mnist$train$y, main = "PCA output"
```



虽然前两个主成分只有占了 17% 的方差，我们可以看到高维的手写数字在两个特征值最大主成分方向上的投影分布非常规律。相同的数字聚集成了条状，不同数字基本上在这个子空间内所分布的区域处于可辨别的程度。

我们再尝试将其投影到非特征值最大的两个主成分方向上，

```
plot(pcamnist$x[,5:6], pch = as.character(mnist$train$y), col = mnist$train$y, main = "PCA output"
```



我们可以看到，虽然我们能够得到与原来类似的结果，但是其不同标签数字之间的分离效果没有特征值最大的主成分好。这两个投影方向下的投影依旧有与最大两个方向投影的相似之处，比如数字 1 虽然与其他数字重叠，其自己还是基本上保持了原来的条形形状。

2 问题 2

因子模型的基本形式如下

$$X - \mu = LF + \varepsilon$$

L 为载荷矩阵，F 为公共因子，ε 为噪声

考虑因子模型三种计算载荷矩阵的方法

2.1 准备工作数据

首先清理环境并安装所需软件包，设置工作路径，并读取数据。

```
library(psych)
setwd("C:/Users/75401/Documents/GitHub/Statistical_Consultary/SecondHomework")
data <- read.table('yzfx.txt',sep = '',header = FALSE,row.names = 1)
result <- list("loading matrix"= 1, "noise" = 1,"matrix of error" = 1)
elements <- c(" 人均粮食支出"," 人均副食支出"," 人均烟酒茶支出"," 人均其他副食支出",
              " 人均衣着商品支出"," 人均日用品支出"," 人均燃料支出"," 人均非商品支出")
colnames(data) <- elements
```

对数据进行标准化

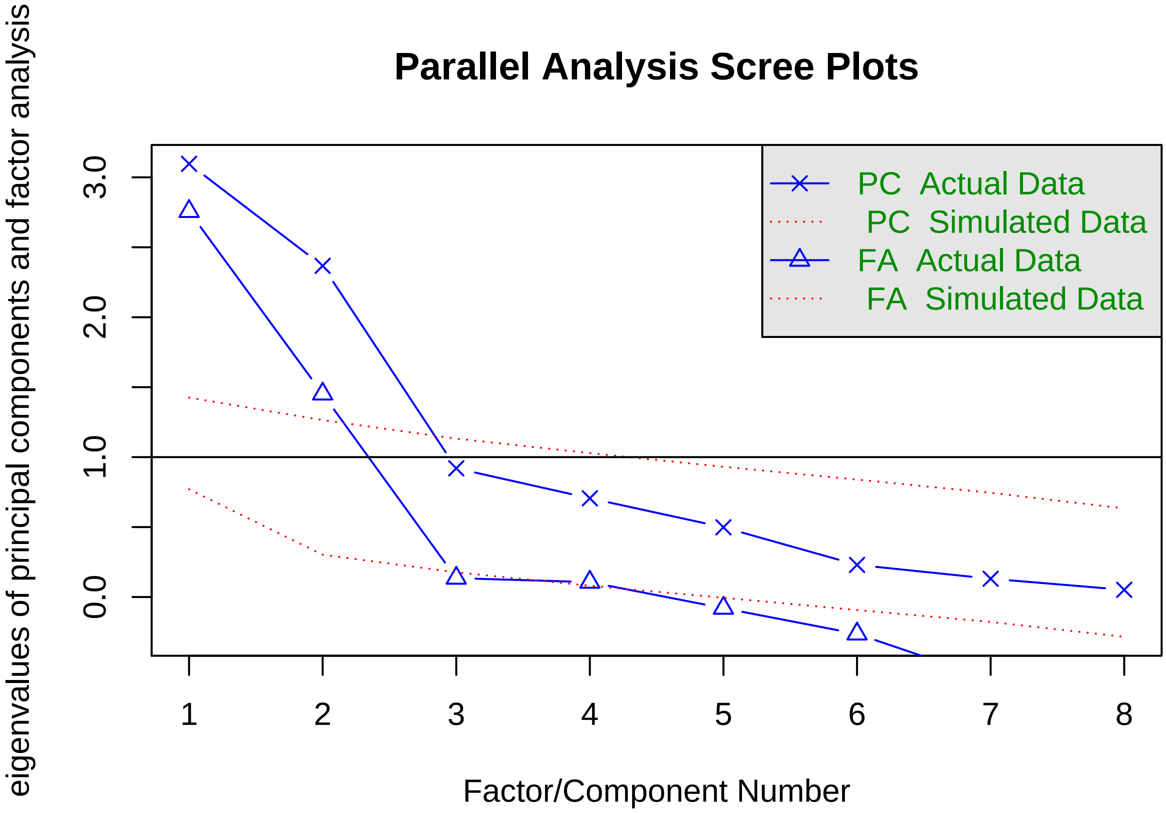
```
data <- scale(data, center = T, scale = T)
```

2.2 实验过程

2.2.1 判断选取因子的数量

通过碎石图判断选取因子的数量

```
fa.parallel(data, n.obs = 112, fa = "both", n.iter = 100)
```




```
## Parallel analysis suggests that the number of factors =  2  and the number of components =  2
```

由图像可看出应选取前两个因子或三个因子，对于因子的数量的选取将在下方的实验中进行尝试

2.2.2 主成分分解法

先对数据作主成分分析，算出其主成分

```
pc <- princomp(data,cor=TRUE)
summary(pc,loadings=T)
```

```
## Importance of components:
##                           Comp.1    Comp.2    Comp.3     Comp.4     Comp.5
## Standard deviation     1.759627 1.5385783 0.9591597 0.84019364 0.70600448
## Proportion of Variance 0.387036 0.2959029 0.1149984 0.08824067 0.06230529
## Cumulative Proportion  0.387036 0.6829389 0.7979373 0.88617801 0.94848330
```

```
##                              Comp.6     Comp.7      Comp.8
## Standard deviation     0.47946669 0.36162932 0.226868982
## Proportion of Variance 0.02873604 0.01634697 0.006433692
## Cumulative Proportion  0.97721934 0.99356631 1.000000000
##
## Loadings:
##                    Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8
## 人均粮食支出        0.250  0.241  0.694  0.377  0.502
## 人均副食支出        0.519                0.225 -0.424         0.282  0.643
## 人均烟酒茶支出            -0.475  0.578        -0.510  0.173 -0.381
## 人均其他副食支出    0.254 -0.538         0.231        -0.399  0.472 -0.458
## 人均衣着商品支出          -0.575        -0.285  0.516 -0.146 -0.159  0.521
## 人均日用品支出      0.493 -0.135 -0.145 -0.224  0.177  0.755        -0.244
## 人均燃料支出        0.317  0.261  0.286 -0.768        -0.355  0.131
## 人均非商品支出      0.509        -0.271  0.177        -0.305 -0.708 -0.181
```

2.2.2.1 选取两个主成分作为因子

```
l1 <- sqrt(0.387036*8)*pc$loadings[,1]
l2 <- sqrt(0.2959029*8)*pc$loadings[,2]
L <- data.frame(l1,l2)
fai <- c()
tl <- t(L)
for(i in 1:dim(L)[1]){
    p <- 1 - t(tl[,i]) %*% tl[,i]
    fai <- c(fai , p)
}
L.matrix <- as.matrix(L)
E <- var(data) - L.matrix %*% t(L.matrix) - diag(fai)
result$'loading matrix' <- L
result$'noise' <- fai
result$'matrix of error' <- E
result
```

```
## $`loading matrix`
##                           l1         l2
## 人均粮食支出        0.43921474 0.37116382
```

```
## 人均副食支出       0.91365872  0.05786194
## 人均烟酒茶支出    -0.03251806 -0.73149938
## 人均其他副食支出   0.44710649 -0.82787939
## 人均衣着商品支出   0.03817479 -0.88537282
## 人均日用品支出     0.86690340 -0.20720886
## 人均燃料支出       0.55805995  0.40108027
## 人均非商品支出     0.89623367  0.13398155
##
## $noise
## [1] 0.6693278 0.1618797 0.4638512 0.1147115 0.2146576 0.2055430 0.5277037
## [8] 0.1788142
##
## $`matrix of error`
##                   人均粮食支出  人均副食支出 人均烟酒茶支出 人均其他副食支出
## 人均粮食支出       2.220446e-16 -8.912192e-02   2.312498e-01       0.04964943
## 人均副食支出      -8.912192e-02  2.220446e-16   4.913455e-02       0.03833099
## 人均烟酒茶支出     2.312498e-01  4.913455e-02   2.220446e-16      -0.05772503
## 人均其他副食支出   4.964943e-02  3.833099e-02  -5.772503e-02       0.00000000
## 人均衣着商品支出   2.249084e-02 -1.399532e-01  -1.496455e-01      -0.05162569
## 人均日用品支出    -1.050520e-01 -6.893027e-02  -9.055352e-02      -0.09122478
## 人均燃料支出      -4.527617e-02 -1.194890e-01   1.724512e-01      -0.08874031
## 人均非商品支出    -1.246908e-01  8.347607e-03  -1.312069e-01       0.02296595
##                  人均衣着商品支出 人均日用品支出 人均燃料支出 人均非商品支出
## 人均粮食支出           0.022490841  -1.050520e-01  -0.04527617  -1.246908e-01
## 人均副食支出          -0.139953215  -6.893027e-02  -0.11948900   8.347607e-03
## 人均烟酒茶支出        -0.149645511  -9.055352e-02   0.17245119  -1.312069e-01
## 人均其他副食支出      -0.051625694  -9.122478e-02  -0.08874031   2.296595e-02
## 人均衣着商品支出       0.000000000   6.357820e-02   0.12552436   3.175946e-03
## 人均日用品支出         0.063578202   2.220446e-16   0.01614460  -4.759997e-02
## 人均燃料支出           0.125524361   1.614460e-02   0.00000000  -1.550216e-01
## 人均非商品支出         0.003175946  -4.759997e-02  -0.15502156   2.220446e-16
```

从残差矩阵可以看出因子分析的效果较好

2.2.2.2 选取三个主成分作为因子

```r
l1 <- sqrt(0.387036*8)*pc$loadings[,1]
l2 <- sqrt(0.2959029*8)*pc$loadings[,2]
l3 <- sqrt(0.1149984*8)*pc$loadings[,3]
L <- data.frame(l1,l2,l3)
fai <- c()
tl <- t(L)
for(i in 1:dim(L)[1]){
    p <- 1 - t(tl[,i]) %*% tl[,i]
    fai <- c(fai , p)
}
L.matrix <- as.matrix(L)
E <- var(data) - L.matrix %*% t(L.matrix) - diag(fai)
result$'loading matrix' <- L
result$'noise' <- fai
result$'matrix of error' <- E
result
```

```
## $`loading matrix`
##                            l1          l2          l3
## 人均粮食支出       0.43921474  0.37116382  0.66557829
## 人均副食支出       0.91365872  0.05786194 -0.06835051
## 人均烟酒茶支出    -0.03251806 -0.73149938  0.55422061
## 人均其他副食支出   0.44710649 -0.82787939 -0.02088723
## 人均衣着商品支出   0.03817479 -0.88537282 -0.04612331
## 人均日用品支出     0.86690340 -0.20720886 -0.13941216
## 人均燃料支出       0.55805995  0.40108027  0.27469437
## 人均非商品支出     0.89623367  0.13398155 -0.26020007
##
## $noise
## [1] 0.2263334 0.1572079 0.1566907 0.1142752 0.2125303 0.1861072 0.4522467
## [8] 0.1111101
##
## $`matrix of error`
##                    人均粮食支出  人均副食支出 人均烟酒茶支出 人均其他副食支出
## 人均粮食支出       2.220446e-16 -4.362930e-02  -1.376274e-01       0.06355151
## 人均副食支出      -4.362930e-02  2.220446e-16   8.701582e-02       0.03690333
```

```
## 人均烟酒茶支出    -1.376274e-01  8.701582e-02   2.220446e-16       -0.04614890
## 人均其他副食支出   6.355151e-02  3.690333e-02  -4.614890e-02        0.00000000
## 人均衣着商品支出   5.318951e-02 -1.431058e-01  -1.240830e-01       -0.05258908
## 人均日用品支出    -1.226234e-02 -7.845916e-02  -1.328843e-02       -0.09413672
## 人均燃料支出      -2.281068e-01 -1.007135e-01   2.020991e-02       -0.08300271
## 人均非商品支出     4.849274e-02 -9.437202e-03   1.300135e-02        0.01753109
##                  人均衣着商品支出 人均日用品支出 人均燃料支出 人均非商品支出
## 人均粮食支出           0.053189514  -1.226234e-02  -0.22810678   4.849274e-02
## 人均副食支出          -0.143105767  -7.845916e-02  -0.10071350  -9.437202e-03
## 人均烟酒茶支出        -0.124083023  -1.328843e-02   0.02020991   1.300135e-02
## 人均其他副食支出      -0.052589082  -9.413672e-02  -0.08300271   1.753109e-02
## 人均衣着商品支出       0.000000000   5.714805e-02   0.13819417  -8.825343e-03
## 人均日用品支出         0.057148052   2.220446e-16   0.05444033  -8.387502e-02
## 人均燃料支出           0.138194175   5.444033e-02   0.00000000  -8.354606e-02
## 人均非商品支出        -0.008825343  -8.387502e-02  -0.08354606   2.220446e-16
```

从残差矩阵可以看出因子分析的效果较好

2.2.3 主成分迭代法

2.2.3.1 取两个因子

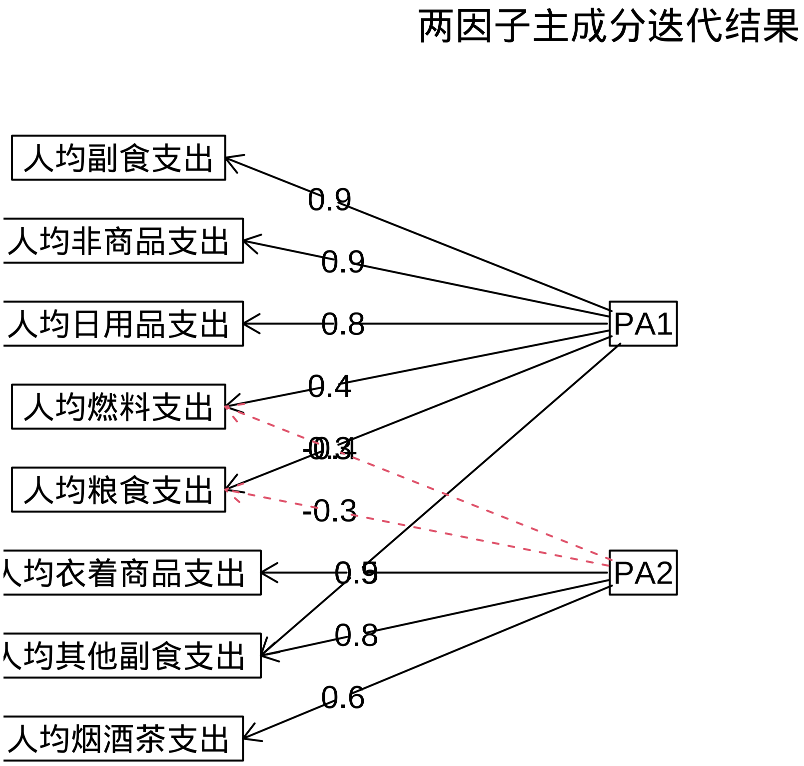
```
pa<-fa(data,nfactors=2,rotate="none",fm="pa")
pa
```

```
## Factor Analysis using method =  pa
## Call: fa(r = data, nfactors = 2, rotate = "none", fm = "pa")
## Standardized loadings (pattern matrix) based upon correlation matrix
##                   PA1   PA2   h2    u2 com
## 人均粮食支出      0.31 -0.31 0.19 0.807 2.0
## 人均副食支出      0.90 -0.17 0.84 0.156 1.1
## 人均烟酒茶支出    0.01  0.59 0.34 0.655 1.0
## 人均其他副食支出 0.52  0.80 0.91 0.091 1.7
## 人均衣着商品支出 0.10  0.85 0.73 0.265 1.0
## 人均日用品支出    0.83  0.11 0.70 0.296 1.0
## 人均燃料支出      0.42 -0.36 0.31 0.690 2.0
## 人均非商品支出    0.87 -0.23 0.81 0.190 1.1
```

```
##
##                         PA1  PA2
## SS loadings            2.83 2.02
## Proportion Var         0.35 0.25
## Cumulative Var         0.35 0.61
## Proportion Explained   0.58 0.42
## Cumulative Proportion 0.58 1.00
##
## Mean item complexity =  1.4
## Test of the hypothesis that 2 factors are sufficient.
##
## The degrees of freedom for the null model are  28  and the objective function was  5.61 with Ch
## The degrees of freedom for the model are 13  and the objective function was  1.35
##
## The root mean square of the residuals (RMSR) is  0.07
## The df corrected root mean square of the residuals is  0.1
##
## The harmonic number of observations is  30 with the empirical chi square  7.44  with prob <  0.
## The total number of observations was  30  with Likelihood Chi Square =  32.6  with prob <  0.00
##
## Tucker Lewis Index of factoring reliability =  0.607
## RMSEA index =  0.222  and the 90 % confidence intervals are  0.131 0.327
## BIC =  -11.61
## Fit based upon off diagonal values = 0.97
## Measures of factor score adequacy
##                                                    PA1  PA2
## Correlation of (regression) scores with factors   0.96 0.98
## Multiple R square of scores with factors          0.93 0.95
## Minimum correlation of possible factor scores     0.86 0.90
```

因子分析图如下所示

```
fa.diagram(pa,simple=F,main='两因子主成分迭代结果')
```



2.2.3.2 取三个因子

```
pa<-fa(data,nfactors=3,rotate="none",fm="pa")
```

```
## maximum iteration exceeded
```

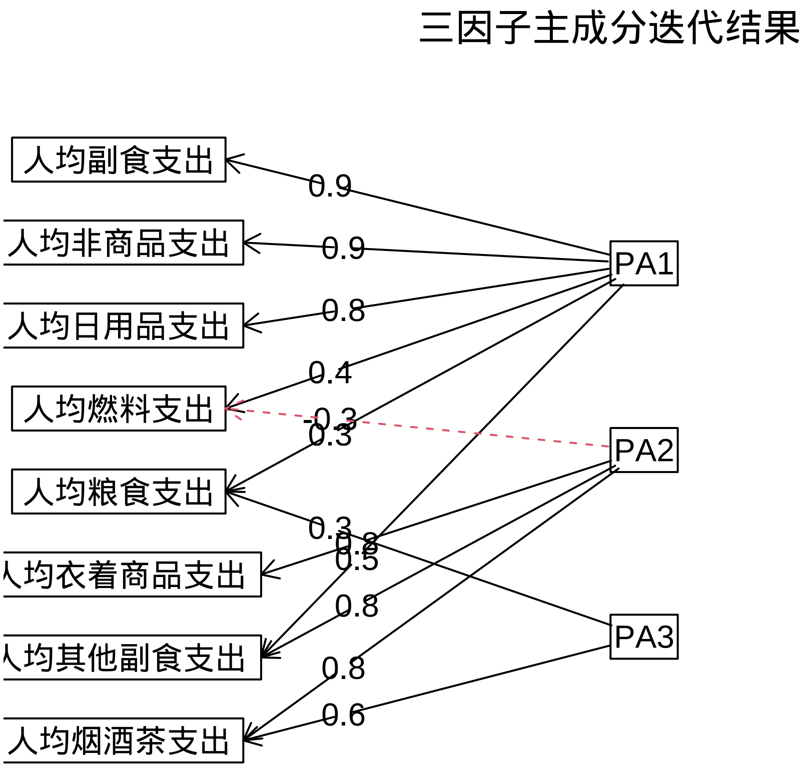
```
pa
```

```
## Factor Analysis using method =  pa
## Call: fa(r = data, nfactors = 3, rotate = "none", fm = "pa")
## Standardized loadings (pattern matrix) based upon correlation matrix
##                  PA1   PA2   PA3   h2    u2 com
## 人均粮食支出      0.33 -0.29  0.30 0.28 0.715 3.0
## 人均副食支出      0.90 -0.14  0.12 0.85 0.151 1.1
## 人均烟酒茶支出    0.01  0.77  0.56 0.91 0.092 1.8
## 人均其他副食支出 0.50  0.77 -0.11 0.87 0.132 1.8
```

```
## 人均衣着商品支出 0.09  0.82 -0.24 0.74 0.264 1.2
## 人均日用品支出   0.83  0.11 -0.09 0.71 0.293 1.1
## 人均燃料支出     0.43 -0.34  0.20 0.34 0.660 2.3
## 人均非商品支出   0.89 -0.22 -0.16 0.86 0.139 1.2
##
##                        PA1  PA2  PA3
## SS loadings           2.85 2.14 0.57
## Proportion Var        0.36 0.27 0.07
## Cumulative Var        0.36 0.62 0.69
## Proportion Explained  0.51 0.39 0.10
## Cumulative Proportion 0.51 0.90 1.00
##
## Mean item complexity =  1.7
## Test of the hypothesis that 3 factors are sufficient.
##
## The degrees of freedom for the null model are  28  and the objective function was  5.61 with Ch
## The degrees of freedom for the model are 7  and the objective function was  0.96
##
## The root mean square of the residuals (RMSR) is  0.05
## The df corrected root mean square of the residuals is  0.1
##
## The harmonic number of observations is  30 with the empirical chi square  3.87  with prob <  0.
## The total number of observations was  30  with Likelihood Chi Square =  22.61  with prob <  0.0
##
## Tucker Lewis Index of factoring reliability =  0.398
## RMSEA index =  0.271  and the 90 % confidence intervals are  0.155 0.409
## BIC =  -1.2
## Fit based upon off diagonal values = 0.99
## Measures of factor score adequacy
##                                                    PA1  PA2  PA3
## Correlation of (regression) scores with factors   0.97 0.97 0.88
## Multiple R square of scores with factors          0.94 0.94 0.78
## Minimum correlation of possible factor scores     0.87 0.87 0.56
```

因子分析图如下所示

```r
fa.diagram(pa,simple=F,main='三因子主成分迭代结果')
```



2.2.4 极大似然估计法

2.2.4.1 取两个因子

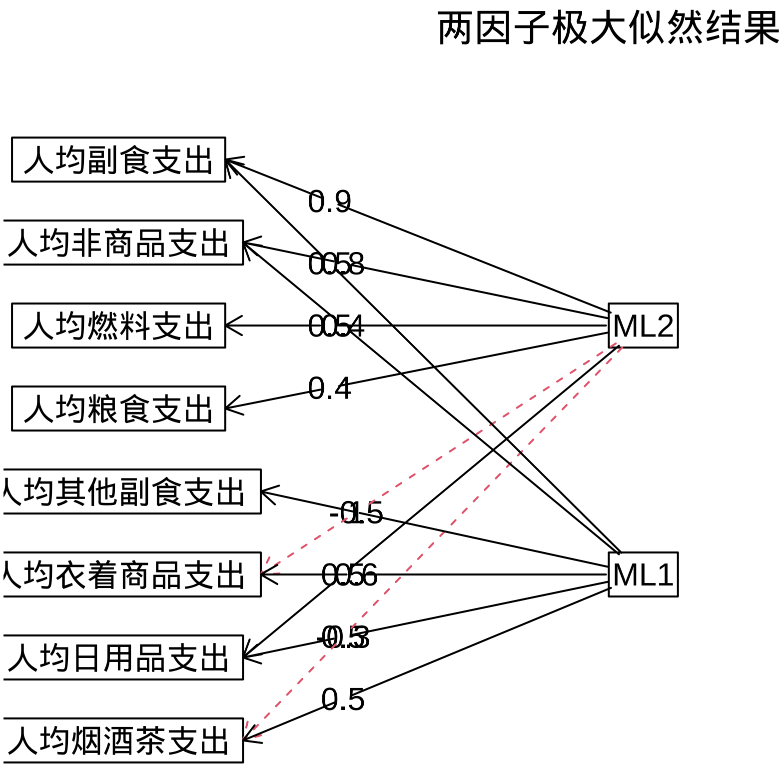
```r
ml<-fa(data,nfactors=2,rotate="none",fm="ml")
ml
```

```
## Factor Analysis using method =  ml
## Call: fa(r = data, nfactors = 2, rotate = "none", fm = "ml")
## Standardized loadings (pattern matrix) based upon correlation matrix
##                      ML2   ML1   h2    u2 com
## 人均粮食支出        0.41 -0.02 0.16 0.835 1.0
## 人均副食支出        0.86  0.49 0.98 0.021 1.6
## 人均烟酒茶支出     -0.33  0.50 0.36 0.639 1.7
```

```
## 人均其他副食支出 -0.10  0.99 1.00 0.005 1.0
## 人均衣着商品支出 -0.54  0.65 0.71 0.291 1.9
## 人均日用品支出    0.53  0.53 0.56 0.442 2.0
## 人均燃料支出      0.55 -0.11 0.31 0.687 1.1
## 人均非商品支出    0.75  0.39 0.72 0.281 1.5
##
##                        ML2  ML1
## SS loadings           2.45 2.35
## Proportion Var        0.31 0.29
## Cumulative Var        0.31 0.60
## Proportion Explained  0.51 0.49
## Cumulative Proportion 0.51 1.00
##
## Mean item complexity =  1.5
## Test of the hypothesis that 2 factors are sufficient.
##
## The degrees of freedom for the null model are  28  and the objective function was  5.61 with Ch
## The degrees of freedom for the model are 13  and the objective function was  1.23
##
## The root mean square of the residuals (RMSR) is  0.09
## The df corrected root mean square of the residuals is  0.13
##
## The harmonic number of observations is  30 with the empirical chi square  12.51  with prob <  0
## The total number of observations was  30  with Likelihood Chi Square =  29.82  with prob <  0.0
##
## Tucker Lewis Index of factoring reliability =  0.663
## RMSEA index =  0.205  and the 90 % confidence intervals are  0.111 0.312
## BIC =  -14.4
## Fit based upon off diagonal values = 0.95
## Measures of factor score adequacy
##                                                    ML2  ML1
## Correlation of (regression) scores with factors   0.99 1.00
## Multiple R square of scores with factors          0.98 1.00
## Minimum correlation of possible factor scores     0.95 0.99
```

因子分析图如下所示

```
fa.diagram(ml,simple=F,main='两因子极大似然结果')
```



2.2.4.2 取三个因子

```
ml<-fa(data,nfactors=3,rotate="none",fm="pa")
```

```
## maximum iteration exceeded
```

```
ml
```

```
## Factor Analysis using method =  pa
## Call: fa(r = data, nfactors = 3, rotate = "none", fm = "pa")
## Standardized loadings (pattern matrix) based upon correlation matrix
##                    PA1   PA2   PA3   h2    u2 com
## 人均粮食支出      0.33 -0.29  0.30 0.28 0.715 3.0
## 人均副食支出      0.90 -0.14  0.12 0.85 0.151 1.1
## 人均烟酒茶支出    0.01  0.77  0.56 0.91 0.092 1.8
```

```
## 人均其他副食支出 0.50  0.77 -0.11 0.87 0.132 1.8
## 人均衣着商品支出 0.09  0.82 -0.24 0.74 0.264 1.2
## 人均日用品支出   0.83  0.11 -0.09 0.71 0.293 1.1
## 人均燃料支出     0.43 -0.34  0.20 0.34 0.660 2.3
## 人均非商品支出   0.89 -0.22 -0.16 0.86 0.139 1.2
##
##                        PA1  PA2  PA3
## SS loadings           2.85 2.14 0.57
## Proportion Var        0.36 0.27 0.07
## Cumulative Var        0.36 0.62 0.69
## Proportion Explained  0.51 0.39 0.10
## Cumulative Proportion 0.51 0.90 1.00
##
## Mean item complexity =  1.7
## Test of the hypothesis that 3 factors are sufficient.
##
## The degrees of freedom for the null model are  28  and the objective function was  5.61 with Ch
## The degrees of freedom for the model are 7  and the objective function was  0.96
##
## The root mean square of the residuals (RMSR) is  0.05
## The df corrected root mean square of the residuals is  0.1
##
## The harmonic number of observations is  30 with the empirical chi square  3.87  with prob <  0.
## The total number of observations was  30  with Likelihood Chi Square =  22.61  with prob <  0.0
##
## Tucker Lewis Index of factoring reliability =  0.398
## RMSEA index =  0.271  and the 90 % confidence intervals are  0.155 0.409
## BIC =  -1.2
## Fit based upon off diagonal values = 0.99
## Measures of factor score adequacy
##                                                    PA1  PA2  PA3
## Correlation of (regression) scores with factors   0.97 0.97 0.88
## Multiple R square of scores with factors          0.94 0.94 0.78
## Minimum correlation of possible factor scores     0.87 0.87 0.56
```

因子分析图如下所示

```r
fa.diagram(pa,simple=F,main='三因子极大似然结果')
```