CS 325 Spring 2018 – HW 6

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Problem 1:

a) The distance of the shortest path from G to C is 16.

The code is:

```
max dc
ST
        dg = 0
        da - dh <= 4
        da - df <= 5
        db - dh <= 9
        db - da <= 8
        db - df <= 7
        dc - df <= 3
        dc - db <= 4
        dd - dc <= 3
        dd - dg <= 2
        dd - de <= 9
        de - db <= 10
        de - dd <= 25
        de - df <= 2
        df - da <= 10
        df - dd <= 18
        dg - de <= 7
        dh - dg <= 3
END
```

The output is:

```
LP OPTIMUM FOUND AT STEP      6

       OBJECTIVE FUNCTION VALUE

       1)      16.00000

  VARIABLE        VALUE          REDUCED COST
        DC        16.000000          0.000000
        DG         0.000000          0.000000
        DA         4.000000          0.000000
        DH         3.000000          0.000000
        DF        13.000000          0.000000
        DB        12.000000          0.000000
        DD         0.000000          0.000000
        DE         0.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          1.000000
        3)         3.000000          0.000000
        4)        14.000000          0.000000
        5)         0.000000          1.000000
        6)         0.000000          0.000000
        7)         8.000000          0.000000
        8)         0.000000          0.000000
        9)         0.000000          1.000000
       10)        19.000000          0.000000
       11)         2.000000          0.000000
       12)         9.000000          0.000000
       13)        22.000000          0.000000
       14)        25.000000          0.000000
       15)        15.000000          0.000000
       16)         1.000000          0.000000
       17)         5.000000          0.000000
       18)         7.000000          0.000000
       19)         0.000000          1.000000

NO. ITERATIONS=       6
```

b)

The distance of shortest paths from G to A is 7.

The distance of shortest paths from G to B is 12.

The distance of shortest paths from G to D is 2.

The distance of shortest paths from G to E is 19.

The distance of shortest paths from G to F is 17.

The distance of shortest paths from G to H is 3.

The code is:

```
max da + db + dd + de + df + dh
ST
        dg = 0
        da - dh <= 4
        da - df <= 5
        db - dh <= 9
        db - da <= 8
        db - df <= 7
        dc - df <= 3
        dc - db <= 4
        dd - dc <= 3
        dd - dg <= 2
        dd - de <= 9
        de - db <= 10
        de - dd <= 25
        de - df <= 2
        df - da <= 10
        df - dd <= 18
        dg - de <= 7
        dh - dg <= 3
END
```

The output is:

```
 LP OPTIMUM FOUND AT STEP      0

        OBJECTIVE FUNCTION VALUE

        1)      60.00000

  VARIABLE        VALUE          REDUCED COST
        DA         7.000000          0.000000
        DB        12.000000          0.000000
        DD         2.000000          0.000000
        DE        19.000000          0.000000
        DF        17.000000          0.000000
        DH         3.000000          0.000000
        DG         0.000000          0.000000
        DC        16.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          6.000000
        3)         0.000000          3.000000
        4)        15.000000          0.000000
        5)         0.000000          1.000000
        6)         3.000000          0.000000
        7)        12.000000          0.000000
        8)         4.000000          0.000000
        9)         0.000000          0.000000
       10)        17.000000          0.000000
       11)         0.000000          1.000000
       12)        26.000000          0.000000
       13)         3.000000          0.000000
       14)         8.000000          0.000000
       15)         0.000000          1.000000
       16)         0.000000          2.000000
       17)         3.000000          0.000000
       18)        26.000000          0.000000
       19)         0.000000          5.000000

 NO. ITERATIONS=       0
```

Problem 2:

Profit of Silk tie: $6.7 – 0.125 * $20 - $0.75 = $3.45

Profit of Poly tie: $3.55 – 0.08 * $6 - $0.75 = $2.32

Profit of Blend1 tie: $4.31 – 0.05 * $6 – 0.05 * $9 – $0.75 = $2.81

Profit of Blend2 tie: $4.81 – 0.03* $6 – 0.07 * $9 – $0.75 = $3.25

Objective function: Maximize Z = 3.45 * s + 2.32 * p + 2.81 * b + 3.25 * c

Subject to:

0.125 * s <= 1000

0.08 * p + 0.05 * b + 0.03 * c <= 2000

0.05 * b + 0.07 * c <= 1250

s >= 6000

s <= 7000

p >= 10000

p <= 14000

b >= 13000

b <= 16000

c >= 6000

c <= 8500

The code is:

```
max 3.45s + 2.32p + 2.81b + 3.25c
ST
        0.125s <= 1000
        0.08p + 0.05b + 0.03c <= 2000
        0.05b + 0.07c <= 1250
        s >= 6000
        s <= 7000
        p >= 10000
        p <= 14000
        b >= 13000
        b <= 16000
        c >= 6000
        c <= 8500
END
```

The output is:

```
 LP OPTIMUM FOUND AT STEP      4

        OBJECTIVE FUNCTION VALUE

        1)      120196.0

  VARIABLE        VALUE          REDUCED COST
         S      7000.000000          0.000000
         P     13625.000000          0.000000
         B     13100.000000          0.000000
         C      8500.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)       125.000000          0.000000
        3)         0.000000         29.000000
        4)         0.000000         27.200001
        5)      1000.000000          0.000000
        6)         0.000000          3.450000
        7)      3625.000000          0.000000
        8)       375.000000          0.000000
        9)       100.000000          0.000000
       10)      2900.000000          0.000000
       11)      2500.000000          0.000000
       12)         0.000000          0.476000

 NO. ITERATIONS=       4
```

Therefore, the maximum profit is $120,196. In order to reach maximum profit, we should make 7000 Silk ties per month, 13,625 Poly ties per month, 13,100 Blend1 ties per month, and 8,500 Blend2 ties per month.

Problem 3:

Part A:

Objective function: Minimize 10cp11 +15cp12 + 11cp21 + 8cp22 + 13cp31 + 8cp32 + 9cp33 + 14cp42 + 8cp43 + 5cw11 + 6cw12 + 7cw13 + 10cw14 + 12cw23 + 8cw24 + 10cw25 + 14cw26 + 14cw34 + 12cw35 + 12cw36 + 6cw37

Subject to:

cp11 + cp12 <= 150

cp21 + cp22 <= 450

cp31 + cp32 + cp33 <=250

cp42 + cp43 <=150

cw11 >= 100

cw12 >= 150

cw13 + cw23 >= 100

cw14 + cw24 + cw34 >= 200

cw25 + cw35 >= 200

cw26 + cw36 >= 150

cw37 >= 100

cp11 + cp21 + cp31 – cw11 – cw12 – cw13 – cw14 = 0

cp12 + cp22 + cp32 + cp42 – cw23 – cw24 – cw25 – cw26 = 0

cp33 + cp43 – cw34 – cw35 – cw36 – cw37 = 0

cp11, cp12, cp21, cp22, cp31, cp32, cp33, cp42, cp43, cw11, cw12, cw13, cw14, cw23, cw24, cw25, cw26, cw34, cw35, cw36, cw37 >= 0

The code is:

```
min 10cp11 + 15cp12 + 11cp21 + 8cp22 + 13cp31 + 8cp32 + 9cp33 +
14cp42 + 8cp43 + 5cw11 + 6cw12 + 7cw13 + 10cw14 + 12cw23 + 8cw24 +
10cw25 + 14cw26 + 14cw34 + 12cw35 + 12cw36 + 6cw37
ST
        cp11 + cp12 <= 150
        cp21 + cp22 <= 450
        cp31 + cp32 + cp33 <= 250
        cp42 + cp43 <= 150
        cw11 >= 100
        cw12 >= 150
        cw13 + cw23 >= 100
        cw14 + cw24 + cw34 >= 200
        cw25 + cw35 >= 200
        cw26 + cw36 >= 150
        cw37 >= 100
        cp11 + cp21 + cp31 - cw11 - cw12 - cw13 - cw14 = 0
        cp12 + cp22 + cp32 + cp42 - cw23 - cw24 - cw25 - cw26 = 0
        cp33 + cp43 - cw34 - cw35 - cw36 - cw37 = 0
        cp11 >= 0
        cp12 >= 0
        cp21 >= 0
        cp22 >= 0
        cp31 >= 0
        cp32 >= 0
        cp33 >= 0
        cp42 >= 0
        cp43 >= 0
        cw11 >= 0
        cw12 >= 0
        cw13 >= 0
        cw14 >= 0
        cw23 >= 0
        cw24 >= 0
        cw25 >= 0
        cw26 >= 0
        cw34 >= 0
        cw35 >= 0
        cw36 >= 0
        cw37 >= 0
END
```

The output is:

```
LP OPTIMUM FOUND AT STEP     13

       OBJECTIVE FUNCTION VALUE

       1)      17100.00

 VARIABLE        VALUE          REDUCED COST
     CP11       150.000000          0.000000
     CP12         0.000000          8.000000
     CP21       200.000000          0.000000
     CP22       250.000000          0.000000
     CP31         0.000000          2.000000
     CP32       150.000000          0.000000
     CP33       100.000000          0.000000
     CP42         0.000000          7.000000
     CP43       150.000000          0.000000
     CW11       100.000000          0.000000
     CW12       150.000000          0.000000
     CW13       100.000000          0.000000
     CW14         0.000000          5.000000
     CW23         0.000000          2.000000
     CW24       200.000000          0.000000
     CW25       200.000000          0.000000
     CW26         0.000000          1.000000
     CW34         0.000000          7.000000
     CW35         0.000000          3.000000
     CW36       150.000000          0.000000
     CW37       100.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          1.000000
       3)         0.000000          0.000000
       4)         0.000000          0.000000
       5)         0.000000          1.000000
       6)         0.000000        -16.000000
       7)         0.000000        -17.000000
       8)         0.000000        -18.000000
       9)         0.000000        -16.000000
      10)         0.000000        -18.000000
      11)         0.000000        -21.000000
      12)         0.000000        -15.000000
      13)         0.000000        -11.000000
      14)         0.000000         -8.000000
      15)         0.000000         -9.000000
      16)       150.000000          0.000000
      17)         0.000000          0.000000
      18)       200.000000          0.000000
      19)       250.000000          0.000000
      20)         0.000000          0.000000
      21)       150.000000          0.000000
      22)       100.000000          0.000000
      23)         0.000000          0.000000
      24)       150.000000          0.000000
      25)       100.000000          0.000000
      26)       150.000000          0.000000
      27)       100.000000          0.000000
      28)         0.000000          0.000000
      29)         0.000000          0.000000
      30)       200.000000          0.000000
      31)       200.000000          0.000000
      32)         0.000000          0.000000
      33)         0.000000          0.000000
      34)         0.000000          0.000000
      35)       150.000000          0.000000
      36)       100.000000          0.000000

 NO. ITERATIONS=      13
```

Therefore, the minimum cost is 17,100 and the optimal solution is as below.

P1 -> W1: 150   P2 -> W1: 200

P2 -> W2: 250   P3 -> W2: 150

P3 -> W3: 100   P4 -> W3: 150

W1 -> R1: 100   W1 -> R2: 150   W1 -> R3: 100

W2 -> R4: 200   W2 -> R5: 200

W3 -> R6: 150   W3 -> R7: 100

Part B:

It's not feasible.

The code is:

```
min 10cp11 + 11cp21 + 13cp31 + 9cp33 + 8cp43 + 5cw11 + 6cw12 + 7cw13 +
10cw14 + 14cw34 + 12cw35 + 12cw36 + 6cw37
ST
        cp11 <= 150
        cp21 <= 450
        cp31 + cp33 <= 250
        cp43 <= 150
        cw11 >= 100
        cw12 >= 150
        cw13 >= 100
        cw14 + cw34 >= 200
        cw35 >= 200
        cw36 >= 150
        cw37 >= 100
        cp11 + cp21 + cp31 - cw11 - cw12 - cw13 - cw14 = 0
        cp33 + cp43 - cw34 - cw35 - cw36 - cw37 = 0
        cp11 >= 0
        cp21 >= 0
        cp31 >= 0
        cp33 >= 0
        cp43 >= 0
        cw11 >= 0
        cw12 >= 0
        cw13 >= 0
        cw14 >= 0
        cw34 >= 0
        cw35 >= 0
        cw36 >= 0
        cw37 >= 0
END
```

The output is:

```
NO FEASIBLE SOLUTION AT STEP          10.
SUM OF INFEASIBILITIES=       50.0000000000000
VIOLATED ROWS HAVE  NEGATIVE SLACK,   OR
(EQUALITY ROWS) NONZERO  SLACKS.   ROWS
CONTRIBUTING TO  INFEASIBILITY  HAVE  A
NONZERO  DUAL  PRICE.   USE THE "DEBUG"
COMMAND FOR MORE INFORMATION.
```

Part C:

Yes, it's feasible. We need to add the constraint: cp12 + cp22 + cp32 + cp42 <= 100

The code is:

```
min 10cp11 + 15cp12 + 11cp21 + 8cp22 + 13cp31 + 8cp32 + 9cp33 +
14cp42 + 8cp43 + 5cw11 + 6cw12 + 7cw13 + 10cw14 + 12cw23 + 8cw24 +
10cw25 + 14cw26 + 14cw34 + 12cw35 + 12cw36 + 6cw37
ST
        cp11 + cp12 <= 150
        cp21 + cp22 <= 450
        cp31 + cp32 + cp33 <= 250
        cp42 + cp43 <= 150
        cw11 >= 100
        cw12 >= 150
        cw13 + cw23 >= 100
        cw14 + cw24 + cw34 >= 200
        cw25 + cw35 >= 200
        cw26 + cw36 >= 150
        cw37 >= 100
        cp11 + cp21 + cp31 - cw11 - cw12 - cw13 - cw14 = 0
        cp12 + cp22 + cp32 + cp42 - cw23 - cw24 - cw25 - cw26 = 0
        cp33 + cp43 - cw34 - cw35 - cw36 - cw37 = 0
        cp12 + cp22 + cp32 + cp42 <= 100
        cp11 >= 0
        cp21 >= 0
        cp31 >= 0
        cp33 >= 0
        cp43 >= 0
        cw11 >= 0
        cw12 >= 0
        cw13 >= 0
        cw14 >= 0
        cw34 >= 0
        cw35 >= 0
        cw36 >= 0
        cw37 >= 0
END
```

The output is:

```
 LP OPTIMUM FOUND AT STEP     15

        OBJECTIVE FUNCTION VALUE

        1)      18300.00

  VARIABLE        VALUE          REDUCED COST
      CP11       150.000000          0.000000
      CP12         0.000000          8.000000
      CP21       350.000000          0.000000
      CP22       100.000000          0.000000
      CP31         0.000000          4.000000
      CP32         0.000000          2.000000
      CP33       250.000000          0.000000
      CP42         0.000000          9.000000
      CP43       150.000000          0.000000
      CW11       100.000000          0.000000
      CW12       150.000000          0.000000
      CW13       100.000000          0.000000
      CW14       150.000000          0.000000
      CW23         0.000000          7.000000
      CW24        50.000000          0.000000
      CW25        50.000000          0.000000
      CW26         0.000000          4.000000
      CW34         0.000000          4.000000
      CW35       150.000000          0.000000
      CW36       150.000000          0.000000
      CW37       100.000000          0.000000
```

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	1.000000
3)	0.000000	0.000000
4)	0.000000	2.000000
5)	0.000000	3.000000
6)	0.000000	-16.000000
7)	0.000000	-17.000000
8)	0.000000	-18.000000
9)	0.000000	-21.000000
10)	0.000000	-23.000000
11)	0.000000	-23.000000
12)	0.000000	-17.000000
13)	0.000000	-11.000000
14)	0.000000	-13.000000
15)	0.000000	-11.000000
16)	0.000000	5.000000
17)	150.000000	0.000000
18)	350.000000	0.000000
19)	0.000000	0.000000
20)	250.000000	0.000000
21)	150.000000	0.000000
22)	100.000000	0.000000
23)	150.000000	0.000000
24)	100.000000	0.000000
25)	150.000000	0.000000
26)	0.000000	0.000000
27)	150.000000	0.000000
28)	150.000000	0.000000
29)	100.000000	0.000000

NO. ITERATIONS= 15

Therefore, the minimum cost is 18,300 and the optimal solution is as below.

P1 -> W1: 150   P2 -> W1: 350

P2 -> W2: 100

P3 -> W3: 250   P4 -> W3: 150

W1 -> R1: 100   W1 -> R2: 150   W1 -> R3: 100   W1 -> R4: 150

W2 -> R4: 50   W2 -> R5: 50

W3 -> R5: 150   W3 -> R6: 150   W3 -> R7: 100

Problem 4:

a)

Let n1 be the number of coin 1, n2 be the number of coin 5, n3 be the number of coin 10, n4 be the number of coin 25

Objective function: Minimize n1 + n2 + n3 + n4

Subject to:

n1 + 5n2 + 10n3 + 25n4 = 202

n1, n2, n3, n4 >= 0

The code is:

```
min n1 + n2 + n3 + n4
ST
        n1 + 5n2 + 10n3 + 25n4 = 202
        n1 >= 0
        n2 >= 0
        n3 >= 0
        n4 >= 0
END
GIN n1
GIN n2
GIN n3
GIN n4
```

The output is:

```
        OBJECTIVE FUNCTION VALUE

        1)      10.00000

  VARIABLE        VALUE          REDUCED COST
        N1         2.000000          1.000000
        N2         0.000000          1.000000
        N3         0.000000          1.000000
        N4         8.000000          1.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          0.000000
        3)         2.000000          0.000000
        4)         0.000000          0.000000
        5)         0.000000          0.000000
        6)         8.000000          0.000000

 NO. ITERATIONS=       31
 BRANCHES=    6 DETERM.=  1.000E    0
```

Therefore, the minimum number of coins is 10. The number of denomination 1 is 2 and the number of denomination 25 is 8.

b)

Let n1 be the number of coin 1, n2 be the number of coin 3, n3 be the number of coin 7, n4 be the number of coin 12, n5 be the number of coin 27.

Objective function: Minimize n1 + n2 + n3 + n4 + n5

Subject to:

n1 + 3n2 + 7n3 + 12n4 + 27n5 = 293

n1, n2, n3, n4, n5 >= 0

The code is:

```
min n1 + n2 + n3 + n4 + n5
ST
        n1 + 3n2 + 7n3 + 12n4 + 27n5 = 293
        n1 >= 0
        n2 >= 0
        n3 >= 0
        n4 >= 0
        n5 >= 0
END
GIN n1
GIN n2
GIN n3
GIN n4
GIN n5
```

The output is:

```
        OBJECTIVE FUNCTION VALUE

        1)      14.00000

  VARIABLE        VALUE          REDUCED COST
        N1         0.000000          1.000000
        N2         0.000000          1.000000
        N3         2.000000          1.000000
        N4         3.000000          1.000000
        N5         9.000000          1.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          0.000000
        3)         0.000000          0.000000
        4)         0.000000          0.000000
        5)         2.000000          0.000000
        6)         3.000000          0.000000
        7)         9.000000          0.000000

 NO. ITERATIONS=      98
 BRANCHES=   34 DETERM.=   1.000E    0
```

Therefore, the minimum number of coins is 14. The number of denomination 7 is 2; the number of denomination 12 is 3; the number of denomination 27 is 9.