

### 二分法实战练习精讲

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### 三种类型二分

在排序的输入集上二分 在未排序的输入集上二分 在答案集上二分



# 在排序输入集二分——排序矩阵找数问题

https://www.lintcode.com/problem/search-a-2d-matrix

一个矩阵每一行都有序,每一行都比前面一行所有数要大 在矩阵中寻找一个数是否存在



## 二维坐标转换一维坐标

每一行都比上一行大 因此矩阵从上到下从左到右列出来是有序的

n行m列的矩阵: **x, y** → **x** \* **m** + **y** 

#### 套用二分法模板,使用子函数 get 进行坐标转换



```
def searchMatrix(self, matrix, target):
    if not matrix or not matrix[0]:
        return False
    n, m = len(matrix), len(matrix[0])
    start, end = 0, n * m - 1
    while start + 1 < end:
        mid = (start + end) // 2
        if self.get(matrix, mid) < target:</pre>
            start = mid
        else:
            end = mid
    if self.get(matrix, start) == target:
        return True
    if self.get(matrix, end) == target:
        return True
    return False
def get(self, matrix, index):
    x = index // len(matrix[0])
    y = index % len(matrix[0])
    return matrix[x][y]
```

```
ublic boolean searchMatrix(int[][] matrix, int ta<u>rget) {</u>
   if (matrix == null || matrix.length == 0) {
       return false:
      (matrix[0] == null || matrix[0].length == 0) {
       return false;
   int n = matrix.length, m = matrix[0].length;
   int start = 0, end = n * m - 1;
   while (start + 1 < end){
       int mid = start + (end - start) / 2;
       if (get(matrix, mid) < target) {</pre>
           start = mid;
       } else {
           end = mid:
      (get(matrix, start) == target) {
       return true;
      (get(matrix, end) == target) {
       return true;
   return false;
private int get(int[][] matrix, int index) {
   int x = index / matrix[0].length;
   int y = index % matrix[0].length;
   return matrix[x][y];
```



## 排序矩阵找数问题 II

https://www.lintcode.com/problem/search-a-2d-matrix-ii

每一行和每一列分别都是排好序的且**严格递增** 不保证下一行都比上一行的数大

#### 从左下或者右上出发,找6

[3, 4, 5, 6, 7],

[<del>5</del>, <del>7</del>, <del>9</del>, 11, 12],

[<del>8</del>, 10, 12, 14, 16]



[<del>3</del>, **4**, 5, 6, 7],

[<del>5</del>, <del>7</del>, 9, 11, 12],

[<del>8, 10, 12, 14, 16</del>]

[3, **4**, **5**, 6, 7],

[<del>5</del>, <del>7</del>, <del>9</del>, 11, 12],

[<del>8</del>, 10, 12, 14, 16]

5 < 6

#### 记住就好



```
def searchMatrix(self, matrix, target):
    if not matrix or not matrix[0]:
        return 0
    n, m = len(matrix), len(matrix[0])
    x, y = n - 1, 0
    count = 0
    while (x \ge 0 \text{ and } y < m):
        if matrix[x][y] == target:
            x -= 1
            y += 1
            count += 1
        elif matrix[x][y] > target:
            x -= 1
        else:
            y += 1
    return count
```

```
public int searchMatrix(int[][] matrix, int target) {
   if (matrix == null || matrix.length == 0) {
       return 0:
   if (matrix[0] == null || matrix[0].length == 0) {
       return 0;
   int n = matrix.length;
   int m = matrix[0].length;
   int i = n - 1, j = 0, count = 0;
   while (i > -1 \&\& j < m) {
       if (matrix[i][j] == target) {
            count++;
       } else if (matrix[i][j] < target) {</pre>
            j++;
       } else {
    return count;
```



# 在未排序输入集二分——包含全部黑色像素的最小矩阵

https://www.lintcode.com/problem/smallest-rectangle-enclosing-black-pixels/

矩阵中有黑色像素和白色像素分别用 1 和 0 表示 找到最小的矩阵,能把所有的 1 都框起来

假设所有的 1 是连通的

#### 入口函数的要精简



```
def minArea(self, image, x, y):
    if not image or not image[0]:
        return 0
    n, m = len(image), len(image[0])
    left = self.find_first(image, 0, y, self.check_column)
    right = self.find_last(image, y, m - 1, self.check_column)
    up = self.find_first(image, 0, x, self.check_row)
    down = self.find_last(image, x, n - 1, self.check_row)
    return (right - left + 1) * (down - up + 1)
```

#### 四个二分合并为两个二分



```
def find_first(self, image, start, end, check_func):
    while start + 1 < end:
        mid = (start + end) // 2
        if check_func(image, mid):
            end = mid
        else:
            start = mid
    if check_func(image, start):
        return start
    return end
def find_last(self, image, start, end, check_func):
    while start + 1 < end:
        mid = (start + end) // 2
        if check_func(image, mid):
            start = mid
        else:
            end = mid
    if check_func(image, end):
        return end
    return start
```

```
def check_column(self, image, col):
    for i in range(len(image)):
        if image[i][col] == '1':
            return True
    return False
def check_row(self, image, row):
    for j in range(len(image[0])):
        if image[row][j] == '1':
            return True
    return False
```



## 在答案集二分—— 抄书问题

https://www.lintcode.com/problem/copy-books/

k 个抄写员抄n本书,每个抄写员必须连续抄写挨着的几本书 问所有抄写员同时抄写,最少什么时候抄完 pages = [3, 2, 4], k = 2 可以在 5个时间单位内抄完

#### 抄书问题



```
public int copyBooks(int[] pages, int k) {
    if (pages == null || pages.length == 0) {
        return 0;
    if (k == 0)
        return -1;
    int start = 0, end = Integer.MAX_VALUE;
    while (start + 1 < end) {</pre>
        int mid = start + (end - start) / 2;
        if (getNumberOfCopiers(pages, mid) <= k) {</pre>
            end = mid;
        } else {
            start = mid;
    if (getNumberOfCopiers(pages, start) <= k) {</pre>
        return start;
    return end;
```

```
private int getNumberOfCopiers(int[] pages, int limit) {
   int copiers = 0;
   int lastCopied = limit;
   for (int page : pages) {
       if (page > limit) {
            return Integer.MAX_VALUE;
       if (lastCopied + page > limit) {
            copiers++;
            lastCopied = 0;
        lastCopied += page;
   return copiers;
```



## 其他答案

https://www.jiuzhang.com/solution/copy-books/本题还可以使用动态规划实现