Problem Set 1

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Question 1

Exploration of collections of bernoulli variables

```r
set.seed(12311)
x1 <- matrix(rbinom(1000,1,.5),100,10)
```

> Let's pretend that x1 is item response data from a test. So 1s and 0s are correct/incorrect responses (rows are people and columns are items).

> For fun we can look at the correlations across items and the variation in row sums (ie, total scores)

```r
cor(x1)
```

```
##               [,1]         [,2]         [,3]          [,4]         [,5]
##  [1,]  1.000000000  0.008410789 -0.009896948 -0.3032770731  0.094629567
##  [2,]  0.008410789  1.000000000  0.089886562 -0.0669602615 -0.057570773
##  [3,] -0.009896948  0.089886562  1.000000000  0.0295469723 -0.125809070
##  [4,] -0.303277073 -0.066960261  0.029546972  1.0000000000  0.089214650
##  [5,]  0.094629567 -0.057570773 -0.125809070  0.0892146505  1.000000000
##  [6,]  0.237526763  0.130744090 -0.001602564 -0.0525279507  0.035484609
##  [7,]  0.081814407 -0.074443750  0.047043222 -0.0988646639  0.060409150
##  [8,]  0.098085811 -0.016333199  0.059259270  0.0455242322 -0.103165975
##  [9,] -0.149960697 -0.107907043  0.053719716 -0.0004168548 -0.001638407
## [10,]  0.025551766  0.179665184  0.060860872  0.0365014114 -0.058030861
##               [,6]         [,7]        [,8]          [,9]       [,10]
##  [1,]  0.237526763  0.081814407  0.09808581 -0.1499606967  0.02555177
##  [2,]  0.130744090 -0.074443750 -0.01633320 -0.1079070433  0.17966518
##  [3,] -0.001602564  0.047043222  0.05925927  0.0537197158  0.06086087
##  [4,] -0.052527951 -0.098864664  0.04552423 -0.0004168548  0.03650141
##  [5,]  0.035484609  0.060409150 -0.10316597 -0.0016384067 -0.05803086
##  [6,]  1.000000000  0.087597723  0.09929932 -0.1904608106 -0.05925927
##  [7,]  0.087597723  1.000000000  0.02350749  0.0090628774  0.05755282
##  [8,]  0.099299317  0.023507488  1.00000000  0.0370118105  0.08043217
##  [9,] -0.190460811  0.009062877  0.03701181  1.0000000000  0.08500515
## [10,] -0.059259270  0.057552816  0.08043217  0.0850051472  1.00000000
```

```r
var(rowSums(x1))
```

```
## [1] 2.706667
```

Q. If you considered the 1s/0s correct and incorrect responses to test items (where the rows are people and the columns are items), does this seem like it could have come from a realistic scenario? How might we know?

> Feel free to ignore this chunk of code (skip ahead to below question). I'm going to generate a new set of data.

```r
set.seed(12311)
th<-matrix(rnorm(100),100,10,byrow=FALSE)
diff<-matrix<-matrix(rnorm(10),100,10,byrow=TRUE)
kern<- exp(th - diff)
pr<-kern/(1+kern)
test<-matrix(runif(1000),100,10)
x2<-ifelse(pr>test,1,0)
```

```r
cor(x2)
```

```
##              [,1]       [,2]       [,3]        [,4]       [,5]        [,6]
##  [1,] 1.00000000 0.05225302 0.31098566  0.08849268 0.17610919  0.11892066
##  [2,] 0.05225302 1.00000000 0.26505302  0.03714521 0.12156613  0.19791667
##  [3,] 0.31098566 0.26505302 1.00000000  0.17914327 0.17914327  0.06001200
##  [4,] 0.08849268 0.03714521 0.17914327  1.00000000 0.05582923  0.13169665
##  [5,] 0.17610919 0.12156613 0.17914327  0.05582923 1.00000000  0.08104409
##  [6,] 0.11892066 0.19791667 0.06001200  0.13169665 0.08104409  1.00000000
##  [7,] 0.17435686 0.25832804 0.22615825  0.10837438 0.06732348  0.17221869
##  [8,] 0.07143616 0.13262503 0.05803086 -0.01959216 0.06204183 -0.04029115
##  [9,] 0.11133735 0.42075805 0.11903823  0.21353066 0.05114993  0.08014439
## [10,] 0.12808759 0.08071308 0.14553789  0.19806502 0.10679082  0.13514748
##              [,7]        [,8]        [,9]       [,10]
##  [1,] 0.17435686  0.07143616  0.11133735  0.12808759
##  [2,] 0.25832804  0.13262503  0.42075805  0.08071308
##  [3,] 0.22615825  0.05803086  0.11903823  0.14553789
##  [4,] 0.10837438 -0.01959216  0.21353066  0.19806502
##  [5,] 0.06732348  0.06204183  0.05114993  0.10679082
##  [6,] 0.17221869 -0.04029115  0.08014439  0.13514748
##  [7,] 1.00000000  0.14204314  0.15182598  0.21266889
##  [8,] 0.14204314  1.00000000  0.33582739  0.13068593
##  [9,] 0.15182598  0.33582739  1.00000000 -0.01399044
## [10,] 0.21266889  0.13068593 -0.01399044  1.00000000
```

```r
var(rowSums(x2))
```

```
## [1] 5.111111
```

##Q. Now, let's ask the same question of the new matrix x2. Does it seem like realistic item response data? Specifically, how does it compare to the first matrix x1 in terms of whether it seems like a realistic set of item responses? What characteristics influence your opinion on this point?

##Q. How would you characterize the key difference between x1 and x2 in terms of what we can observe if we blind ourselves to the data generating process?

Question 2

```
load(here("data","ps1-logreg.Rdata"))
```

```
m1 <- glm(y1 ~ x,df, family = "binomial")
m2 <- glm(y2 ~ x,df, family = "binomial")
```

```
summary(m1)
```

```
##
## Call:
## glm(formula = y1 ~ x, family = "binomial", data = df)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.0732  -1.0230   0.3943   1.0042   2.3292
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  0.06109    0.06940    0.88    0.379
## x            0.99636    0.08424   11.83   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 1386.1  on 999  degrees of freedom
## Residual deviance: 1204.9  on 998  degrees of freedom
## AIC: 1208.9
##
## Number of Fisher Scoring iterations: 3
```

```
summary(m2)
```

```
##
## Call:
## glm(formula = y2 ~ x, family = "binomial", data = df)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.8981  -0.6553   0.4267   0.6887   2.2202
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  1.44146    0.09658   14.93   <2e-16 ***
## x            1.43951    0.10921   13.18   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 1152.34  on 999  degrees of freedom
## Residual deviance:  896.89  on 998  degrees of freedom
## AIC: 900.89
##
## Number of Fisher Scoring iterations: 5
```

(A) How would you compare the association between y1 or y2 & x?

(B) How would you interpret the regression coefficients from (say) m1?

For m1: odds $= e^{\hat{\beta}_1} = 2.7084053$

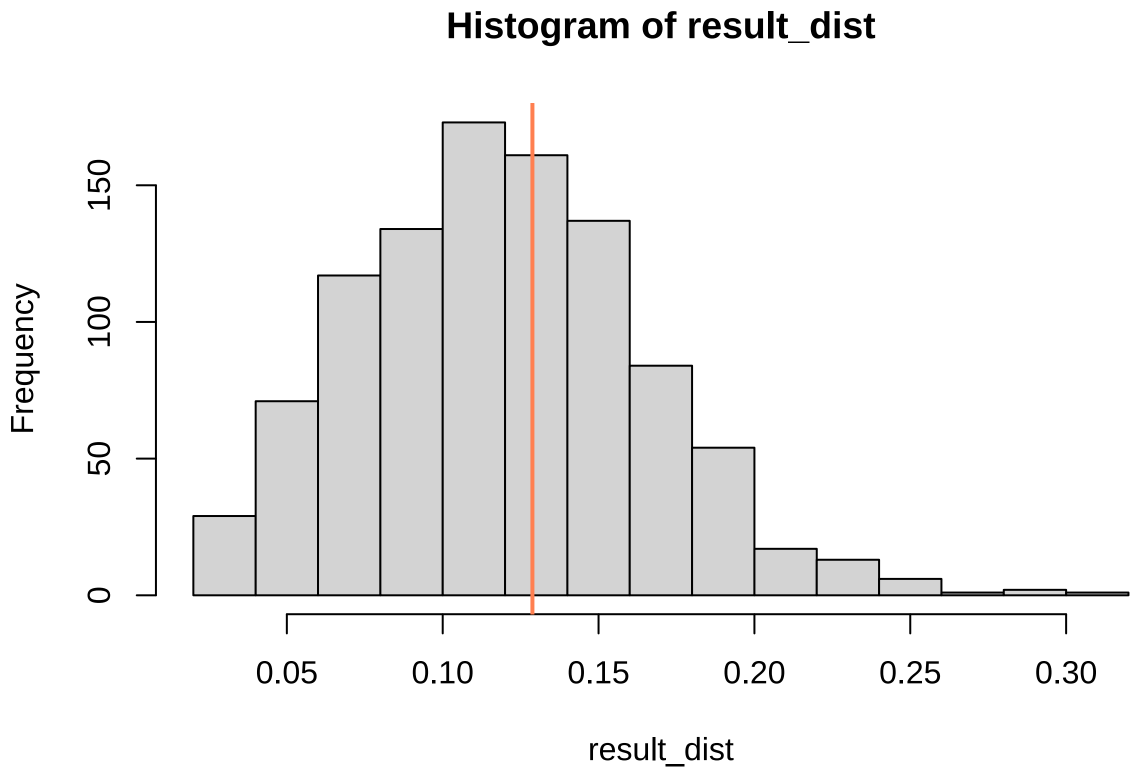
(C) Do m1 and m2 show equivalent model fit? Can you notice anything peculiar about either y1 or y2 (in terms of their association with x)? [Note: This one is sneaky. I'd encourage you to avoid fit statistics and look at techniques for model diagnostics (e.g., residuals).]

Bonus Question

```r
d <- 5 # distance
l <- 1 # length of needle
simulate_n <- 1000 # simulate how many times?
sample_n <- 50 # sample size in each simulation?
```

```r
intersect_function <- function(x) {

  if (min(x1, x2) <= x & max(x1, x2) >= x) {return(1)}
  else {return(0)}

}
```

```r
result_dist <- c()
set.seed(252)

for (i in 1:simulate_n) {
  result_sample <- c() # list of 0 and 1 for each j
  for (j in 1:sample_n){
    result_temp <- c()

    # create coordinates for the needle ends

    x1 <- runif(1, -100, 100) # a random number from -100 to 100
    y1 <- runif(1, -100, 100)
    theta <- runif(1, 0, pi) # a random angle
    x2 <- x1 + l * cos(theta) # find the other coordinate
    y2 <- y1 + l * sin(theta)

    # create parallel lines
    initial_line <- runif(1, -120, -110)
    line_list <- seq(from = initial_line, to = -initial_line, by = d) # create list of parallel lines

    # check intersection
    intersect_yes <- sapply(line_list, intersect_function) # for each line, does it intersect with the
    if (sum(intersect_yes) != 0) {
      result_temp <- append(result_temp, 1)
    }

    else {result_temp <- append(result_temp, 0)} # whether j intersects

    result_sample <- append(result_sample,result_temp)

  }
result_dist <- append(result_dist,mean(result_sample))

}

hist(result_dist)
abline(v = mean(result_dist),col = "coral", lwd = 2)
```



```r
mosaic::favstats(result_dist)
```

```
## Registered S3 method overwritten by 'mosaic':
##   method                           from
##   fortify.SpatialPolygonsDataFrame ggplot2
```

```
##   min  Q1 median   Q3  max    mean         sd    n missing
##  0.02 0.1   0.12 0.16 0.32 0.12878 0.04661206 1000       0
```