Question 1.

There are 3 target classes in the dataset: house, condo, and apartment.

There are 8 features in the dataset. Local price, land area, living area, and age of home are continuous-valued features, we use formula 1 to calculate the conditional probability

$$P(X_j | C = c_i) = \frac{1}{\sqrt{2\pi}\sigma_{ji}} \exp\left(-\frac{(X_j - u_{ji})^2}{2\sigma_{ji}^2}\right) \quad (1)$$

u_{ji} is mean of the feature value, σ_{ji} is the standard deviation of the feature values. I use the sample formula to calculate it.

The other features are categorical features. The conditional probability is calculated according to formula 2

$$P(X_j = a_{jk} | C = c_i) = \frac{n_c + mp}{n + m} \quad (2)$$

Here n_c is the number of training examples for feature A_{jk} in class Ci. And p is prior estimate, m is weight of prior, n is number of training examples for class Ci

1. When C = apartment (n=7) :

(1) Conditional probability of "local price":

Price for apartment: 4.9176, 4.5573, 5.0597, 14.4598, 5.05, 8.2464, 9.0384

Mean of price: 7.332743;

Standard deviation of price: 3.615997

$$P(price\ |apartment) = \frac{1}{\sqrt{2\pi}*3.62} \exp\left(-\frac{(price - 7.33)^2}{2*3.62^2}\right) = 0.11 * \exp\left(-\frac{(price - 7.33)^2}{26.2}\right)$$

(2) Conditional probability of "Bathrooms":
Bathrooms: 1, 1, 1, 2.5, 1, 1.5, 1
Here m = 3, p =1/3, so
$$P(bathroom = 1\ |apartment) = \frac{5+1}{7+3} = 0.6$$
$$P(bathroom = 1.5\ |apartment) = \frac{1+1}{7+3} = 0.2$$
$$P(bathroom = 2.5\ |apartment) = \frac{1+1}{7+3} = 0.2$$

(3) Conditional probability of "Land Area":

Mean = 6.1; standard deviation = 3.26

$$P(landarea\ |apartment) = 0.12 * \exp\left(-\frac{(landarea - 6.1)^2}{21.24}\right)$$

(4) Conditional probability of "Living area"

Mean = 1.5; standard deviation = 0.7

$$P(livingarea \mid apartment) = 0.57 * \exp\left(-\frac{(livingarea-1.5)^2}{0.99}\right)$$

(5) Conditional probability of "Garages"

Garages are: 1   1   1   2   0   2   1.5

Here m = 4, p = 1/4, so

P(garage = 0 | apartment) = 2/11

P(garage = 1 | apartment) = 4/11

P(garage = 1.5 | apartment) = 2/11

P(garage = 2 | apartment) = 3/11

(6) Conditional probability of " Rooms"

Rooms are: 7   6   6   9   5   8   7

Here m = 6, p = 1/6, so

P(room = 5 | apartment) = 2/13

P(room = 6 | apartment) = 3/13

P(room = 7 | apartment) = 3/13

P(room = 8 | apartment) = 2/13

P(room = 9 | apartment) = 2/13

P(room = 10 | apartment) = 1/13

(7) Conditional probability of "Bedrooms"

Rooms are: 4   3   3   5   2   4   3

Here m = 4, p = 1/4, so

P(room = 2 | apartment) = 2/11

P(room = 3 | apartment) = 4/11

P(room = 4 | apartment) = 3/11

P(room = 5 | apartment) = 2/11

(8) Conditional probability of "Age of home"

Mean = 38.7, standard deviation = 14.7

$$P(age \mid apartment) = 0.027 * \exp\left(-\frac{(age-38.7)^2}{431.14}\right)$$

2. When C = condo (n=6) :

(1) Conditional probability of "local price":

Price for apartment: 4.5429   3.891   5.898   16.4202 5.9592   7.7841

Mean of price: 7.4159;

Standard deviation of price: 4.611247858

$$P(price\ |apartment) = \frac{1}{\sqrt{2\pi} * 3.62} \exp\left(-\frac{(price - 7.42)^2}{2 * 4.61^2}\right) = 0.09 * \exp\left(-\frac{(price - 77.41)^2}{42.5}\right)$$

(2) Conditional probability of "Bathrooms":

Bathrooms: 1   1   1   2.5   1   1.5

Here m = 3, p =1/3, so

$$P(bathroom = 1\ |apartment) = \frac{4+1}{6+3} = 5/9$$

$$P(bathroom =\ 1.5\ |apartment) = \frac{1+1}{6+3} = 2/9$$

$$P(bathroom =\ 2.5\ |apartment) = \frac{1+1}{6+3} = 2/9$$

(3) Conditional probability of "Land Area":

Mean = 6.024666667; standard deviation = 2.544777371

$$P(landarea\ |apartment) = 0.16 * \exp\left(-\frac{(landarea - 6.02)^2}{12.95}\right)$$

(4) Conditional probability of "Living area"

Mean = 1.55; standard deviation = 0.92

$$P(livingarea\ |apartment) = 0.43 * \exp\left(-\frac{(livingarea - 1.55)^2}{1.7}\right)$$

(5) Conditional probability of "Garages"

Garages are: 1 1   1   2   2   1

Here m = 4, p =1/4, so

P(garage = 0 | apartment) = 1/10

P(garage = 1 | apartment) = 5/10

P(garage = 1.5 | apartment) = 1/10

P(garage = 2 | apartment) = 3/10

(6) Conditional probability of " Rooms"

Rooms are: 6   6   7   10   6   6

Here m = 6, p =1/6, so

P(room = 5 | apartment) = 1/12

P(room = 6 | apartment) = 5/12

P(room = 7 | apartment) = 2/12

P(room = 8 | apartment) = 1/12

P(room = 9 | apartment) = 1/12

P(room = 10 | apartment) = 2/12

(7) Conditional probability of "Bedrooms"

Rooms are: 3   3   3   5   3   3

Here m = 4, p =1/4, so

P(room = 2 | apartment) = 1/10

P(room = 3 | apartment) = 6/10

P(room = 4 | apartment) = 1/10

P(room = 5 | apartment) = 2/10

(8) Conditional probability of "Age of home"

Mean = 39.67, standard deviation = 13.96

$$P(age\,|apartment) = 0.027 * \exp\left(-\frac{(age-\,39.67)^2}{389.333}\right)$$

3. When C = house (n=7) :

    (1) Conditional probability of "local price":

Price for apartment: 5.0208   5.6039 5.8282 5.3003 6.2712 5.6039 6.6969

Mean of price: 5.76;

Standard deviation of price: 0.57

$$P(price\,|apartment) = \frac{1}{\sqrt{2\pi} * 0.57}\exp\left(-\frac{(price-\,5.76)^2}{2*0.57^2}\right) = 0.7 * \exp\left(-\frac{(price-\,5.76)^2}{0.65}\right)$$

    (2) Conditional probability of "Bathrooms":

    Bathrooms: 1   1   1   1   1   1   1.5

    Here m = 3, p =1/3, so

    $P(bathroom = 1\,|apartment) = \frac{6+1}{7+3} = 0.7$

    $P(bathroom =\,1.5\,|apartment) =\,\frac{1+1}{7+3} = 0.2$

    $P(bathroom =\,2.5\,|apartment) =\,\frac{0+1}{7+3} = 0.1$

(3) Conditional probability of "Land Area":

Mean = 6.6309; standard deviation = 2.248973218

$$P(landarea \mid apartment) = 0.18 * \exp\left(-\frac{(landarea - 6.63)^2}{10.11}\right)$$

(4) Conditional probability of "Living area"

Mean = 1.39; standard deviation = 0.21

$$P(livingarea \mid apartment) = 1.87 * \exp\left(-\frac{(livingarea - 1.39)^2}{0.09}\right)$$

(5) Conditional probability of "Garages"

Garages are: 2   0   2   1   1   0   1.5

Here m = 4, p = 1/4, so

P(garage = 0 | apartment) = 3/11

P(garage = 1 | apartment) = 3/11

P(garage = 1.5 | apartment) = 2/11

P(garage = 2 | apartment) = 3/11

(6) Conditional probability of " Rooms"

Rooms are: 7   6   6   6   5   6   7

Here m = 6, p = 1/6, so

P(room = 5 | apartment) = 2/13

P(room = 6 | apartment) = 5/13

P(room = 7 | apartment) = 3/13

P(room = 8 | apartment) = 1/13

P(room = 9 | apartment) = 1/13

P(room = 10 | apartment) = 1/13

(7) Conditional probability of "Bedrooms"

Rooms are: 4   3   3   3   2   3   3

Here m = 4, p = 1/4, so

P(room = 2 | apartment) = 2/11

P(room = 3 | apartment) = 6/11

P(room = 4 | apartment) = 2/11

P(room = 5 | apartment) = 1/11

(8) Conditional probability of "Age of home"

Mean = 34.28571, standard deviation = 12.7

$$P(age \mid apartment) = 0.03 * \exp\left(-\frac{(age - 34.29)^2}{323.81}\right)$$