24Fall MGCR372-001

Operation Management HW2

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September 23, 2024

Problem 1.

(a) Since the service time of each customer is exponentially distributed, the cumulative distribution function is given by:

$$F(x, \lambda) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Each cashier can serve 10 customers per hour in average, then $\frac{1}{\lambda} = 6 \Rightarrow \lambda = \frac{1}{6}$.

$$F(x = 19, \lambda = \frac{1}{6}) = 1 - e^{-\frac{19}{6}} \approx 0.958$$

The probability that a customer's service time is less than 19 minutes is 0.958.

(b) From the description, we have:

$$\mu = \frac{1}{6} \; customers \; per \; minute,$$

$$\lambda = \frac{5}{3} \; customers \; per \; minute,$$

$$\frac{\lambda}{\mu} = 10,$$

$$s = 12$$

Use **M/M/s** spreadsheet, $L_Q = \mathbf{2.2465}$

(c) The average service time is given by:

$$W_{SE} = \frac{1}{\mu} = 6 \; minutes$$

(d) By Little's Law, we have the following equations:

$$\frac{1}{\mu} = W_S - W_Q \tag{1}$$

$$L_S = \lambda \times W_S \tag{2}$$

$$L_Q = \lambda \times W_Q \tag{3}$$

$$\Rightarrow \begin{array}{rl} W_S = & 7.3479 \; minutes \\ L_S = & \lambda \times W_S = 12.2465 \end{array}$$

Therefore, the average total people waiting in the queue is **12.2465**.

(e) There are 12 or more customers in the checkout area means new customer must wait.

Use M/M/s spreadsheet, the probability being asked is given by:

$$P(delay) = 0.4494$$

(f) Consider one line:

$$\lambda = \frac{1}{12} \times \frac{5}{3} = \frac{5}{36} \; customers \; per \; minute$$

$$\mu = \frac{1}{6} \; customers \; per \; minute$$

The average number of customers waiting to be checked in one line is:

$$L_Q = \frac{\lambda}{\mu} \times \frac{\lambda}{\mu - \lambda} = \frac{25}{6} \; customers$$

Therefore the average number of customers waiting in 12 lines is **50**.