MATH 350: Honours Discrete Mathematics. Fall 2024.
Due by 4:05 PM, Tuesday, October 8th on myCourses.

Assignment #3: Spanning trees and bipartite graphs.

1. We say that $F \subseteq E(G)$ is *even-degree* if every vertex of G is incident with an even number of edges in F. Show that if T is a spanning tree of G, there is an even-degree set $F \subseteq E(G)$ with $F \cup E(T) = E(G)$. (*Hint*: First, show that if F_1 and F_2 are both even-degree then so is $F_1 \triangle F_2 := (F_1 - F_2) \cup (F_2 - F_1)$.)

2.

a) Let e be an edge of the complete graph K_n with $n \geq 2$. Show that K_n has exactly $2n^{n-3}$ spanning trees containing e.

b) Let G_n be a simple graph obtained from the complete graph K_n by adding one extra vertex adjacent to exactly two vertices of K_n. Find the number of spanning trees of G_n.

3. Prove that every graph G contains a bipartite subgraph H such that $V(H) = V(G)$, and $\deg_H(v) \geq \deg_G(v)/2$ for every $v \in V(H)$.