Q-Variance: Or a Duet Concerning the Two Chief World Systems

David Orrell, while writing a book on the history of money, came up with the idea of applying quantum ideas to finance. Without looking at any data, he made a prediction about the behavior of volatility. In the spirit of a true scientist and almost unique in finance, he went on to test this prediction. Here, David and **Paul Wilmott** discuss what happened next. There is also a request for assistance from the readers. The full paper follows this dialogue.

PAUL: Hey!
DAVID: Hey!

PAUL: You've been blathering on about a quantum finance model for ages now…have you found any takers?
DAVID: It's what you would call a niche community.

PAUL: I'm not totally surprised. I've seen quite a few attempts to apply physics theories to finance, and they tend to be a bit gratuitous. But I must admit that the quantum thing feels different. I quite like the idea.
DAVID: Most people do not like quantum ideas being used outside of physics! They associate it with subatomic particles and think it can't be applied to anything else. But calculus was first developed to model the motion of planets, and we now use it for modeling things like the economy without confusing people with planets.

Speaking of attempts to apply physics to finance though, Louis Bachelier, of random walk fame, also taught a course at the Sorbonne in the early 1900s on "Probability calculus with applications to financial operations and analogies with certain questions from physics" … Too bad he didn't pick up on some of the new quantum stuff being developed at the same time!
PAUL: What is the background to your using quantum ideas in finance?

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DAVID: I got into it while working on a book about the history of money. Money has dualistic properties — it is associated with both a physical thing, like gold, and an abstract number, like credit — which resemble wave/particle duality in physics. If you are selling your house, you have a vague idea of its value, but when the transaction occurs, that value gets collapsed down to an exact number, like the measurement procedure in quantum physics. In the stock market, the inherent uncertainty in price is captured by the bid/ask spread.

Is there any evidence that you are onto something? Can it be exploited "for pleasure or profit"?

Transactions measure the price, but also perturb the system, which leads to interesting dynamics.

PAUL: Yes, it's the idea of "collapsing" to a known value at the time of a transaction that I like. I dabbled in something similar decades ago, but without using the q word. That collapsed as well.

I know a little bit about your work and we've published some in this magazine. So, my role in this duet is to direct the flow and also to act as Devil's advocate. I don't know whether I'm Sonny and you Cher, or maybe John Travolta and Olivia Newton John? No, I'll be Elton John and you can be Kiki Dee. How's that?

DAVID: Sure, I'll be the clown.

In quantum physics, the probability of, say, a particle's position is represented by a complex-valued wave function which somehow collapses at the time of measurement. No one knows what the wave function is or how it collapses, which is one reason quantum mechanics is often described by physicists as being "fundamentally incomprehensible" (Niels Bohr). In finance, the process is less mysterious — the wave function represents fuzzy information about value, and the price is assigned at the moment of transaction.

PAUL: What does the maths look like? Similar to stochastic calculus or completely different? Can the model be tested against data? Is there any evidence that you are onto something? Can it be exploited "for pleasure or profit"?

DAVID: The maths is different because instead of breaking time down into steps, you look at a single period as a whole. One way to look at it is that the classical model is a zero-order model because it assumes price is at equilibrium but perturbed by random shocks. The quantum model is a first-order model because it allows for linear dynamics which unfold over the period.

PAUL: Ok, I don't get all of that. Except for the bit about looking "at a single period as a whole." That's unusual. We are used to working in continuous time, say, with stochastic differential equations. When we have discrete time, it's usually the binomial model, especially for derivatives valuation, or basic portfolio management. Let's come back to this later; meanwhile, I have a few issues. Although, as they say, it's not you, it's me.

DAVID: The physicist Richard Feynman famously said, "The test of science is its ability to predict." One prediction to come out of the model is something I'm calling q-variance. This is very easy to test, and I know you've tried this yourself, so I'm going to ask interested readers to try the following. Take a long time series of historical data, something like the S&P 500 is good. Break it up into periods T of, say, 20 trading days, and for each period calculate the annualized volatility σ, and the log price change x over the period (which is just the sum of the 20 daily log price changes). Let

$$z = \frac{x}{\sqrt{T}}$$

where T=20/252 is measured in years.

Subtract off the mean of the zs, this amounts to a small horizontal shift which accounts for effects like drift. Now do a scatter plot of σ versus z. You should see a pattern emerge. Now try different period lengths T, from a few days to a year. The pattern should still be there. And if you interpolate through the points, to find the expected volatility, you should get a graph which looks like

$$\sigma(z) = \sqrt{\sigma_0^2 + \frac{z^2}{2}}$$

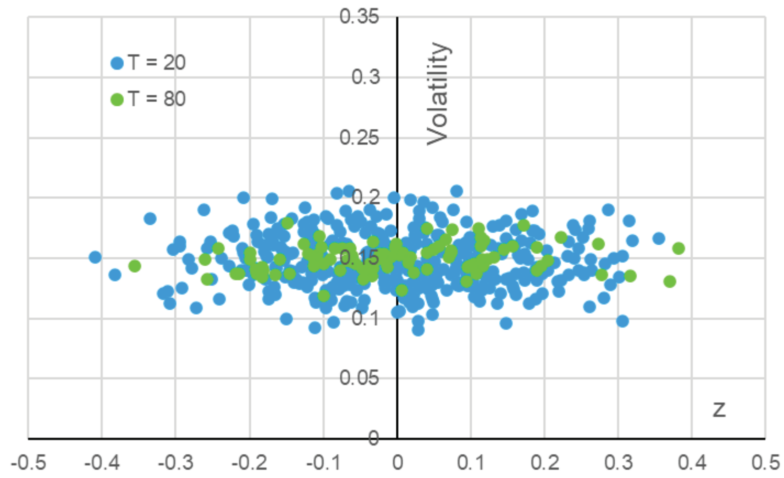
where the sole parameter σ_0 is a minimum volatility. That is q-variance.

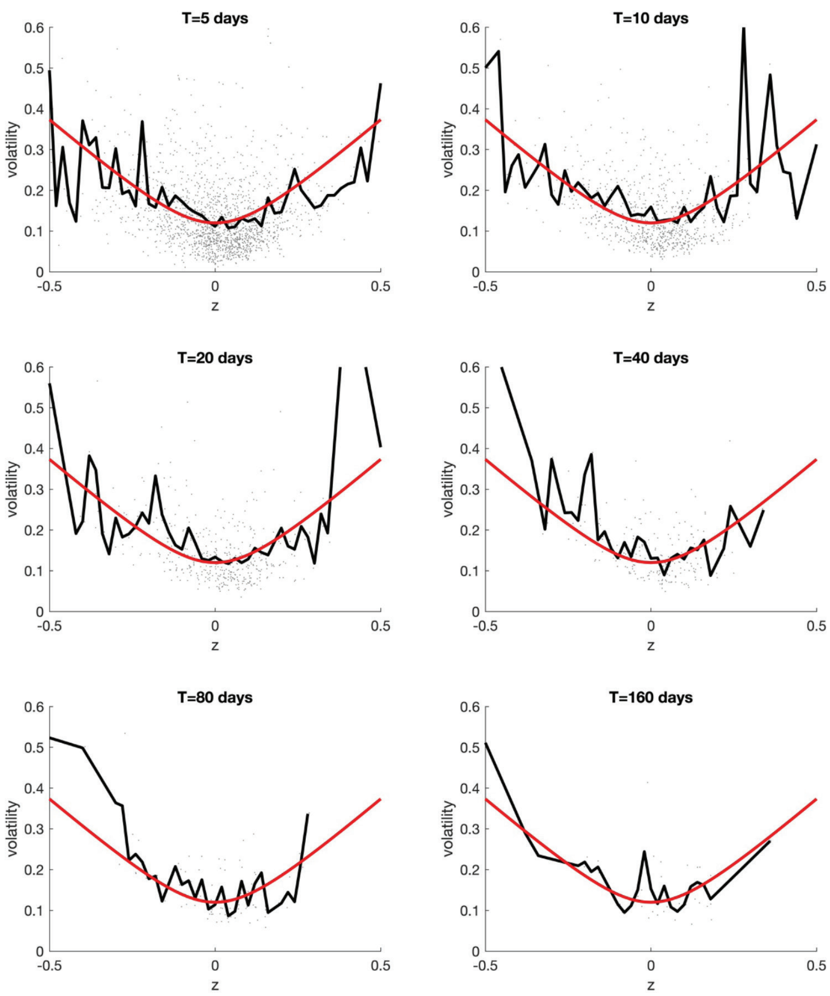
PAUL: You said one "prediction." Do you mean that you came up with a model, it told you something about share price returns, and that prediction turned out to be correct?

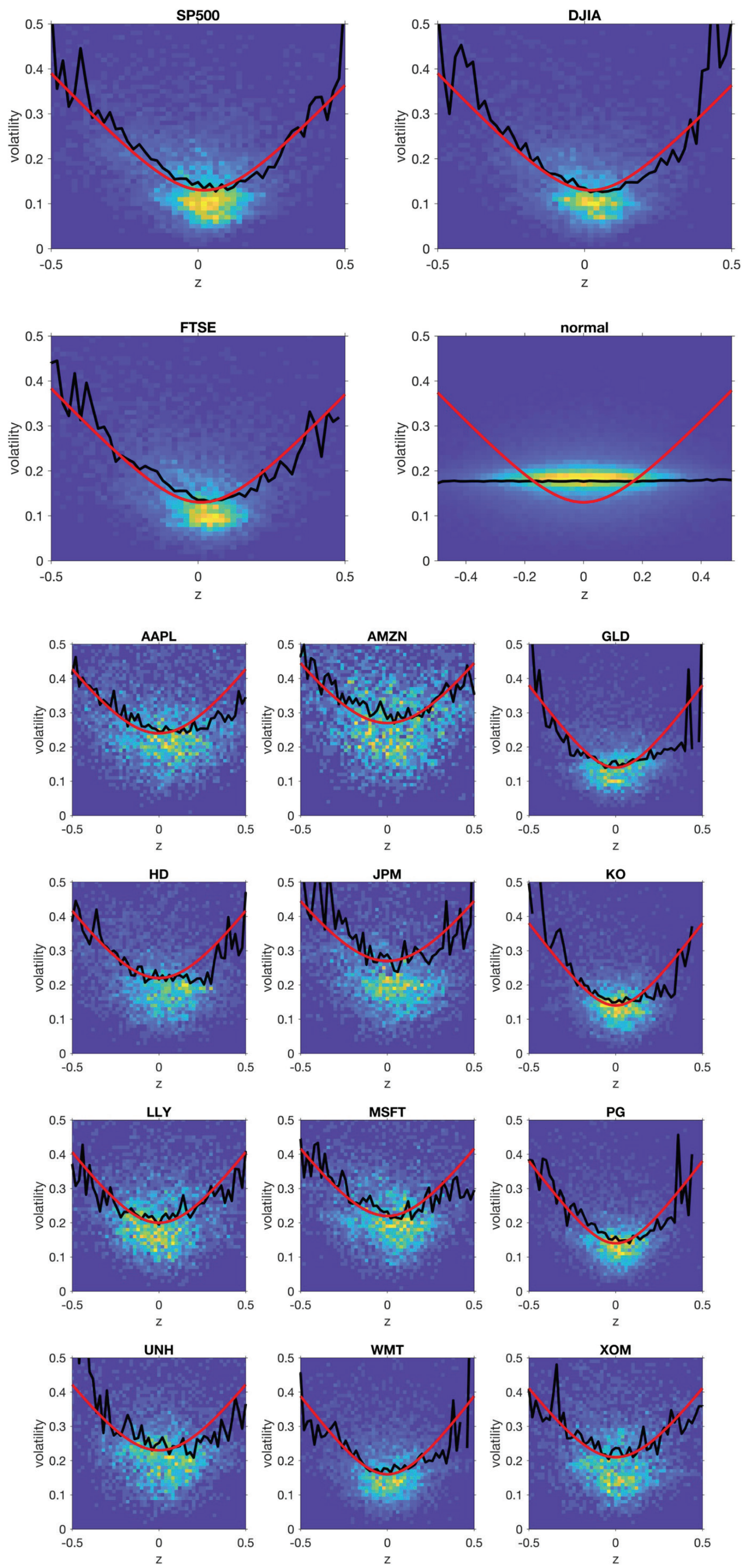
DAVID: Yes. The model behaves like a quantum spring. If you stretch a spring by a certain amount, the energy goes up. In the quantum model, price change and volatility are similarly linked according to a simple equation. So, the prediction came from the model, but it is easy to test empirically.

PAUL: Let me recap that for the readers. In stages.

1. First get your data. Let's say that you have 50 years' worth of daily S&P 500 index prices, that's S_i.
2. Using this data, you are going to calculate a time series of returns and volatilities over periods of T days.
3. Calculate a series of daily log returns, that's $\ln\left(\frac{S_{i+1}}{S_i}\right)$.
4. Using these daily returns, calculate a time series of annualized volatilities over periods of T days. That's just a standard deviation multiplied by $\sqrt{252}$. If you've scaled correctly, you'll get a lot of numbers around the 10%, 15%, etc. level.
5. Now calculate the return over T days. Because we are dealing with log returns, you just have to sum the daily returns over each period of T days.
6. Now scale *these* returns with $\sqrt{\frac{T}{252}}$. This is a new time series, call them z. And subtract off the mean







of these *zs*. The *zs* should now be order of magnitude 10%, 20%, positive and negative.

7. So now you have two time series, volatilities and *zs*, and let's have the periods not overlapping, so you have far fewer data points than in the original S&P 500 index data.
8. Plot vols on the vertical and *zs* on the horizontal.

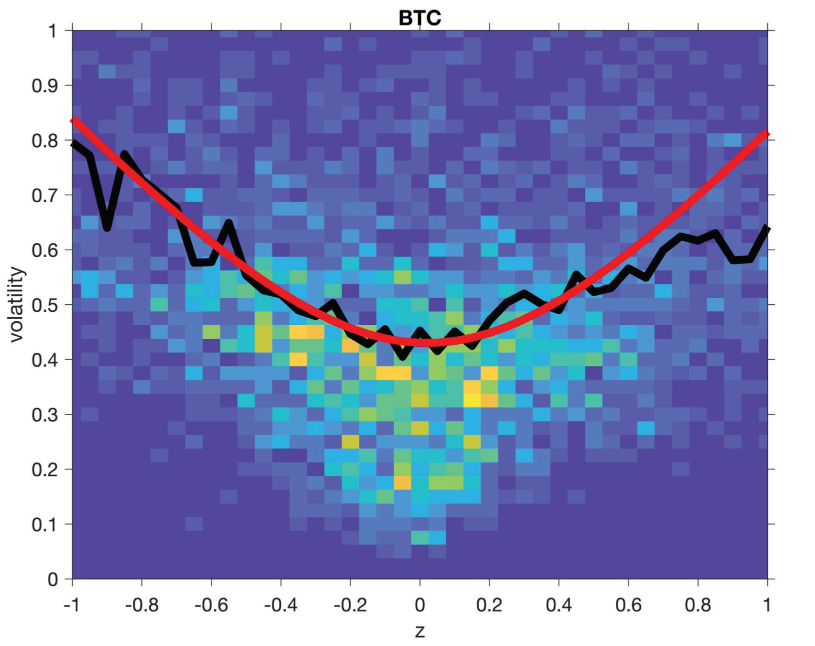
DAVID: That's right. And what you'll see…

PAUL: Easy, Tiger! Let me stop you there! Before you do this with the S&P 500 data, I want to show the readers what this looks like for a simulation of the classical, constant volatility, lognormal random walk.

These are the results for $T = 20$ and $T = 80$, And it will be similar for any *Ts*. There's really nothing much to see here. For both *Ts* there is a spread of *zs* and a spread of vols, just as you'd expect. But no relationship between the vols and the *zs*. Now, what were you saying about the S&P 500 data?

DAVID: If you do exactly the same but with the S&P 500 data, the pattern looks different. Periods with a large price change *z* are associated with high volatility. If you interpolate through the data — best is to bin the data in *z* and compute the average variance for each bin — then the result is a smile-shaped curve that is approximated by the q-variance equation.

PAUL: Show me some pictures! BTW, I don't like the use of the word "smile." That's usually associated implied vol



against strike. And you haven't mentioned options, let alone implied vol and strikes!

DAVID: Here is one that I made earlier. The black lines are the interpolation, the red line is q-variance. Time periods are T=5, 10, 20, 40, 80 and 160 days.

The plots (see Page 37) combine periods T from 1 to 50 weeks for the S&P 500, DJIA, FTSE, and lognormal — heat maps show the density. Data since 1992. You can tell which is the odd one out!

It works for individual stocks, too. Some big names chosen at random:

Crypto isn't immune either. Here is a plot of BTC-USD over the last ten years. Note the scale is twice as large to accommodate the volatility.

PAUL: And it's the same red curve in every case?

DAVID: Yes, the only parameter to change is the minimum volatility. Now, as you pointed out, these plots are for actual volatility, not implied volatility. However, it would seem quite a natural starting point to explain the volatility smile seen with options.

PAUL: I wouldn't put it like that. I'd say that it might be "consistent" with the smile but not an "explanation." The smile comes about because of the price paid for certain options. So, the smile being explained by something usually requires that "something" be known by traders, which, as we've experienced, it doesn't seem to be. It's like saying that your cat is grumpy because its star sign is Cancer.

Every quant has a vol model that is consistent with the smile, that's rather why they exist. And that includes me. The model that, I think, comes closest to "explaining" the smile, in the sense that traders don't need to know the details to value certain options higher, is the memory model developed by me, Alan Lewis and Daniel Duffy. Anyway, park the implied vol thing for now. What you've got is far more interesting, keep away from the implied vol space for the moment, that's far too crowded already in my opinion!

DAVID: Well, I hope there is room for one more theory about implied volatility! And I discuss this a bit in the paper. But this raises an interesting question, which is why q-variance apparently hasn't previously been discussed in the literature, or if it has, it didn't make much of a wave. I'm sure others must have come across it! But I have asked numerous people, including the Wilmott forum brain trust, and so far no leads.

PAUL: Indeed. I've asked too…tumbleweed.

Let's recap why this q-variance property is odd.

Here's my list:

1. The model works over large, discrete and arbitrary timescales, but not in continuous time. A continuous-time limit isn't obvious.
2. The q-variance effect is there for all timescales. This is unusual, in that, most models tend to a normal distribution for returns over long time periods. Take jump-diffusion, for example. That has fat tails for short timescales but they disappear.
3. It's the same quadratic curve regardless of the time period T.
4. And the big one, there is no extra parameter for the z dependence. I've almost managed to get something like q-variance with the memory model, but there are two problems, the z dependence is not as strong and, most importantly, there is another parameter.

I did wonder if the quadratic behavior is some statistical effect, some bias, for example. But it's not there with classical simulations. Maybe a statistician can advise.

Let me know your thoughts on these.

DAVID: Agree with all of these. The model works in terms of discrete transactions, so continuous time isn't really a thing. The fact that the effect is there for all timescales and periods T from a few days to a year or more rules out a lot of explanations. As for there being no extra parameter, that's what makes it a falsifiable prediction.

For example, in the Black-Scholes model, the volatility is a free parameter which can be adjusted to get the price you want. But with q-variance, you can test it, and it will either work or it won't. To be fair, you can find cases or periods where it holds better than others, but in general the pattern is very persistent.

The general problem though is that q-variance doesn't sit easily with random walk theory, at least in any non-contrived form — and I would argue that this helps explain the strange fact that it hasn't been more widely reported. To quote a physicist again (Einstein): "Whether you can observe a thing or not depends on the theory which you use. It is the theory which decides what can be observed." In the quantum model, q-variance is something you want to check, because otherwise the theory doesn't work.

Of course, this only means the quantum model hasn't been falsified, yet. And from a practical standpoint, what really counts isn't the theory behind q-variance, but what the empirical property means for things like asset price behavior. So, any feedback from readers on the verification/application/implications of q-variance would be very useful! And any references.

PAUL: I know there's so much more you want to talk about, and that's why there's plenty more in the following paper. And from my perspective, I'd like to figure out the role of hedging, if any, given the discrete-time nature of your model. But I want to end by echoing your request to the readers to come back with any explanation or references to the q-variance property. In a nutshell, can anyone explain those plots of yours!

DAVID: Consider it a quantum challenge!

Thanks for the discussion — (I've Had) The Time Of My Life (?)