Session Two:

Factor Investing and Smart Beta

CU SEAS IEOR 4723:

Financial Eng. for ESG Finance

Organizational Matters

HW

Homework:

- *Please submit within two weeks of assignment given*
- *Please email your files to me at cs4143@columbia.edu*

Class Finale:

- *Vote today on class project presentations vs final exam*

2.1 *Index Construction Basics*

Definition of an Index

Consider a portfolio of (up to) *N* securities. Every period *t* (for instance, every day) w_{it} portion of the portfolio is invested in security $i=1,\ldots,N$. These portions add up to 100%:

$$\sum_{i=1}^{N} w_{it} = 1.$$

During each period, security *i* earns return r_{it}. Then, at the end of the period, the total portfolio value is

$$I_t = I_{t-1} \sum_{i=1}^{N} w_{it} \,(1 + \, r_{it})$$

We define our "index" as the value of this portfolio I_t.
We end up with different indexes if we choose to interpret "return" r_{it} differently: differentiate between:

- "price return"

$$r_{it} = \frac{P_{it}}{P_{i,t-1}} - 1$$

- "total return" index, where the numerator includes distributions such as cash dividends

Notes:

- *Omitting here a discussion of index adjustments for stock splits, stock dividends, etc. These are straightforward; commercial indexes' methodology documentation usually explains in detail how such adjustments are handled by any particular index:*

  https://www.spglobal.com/spdji/en/documents/methodologies/methodology-sp-equity-indices-policies-practices.pdf

- *Here w_{it} is the portfolio weight of security i during period t. It is more common in literature to use w_{it} to denote the weight held in period i+1 (but determined on previous day's close, in period i)*

Basic Index Weight Choices

Numerous choices exist for how w_{it} can be determined. Richness of these choices leads to the richness of the indexes' universe, even for the same underlying portfolio of securities

Simplest and most common traditional choices:

- **Equal-weighted Index.** All weights are the same:

$$w_{it} = 1/N.$$

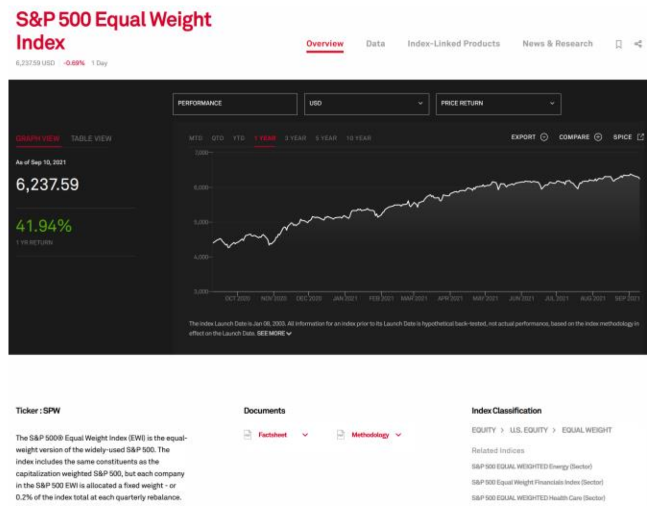
  Many popular equity indexes publish equal-weighted versions. *Example: SPW Index*

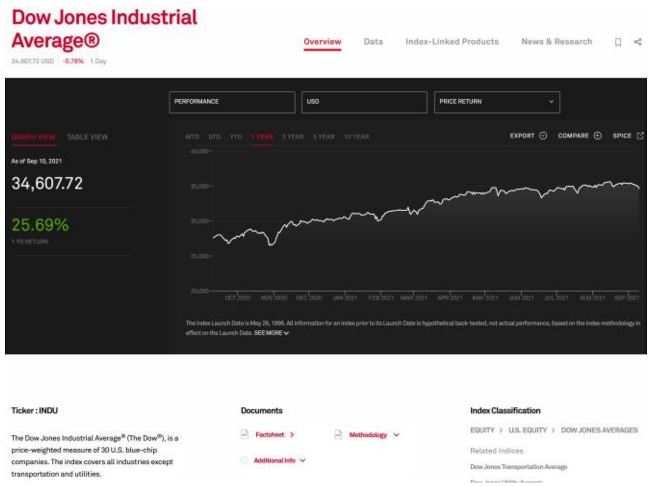
- **Market Cap-weighted Index.** Weights are chosen proportional to each stock's market capitalization (in practice, float). Likely the most popular weighting scheme out there. *Examples: S&P 500, NASDAQ, Eurostoxx 50, etc.*

- **Price-weighted Index.** Weights are proportional to share price:

$$w_{it} = \frac{P_{i,t-1}}{\sum_{j=1}^{N} P_{j,t-1}}$$

(recall that prices at the end of period t P_{it} are not known yet when w_{it} are being determined, which is **before** period i). *Examples: DJIA*





Definition of Alpha and Beta

Alpha and beta refer to the coefficients in

$$r_{it} = \alpha + \beta r_{Mt} + \varepsilon_t \qquad \text{(SML)}$$

where r_{Mt} refers to a broad "market portfolio" return and ε_t to an unbiased idiosyncratic term. This definition expands on (and is inspired by) Capital Asset Pricing Model (CAPM)

- Beta represents the contribution of the broad market to the return of a specific security in question
- Alpha represents excess return specific to the security

(Alternative specifications also common, where risk-free rate is subtracted from both r_{it} and r_{Mt} before estimating the linear relationship above.)

Subsequent studies including Fama&French have demonstrated that broad market r_{Mt} is not the only source of returns that isn't idiosyncratic to the security. To account for those, beta terms for other identifiable risk factors are added to the r_{it} equation above. Subsequent slides discuss the introduction of these "factors"

2.2 *Smart Beta Indexes*

Smart Beta Indexes

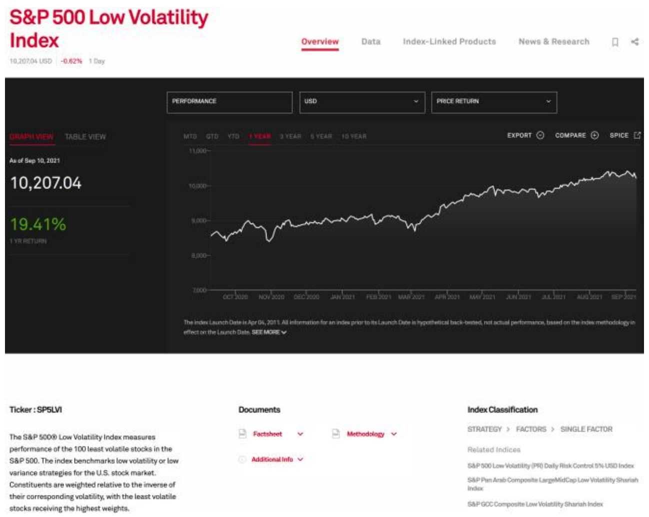
Traditionally index construction focused on the selection criteria for the appropriate universe, e.g. "N largest by market cap stocks traded in the U.S. market" or "N most liquid stocks", etc. (In practice, inclusion/exclusion determined by a committee.) Once the universe of underlying stocks has been determined, one of the simpler traditional schemes apply

As an alternative to traditional choices, other systematic weighting schemes (of varying complexity) have been devised over time . We will refer to indexes that rely on systematic weight rebalancing as "smart beta" indexes

Low-volatility Indexes. A common category of systematically rebalanced indexes (that we refer to here as "smart beta") are "low-volatility" indexes. Probably the simplest version of a low-volatility index is an index that assigns weights that are inversely proportional to constituent stocks' volatilities:

$$w_{it} = 1/\sigma_{it}.$$

A commercially available index is the *S&P Low Volatility Index*



A more sophisticated weighting scheme that furthers the same concept is "Equal Risk Contribution" (ERC), where weights are chosen so that contribution of each member to the portfolio volatility is the same

Other volatility-minimizing approaches to index construction are possible

[Exercise A: Constructing a Low-Volatility Index]

Optimization-Driven Indexes: Global Minimum Variance

Global Minimum Variance index represents a portfolio with weights derived via an optimization problem

$$w = argmin\ \frac{1}{2} \sum_{i,j=1}^{N} \Sigma_{ij} w_i w_j \quad s.t. \sum_{i=1}^{N} w_i = 1.$$

It can be shown that the optimal solution to the above is a vector weights proportional to the sum of rows of Σ^{-1}:

$$const\ *\ \Sigma^{-1}\ \mathbf{1}_N$$

It is common to introduce additional constraints on the weights in the optimization:

- make the portfolio long-only (require non-negative weights)
- require a minimum Herfindahl index level
- require minimum representation constraints by sector: $\kappa_{j,-} \leq \sum_{i \in \mathrm{K}_j} w_i \leq \kappa_{j,+}$

Herfindahl (or Herfindahl-Hirschman) index is a popular measure of concentration defined as

$$HHI = \sum_{i=1}^{N} w_i^2$$

Its inverse may be interpreted as the "effective number of members" in the index

Optimization-Driven Indexes: Most-Diversified Portfolio and Other

Most Diversified Portfolio is a noteworthy systematic allocation approach:

$$w = argmax \ln D(\circ) \qquad s.t. \sum_{i=1}^{N} w_i = 1,\ \ 0 \le w_i \le 1,$$

$$D(w) = \frac{\sum_{i=1}^{N} w_i \sigma_i}{\sqrt{\sum_{i,j=1}^{N} \Sigma_{ij} w_i w_j}}$$

This portfolio maximizes the "diversification ratio" *D(w)*: ratio between the weighted average volatility (numerator) and the total portfolio volatility (denominator). Introduced by Choueifaty and Coignard (2008)

Sharpe Ratio Maximization. Perhaps the most common and popular class of smart beta indexes determines weights by maximizing the portfolio's Sharpe ratio under a variety of constraints

These approaches differ greatly in the assumptions being made about the future expected return and future volatility, particularly volatility

It is common to leverage the factor investing framework (discussed further here) to arrive at the Sharpe ratio numerator estimates

[Exercise B: Brute-force Sharpe Ratio Optimization]

Comparison of Index Weights

Table: difference in weights derived following different systematic approaches
Underlying member universe for each: member stocks of Eurostoxx 50 as of Jan 2010

Columns:

- **CW**: capitalization-weighted
- **MV**: Minimum Variance
- **ERC**: Equal Risk Contribution
- **MDP**: Most Diversified Portfolio
- **MV or MDP x%**: MV or MDP with a lower-bound constraint of x% imposed on weights

Table 7: Composition in % (January 2010)

	CW	MV	ERC	MDP	1/n	MV 10%	MDP 10%	MV 5%	MDP 5%
TOTAL	6.1		2.1		2			5.0	
BANCO SANTANDER	5.8		1.3		2				
TELEFONICA SA	5.0	31.2	3.5		2	10.0		5.0	5.0
SANOFI-AVENTIS	3.6	12.1	4.5	15.5	2	10.0	10.0	5.0	5.0
E.ON AG	3.6		2.1		2				1.4
BNP PARIBAS	3.4		1.1		2				
SIEMENS AG	3.2		1.5		2				
BBVA(BILB-VIZ-ARG)	2.9		1.4		2				
BAYER AG	2.9		2.6	3.7	2	2.2	5.0	5.0	5.0
ENI	2.7		2.1		2				
GDF SUEZ	2.5		2.6	4.5	2		5.4	5.0	5.0
BASF SE	2.5		1.5		2				
ALLIANZ SE	2.4		1.4		2				
UNICREDIT SPA	2.3		1.1		2				
SOC GENERALE	2.2		1.2	3.9	2		3.7		5.0
UNILEVER NV	2.2	11.4	3.7	10.8	2	10.0	10.0	5.0	5.0
FRANCE TELECOM	2.1	14.9	4.1	10.2	2	10.0	10.0	5.0	5.0
NOKIA OYJ	2.1		1.8	4.5	2		4.8		5.0
DAIMLER AG	2.1		1.3		2				
DEUTSCHE BANK AG	1.9		1.0		2				
DEUTSCHE TELEKOM	1.9		3.2	2.6	2	5.7	3.7	5.0	5.0
INTESA SANPAOLO	1.9		1.3		2				
AXA	1.8		1.0		2				
ARCELORMITTAL	1.8		1.0		2				
SAP AG	1.8	21.0	3.4	11.2	2	10.0	10.0	5.0	5.0

	CW	MV	ERC	MDP	1/n	MV 10%	MDP 10%	MV 5%	MDP 5%
RWE AG (NEU)	1.7	2.7	2.7		2	7.0		5.0	
ING GROEP NV	1.6		0.8	0.4	2				
DANONE	1.6	1.9	3.4	1.8	2	8.7	3.3	5.0	5.0
IBERDROLA SA	1.6		2.5		2	5.1		5.0	1.2
ENEL	1.6		2.1		2			5.0	2.9
VIVENDI SA	1.6	2.8	3.1	4.5	2	10.0	5.9	5.0	5.0
ANHEUSER-BUSCH INB	1.6	0.2	2.7	10.9	2	2.1	10.0	5.0	5.0
ASSIC GENERALI SPA	1.6		1.8		2				
AIR LIQUIDE(L')	1.4		2.1		2			5.0	
MUENCHENER RUECKVE	1.3		2.1	2.1	2		3.1	5.0	5.0
SCHNEIDER ELECTRIC	1.3		1.5		2				
CARREFOUR	1.3	1.0	2.7	1.3	2	3.7	2.5	5.0	5.0
VINCI	1.3		1.6		2				
LVMH MOET HENNESSY	1.2		1.8		2				
PHILIPS ELEC(KON)	1.2		1.4		2				
L'OREAL	1.1	0.8	2.8		2	5.5		5.0	5.0
CIE DE ST-GOBAIN	1.0		1.1		2				
REPSOL YPF SA	0.9		2.0		2			5.0	
CRH	0.8		1.7	5.1	2		5.2		5.0
CREDIT AGRICOLE SA	0.8		1.1		2				
DEUTSCHE BOERSE AG	0.7		1.5		2				1.9
TELECOM ITALIA SPA	0.7		2.0		2				2.5
ALSTOM	0.6		1.5		2				
AEGON NV	0.4		0.7		2				
VOLKSWAGEN AG	0.2		1.8	7.1	2		7.4		5.0
Total of components	50	11	50	17	50	14	16	20	23

Source: "Risk-Based Indexation" by Demey, Maillard and Roncalli, 2010

Commercially Available Indexes

Many commercially available indexes (and exchange-traded funds – ETF – that track them) implement the ideas described in this section

The universe of smart beta products available in the market is constantly changing. This is a slightly dated (2012) snapshot; refer to Bloomberg (or a Google search) for updates…

EXHIBIT 2
An Overview of Stock Selection and Weighting Decisions of Some Alternative Equity Indices

Index	Stock Selection	Stock Weighting
Indices that change only the selection compared to standard index		
Broad Dividend Achievers[1]	Positive dividend growth	Market capitalization (or free float)
MSCI High Dividend Yield[2]	High dividend yield, positive dividend-per-share growth, low dividend payout ratio	
FTSE Active Beta Momentum and Value[3]	High price momentum, high book value-to-price ratio, high sales-to-price ratio, high cash flow-to-a price ratio	
Russell High Dividend Yield[4]	Positive free cash flow, positive return on equity, positive forecasted earnings growth, high price momentum, low debt-to-equity ratio, low EPS variability, high dividend yield, high dividend growth	
Russell Defensive[5]	Low leverage, high return on assets, low earnings variability, low total return volatility	
Indices that change only the weighting		
FTSE GWA[6]	Index universe remains the same as the parent market index	Net income, cash flow, book value
MSCI Value Weighted[7]		Sales, earnings, cash earnings, book value
MSCI Risk Weighted[8]		Inverse of historical variance
MSCI Minimum Volatility[9]		Volatility minimization
S&P 500 Equal-weighted[10]		Equal weighted
FTSE EDHEC-Risk Efficient[11]		Sharpe ratio maximization
FTSE TOBAM Max. Diversified[12]		Maximize diversification ratio
Lyxor SmartIX ERC[19]		Set risk contribution of constituents equal
Indices that change both selection and weighting		
Dow Jones Select Dividend[13]	Positive dividend growth, low dividend payout ratio	Dividend
FTSE RAFI[14]	High sales, high cash flow, high book value, high dividend	Sales, cash flow, book value, dividend
Intellidex[15]	High price momentum, earnings momentum, quality, value, management action	Equal weighted
S&P GIVI[16]	Low market beta	Intrinsic value
S&P 500 High Beta[17]	High market beta	Market beta
S&P 500 Low Volatility[18]	Low volatility	Inverse of volatility

[1]http://www.indxis.com/USBroad.html, [2]http://www.msci.com/products/indices/strategy/risk_premia/hdy/,
[3]http://www.ftse.com/Indices/FTSE_ActiveBeta_Index_Series/index.jsp,
[4]http://www.russell.com/indexes/data/dividend/russell-high-dividend-yield-indexes.asp, [5]http://www.russell.com/indexes/data/stability/russell-stability-indexes.asp,
[6]http://www.ftse.com/Indices/FTSE_GWA_Index_Series/index.jsp, [7]http://www.msci.com/products/indices/strategy/risk_premia/value_weighted/,
[8]http://www.msci.com/products/indices/strategy/risk_premia/risk_weighted/, [9]http://www.msci.com/products/indices/strategy/risk_premia/minimum_volatility/,
[10]http://www.standardandpoors.com/indices/sp-500-equal-weight-index/en/us/?indexId=spusa-500-usdew--p-us-l--,
[11]http://www.ftse.com/Indices/FTSE_EDHEC-Risk_Efficient_Index_Series/index.jsp,
[12]http://www.ftse.com/Indices/FTSE_TOBAM_Maximum_Diversification_Index_Series/index.jsp,
[13]https://www.djindexes.com/dividend/,[14]http://www.ftse.com/Indices/FTSE_RAFI_Index_Series/index.jsp,[15]https://indices.nyx.com/fr/directory/intellidex,
[16]http://www.standardandpoors.com/indices/sp-givi-global/en/us/?indexId=sp-givi-global,
[17]http://www.standardandpoors.com/indices/sp-500-high-beta/en/us/?indexId=spusa-500-usdw-hbp-us-l--,
[18]http://www.standardandpoors.com/indices/sp-500-low-volatility/en/us/?indexId=spusa-500-usdw-lop-us-l--
[19]http://www.ftse.com/Indices/Lyxor%20SmartIX%20ERC%20Equity%20Indices/index.jsp

Source: Amenc, Goltz and Lodh (2012)

2.3 *Risk Factor Investing*

Risk Factor Investing

Since the original publication of CAPM, it has been demonstrated empirically that:

Certain unobserved variables other than the "broad market excess return" had significant explanatory power over returns of individual securities

Arbitrage Pricing Theory of Ross (1976) extends the SML equation to

$$r_{it} = \alpha + \sum_{j=1}^{M} \beta_{ij} f_{jt} + \varepsilon_t$$

We refer to these unobservable variables f_{jt} as "***risk factors***"

The list of potential risk factors continues to grow as new risk factors are identified by academic and practitioners' research

Importantly, these risk factors are assigned economic meaning

Well-Established Risk Factors

Most popular/most commonly quoted risk factors are summarized in the MSCI factor investing primer

Exhibit 1: Well-Known Systematic Factors from the Academic Research

Systematic Factors	What It is	Commonly Captured by
Value	➢ Captures excess returns to stocks that have low prices relative to their fundamental value	➢ Book to price, earnings to price, book value, sales, earnings, cash earnings, net profit, dividends, cash flow
Low Size (Small Cap)	➢ Captures excess returns of smaller firms (by market capitalization) relative to their larger counterparts	➢ Market capitalization (full or free float)
Momentum	➢ Reflects excess returns to stocks with stronger past performance	➢ Relative returns (3-mth, 6-mth, 12-mth, sometimes with last 1 mth excluded), historical alpha
Low Volatility	➢ Captures excess returns to stocks with lower than average volatility, beta, and/or idiosyncratic risk	➢ Standard deviation (1-yr, 2-yrs, 3-yrs), Downside standard deviation, standard deviation of idiosyncratic returns, Beta
Dividend Yield	➢ Captures excess returns to stocks that have higher-than-average dividend yields	➢ Dividend yield
Quality	➢ Captures excess returns to stocks that are characterized by low debt, stable earnings growth, and other "quality" metrics	➢ ROE, earnings stability, dividend growth stability, strength of balance sheet, financial leverage, accounting policies, strength of management, accruals, cash flows

Source: https://www.msci.com/documents/1296102/1336482/Foundations_of_Factor_Investing.pdf

Commercial Factor Indexes

A large number of products available in the current market designed to facilitate factor investing. For a (far from exhaustive) example, factor indexes offered by MSCI

We do not take a view on whether some risk factors are "beneficial" and therefore higher exposure to those risk factors necessarily would lead to portfolio outperformance. We will focus on the fact that a set of risk factors is responsible, at least over some time horizon, for a large portion of the security's total return

Exhibit 10: MSCI Family of Factor Indexes

Systematic Factors	MSCI Indexes
Value	➢ **MSCI Value Weighted Indexes:** Capture Value factor by weighting according to four fundamental variables (Sales, Earnings, Cash Flow, Book Value)
Low Size (Small Cap)	➢ **MSCI Equal Weighted Indexes:** Capture low size effect by equally weighting all stocks in a given parent index
Momentum	➢ **MSCI Momentum Indexes:** Reflect the performance of high momentum stocks by weighting based on 6- and 12-month momentum scaled by volatility
Low Volatility	➢ **MSCI Minimum Volatility Indexes:** Reflect empirical portfolio with lowest forecast volatility using minimum variance optimization ➢ **MSCI Risk Weighted Indexes:** Capture low volatility stocks by weighting based on the inverse of historical variance
Dividend Yield	➢ **MSCI High Dividend Yield Indexes:** Select high dividend stocks with screens for quality and potential yield traps
Quality	➢ **MSCI Quality Indexes:** Capture high quality stocks by weighting based on debt-to-equity, return-on-equity, and earnings variability

Source: MSCI

Fama-French Factors

Prof. Kenneth R. French maintains a database of key risk factors Fama and French have identified and continue to study at *https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html*

The key risk factors we will focus in our analyses are the following four (in addition to the “broad market return” risk factor of CAPM):

- **SMB** a.k.a. “Small Minus Big”, representing a discount associated with firm size. It is computed as the average return on nine small stock portfolios minus the average return on nine big stock portfolios
- **HML** a.k.a. “High Minus Low”, representing a premium associated with “value” investment. It is computed as the average return on two value portfolios minus the average return on two growth portfolios
- **RMW** a.k.a. “Robust Minus Weak”, representing a premium associated with higher profitability. It is computed as the average return on two robust operating profitability portfolios minus the average return on two weak operating profitability portfolios
- **CMA** a.k.a. “Conservative Minus Aggressive”, representing a discount associated with aggressive investment as indicated by total asset growth. It is computed as the average return on two conservative investment (low annual asset growth) portfolios minus the average return on two aggressive investment (high annual asset growth) portfolios

Sorted Portfolios Approach

This is a recap from **Session One**. *The specifics of construction of the relevant series described in* *https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_5_factors_2x3.html*

Fama and French create time series for these risk factors following the general method of portfolio sorting:

- On a given date (start of period), rank all member stocks according to a chosen metric e.g. Px/Book ratio
- Group all member stocks into K portfolios according to the ranking e.g. into deciles of the ranking
- Within each portfolio, weights are either equal or proportional to market capitalization
- At the end of the period, record the returns of these K portfolios. Repeat all steps starting from ranking/grouping for the next period

Each risk factor series is then constructed as a difference in returns across some of the portfolios (SMB, HML, RMW, CMA)

Fama-MacBeth Regressions

Fama-MacBeth regressions are a method of constructing risk premium time series associated with specific risk factors

It is a two-step procedure:

1) For each member security, time series of its returns are regressed against the time series of risk factors as calculated by Fama and French (previous section). This regression yields factor loadings for each member security (betas of that stock w.r.t. to specific risk factors)

2) For each date in the sample, a cross-sectional regression is performed: stock returns on a given date are regressed against their factor loadings (betas). This regression yields the risk premia amounts (portions of security returns) associated with each factor on that date

[Exercise C: Estimation of Risk Premia Series]

The exposition of this exercise largely follows "Machine Learning for Factor Investing" by Guillaume Coqueret and Tony Guida

2.4 *Expanding Risk Factor Universe*

Expanding Risk Factor Universe

Avid research by both academics and practitioners led to a proliferation of identified risk factors. This phenomenon is sometimes referred to (disparagingly?) as the "risk factor zoo"

It is however not unreasonable that new data processing techniques (such as deep learning) and a significant increase in data availability led to a large number of new potentially useful risk factors being identified

Successful commercial risk factor models based on a large number of identified risk factors include e.g. Barra Global Equity Model

Deep Learning in Search of Risk Factors. Machine learning is instrumental in the search for new risk factors as it is able to efficiently process a large number of "features"

We no longer need to restrict ourselves to linear dependence in the risk factor definition equation; one can more generally think of risk factors as the space of features in a supervised ("deep" i.e. having hidden layers) learning problem with security returns as output

New studies exist that combine the deep learning approach with the problem of new ESG risk factor search (e.g. de Franco et al, 2020 https://arxiv.org/pdf/2002.07477.pdf)

Alternative Data

While traditional fundamental indexes and smart beta indexes relying on systematic portfolio rebalancing of the same universe of stocks have been popular for years and both offer a rich variety of commercially available indexes, indexes that rely on "alternative data", that is, non-traditional data sources are still rare

Without doubt, the hedge fund industry does make extensive use of "alternative data", but few indexes so far are calculated leveraging those data sources

One way to categorize **Alternative Data** is
(to be discussed in more detail in **Session Four**):

- **Sentiment**: Social-media feeds, news flow, corporate announcements and other published media are monitored and analyzed for clues to sentiment on stocks, products, and the economy. This category includes the analysis of language used by executives on earnings calls, etc.
- **Other Web Scraping**: Involves compiling data from targeted websites in a bid to gain information on brands and products. Includes job listings and employee-satisfaction rankings, which can offer clues to a company's growth prospects
- **Credit Card Data**: Some data providers put together large panels of consumers who agree to share their credit- and debit-card activity. Panels made up of more than 3 million consumers are considered big enough to be useful; these data are used for real-time tracking of retail revenue
- **Satellites and Aerial Surveillance**: Used to track ships en route, monitor crops, and detect activity in ports and oil fields. Also, cars in parking lots are often counted as a proxy for retailer sales activity

Alternative Data / ESG Risk Factors

Examples of available Alternative Data indexes do exist

Particularly noteworthy, indexes of **Sentiment** type that leverage natural language sources (to be discussed in more detail in **Session Five**). Many of these offerings are from recent startups

- *Indexica* offers customized index solutions applying NLP techniques to public text data sources
- *Refinitiv* publishes sector-based news sentiment indices. It is highly likely that we will see numerous new offerings in this field emerge in the near future

ESG Risk Factors. *It is entirely possible to construct risk factors following the Fama-French portfolio sorting approach above for an arbitrary metric used to rank the underlying member securities. For instance, we can use ESG scoring (environmental, social, and corporate governance score, or a combination thereof) to rank the portfolios. For example, we can rank all member securities by carbon emissions and define our risk factor as the difference in returns between "high-carbon polluting" and "low carbon polluting" quartile portfolios.*

In practice, there is no uniform standard approach to ESG scoring; therefore, performance of ESG risk factors is highly dependent on the choice of underlying data and the choice of scoring methodology

Session II Homework

HW

How does the time series of the low-vol index derived in class for Pharma compare to (a) equal-weighted Pharma index? (b) price-weighted Pharma index? Please plot three series together

Please turn in either a Jupyter notebook or a script in a text file