MATH 350: Honours Discrete Mathematics. Fall 2024.
Due by 4:05 PM, Thursday, September 26th on myCourses.

Assignment #2: Trees.

1. Let $d_1, d_2, \ldots, d_n$ be positive integers with $n \geq 2$. Prove that there exists a tree with vertex degrees $d_1, d_2, \ldots, d_n$ if and only if

$$\sum_{i=1}^{n} d_i = 2n - 2.$$

2. Let G be a non-null graph such that for every pair of vertices $u, v \in V(G)$ there exists a path in G from u to v of length at most k. Show that either G contains a cycle of length at most $2k + 1$ or G is a tree.

3. Let T be a tree, and let $T_1, \ldots, T_n$ be connected subgraphs of T so that $V(T_i \cap T_j) \neq \emptyset$ for all i, j with $1 \leq i < j \leq n$. Show that

$$V(T_1 \cap T_2 \cap \ldots \cap T_n) \neq \emptyset.$$

[*Hint*: Delete a leaf and use induction on $|V(T)|$.]