Eindhoven University of Technology

5XWA0 Power system analysis and optimization

Assignment 3

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1 Modelling of Optimization Problem

I.1 The whole system is simulated for **24 time-steps**, with each time-step= $1hour$. So for each time-step, we have the following decision variables as shown in Table:1.

Table 1: Decision variables (Outputs)

Component	**Decision Variables**	**Quantity**
	$P_{i,t}$	2
	$B_{i,f,t}$	2
Generator	$u_{i,t}$	2
	$CSU_{i,t}$	2
	$CSD_{i,t}$	2
	$SOE_{s,t}$	2
ESS	$P_{s,t}^{ch}$	2
	$P_{s,t}^{dis}$	2
	$u_{s,t}^{ESS}$	2
Transmission Lines	$f_{l,t}$	7
Bus	$\theta_{n,t}$	5
System	TSC	1
Total		31

I.2 The ESS has 5 constraints as shown below

$$SOE_{s,t} = SOE_{s,(t-1)} + \eta_s \cdot P_{s,t}^{ch} - \frac{P_{s,t}^{dis}}{\eta_s} \quad \forall\, s, t > 1 \tag{1}$$

$$SOE_{s,t} = SOE_s^{ini} + \eta_s \cdot P_{s,t}^{ch} - \frac{P_{s,t}^{dis}}{\eta_s} \quad \forall\, s, t = 1 \tag{2}$$

$$SOE_s^{min} \leq SOE_{s,t} \leq SOE_s^{max} \quad \forall\, s, t \tag{3}$$

$$P_{s,t}^{ch} \leq P_s^{ESS,max} \cdot u_{s,t}^{ESS} \quad \forall\, s, t \tag{4}$$

$$P_{s,t}^{dis} \leq P_s^{ESS,max} \cdot (1 - u_{s,t}^{ESS}) \quad \forall\, s, t \tag{5}$$

All the variables shown in the above equations follow the symbols and definitions as provided in the assignment.

- The first constraint as given in eq:1, relates the state of the energy (SOE) of the ESS with its previous SOE, charging power, discharging power and efficiency of charging/discharging.
- The second constraint (eq:2) is a special case of constraint one for the first period of the optimization horizon.
- The third constraint (eq:3) keeps the SOE of the ESS within safe limit of operation at any given period of time.

- The fourth constraint (eq:4) restricts the charging power to exceed the maximum power limit of the ESS at any given period of time.
- The fifth constraint (eq:5), like fourth constraint prevents the discharge power of the ESS to reduce beyond the permissible limit.

II The following constraints (eq:6a,6b) take into account the shut-down cost of the conventional generators.

$$CSD_{i,t} \geq SDC_i \cdot (u_{i,(t-1)} - u_{i,t}) \quad \forall\, i, t > 1 \tag{6a}$$

$$CSD_{i,t} \geq SDC_i \cdot (u_i^{init} - u_{i,t}) \quad \forall\, i, t > 1 \tag{6b}$$

The second constraint(eq:6b) given above is a special case of the first constraint (eq:6a), which is applicable during the first period of the optimization horizon. Each generator is associated with a shut-down cost which is incurred, every time a generator is shut downed. This includes cost incurred due to loss of power from already generated steam among other costs.

So as per the constraint equation, the cost incurred by shutting down i^{th} generator in period t ($CSD_{i,t}$) takes the value equal to fixed shut-down cost (SDC_i) of the i^{th} generator if and only the generator status at $t-1$ ($u_{i,t-1}$) was 1 and it shutdowns at period t. In other words $u_{i,t-1} = 1$ and $u_{i,t} = 0$. For rest all cases, the variable $CSD_{i,t}$ takes the value zero.

Though the eq:6a and eq:6b allows the variable $CSD_{i,t}$ to take negative values and it is included in the cost function which is minimized, but as $CSD_{i,t}$ can only be positive, the $CSD_{i,t}$ can only take values equal to SDC_i or zero.

III The total system cost is defined as shown in eq:7.

$$TSC = \sum_t \sum_i \left[\left(\sum_f C_{i,f} \cdot b_{i,f,t} \right) + CSU_{i,t} + CSD_{i,t} \right] \tag{7}$$

The term for operating cost of the generator in the TSC is sum of the products of Cost of energy from the f^{th} "power block" of i^{th} generator and Size of the f^{th} "power block" of i^{th} generator, ie: $\sum_f C_{i,f} \cdot b_{i,f,t}$. Hence the optimization solution will always minimise the total cost of operation the i^{th} generator which will ensure that the previous power blocks are filled totally before moving to the next block as the marginal cost associated with each block increases with the block number. In other words, $C_{i,1} \leq C_{i,2} ... \leq C_{i,N}$ which represents a step- wise increasing constant marginal cost function

The capability of the optimization model to decompose the generating unit power output correctly without the need of additional constraints can be shown with the help of the following example as shown in Table:2. The example takes into account the cost data given for generator 1 for time-step 1. The generator has the ability to generate power in four blocks each amounting to a maximum capacity of 10, 20, 30 and 90 MW respectively. Different combinations for $b_{i,f}$ are tried and their respective total operating costs are calculated.

As it can be seen that though all the options satisfy the constraints, only the option-1, which correctly fills up the power blocks has the minimum operating cost. Hence the optimization model can correctly decompose the generating unit power output without any additional constraints.

IV The power balance at each node(n) of the network for the given time-step=t is given by eq:8.

$$\sum_{k \in K^n} PV_{k,t} + \sum_{q \in Q^n} W_{q,t} + \sum_{s \in S^n} P_{s,t}^{dis} + \sum_{i \in I^n} P_{i,t} + \sum_{l|n=\binom{receiving}{end}} f_{l,t} = \sum_{j \in J^n} D_{j,t} + \sum_{s \in S^n} P_{s,t}^{ch} + \sum_{l|n=\binom{sending}{end}} f_{l,t} \quad \forall\, n, t \tag{8}$$

Table 2: Operating cost of G1 for t=1 for different combinations

i	**f**	$\mathbf{C_{i,f}}$	$\mathbf{B_{i,f}}$	**Option-1**		**Option-2**		**Option-3**	
				$\mathbf{b_{i,f}}$	**cost**	$\mathbf{b_{i,f}}$	**cost**	$\mathbf{b_{i,f}}$	**cost**
1	1	22.964	10	10	229.64	10	229.64	5	114.82
1	2	23.97	20	20	479.4	10	239.7	20	479.4
1	3	27.842	30	30	835.26	20	556.84	30	835.26
1	4	32.015	90	90	2881.35	20	640.3	5	160.075
Total cost for each option					**4425.65**		**1666.48**		**1589.555**

By this equation, the generation and demand at each individual busses are equated. The generation at each node is the sum of power generation of solar, wind, energy storage (discharging), conventional generator and power received through the transmission lines connected to that node. Where as the demand of the node includes the sum of power consumption of load, energy storage (charging) and sending of power to other nodes from that particular node.

If the network constraints are to be neglected, then it will be enough to equate the system's total generation with the system's total power demand. By this approach we are neglecting any losses occurred in the network and also ignore powers in the transmission lines as they do not add or reduce the network's capacity at large. This can be mathematically shown as shown in eq:9.

$$\sum_{n}\left[\sum_{k\in K^n} PV_{k,t} + \sum_{q\in Q^n} W_{q,t} + \sum_{s\in S^n} P^{dis}_{s,t} + \sum_{i\in I^n} P_{i,t}\right] = \sum_{n}\left[\sum_{j\in J^n} D_{j,t} + \sum_{s\in S^n} P^{ch}_{s,t}\right] \quad \forall\, n,t \tag{9}$$

2 Implementation of the Optimization Model using Python

I Implementation

I.1 The parameters are defined in the python notebook under **"Define Parameters"**.

I.2 The initialization of parameters related to the generators and total system load are defined in the python notebook under **"Initialize Parameters"**.

I.3 All the decision variables are defined in the python notebook under **"Define"**.

I.4 The given constraints are are defined in the python notebook under **"Define Constraints"**.

The implemented model is tested with the validation input files and the answers are in tally with the provided solutions.

II Basic Simulation

II.1 [**A**] The total operating cost for a day for the given three cases are as follows:

- **Case 1:** € *2655566.701*
- **Case 2:** € *3406743.352*
- **Case 3:** €*2675324.913*

It can be clearly seen that the **case 1** has the lowest cost as it does not have any constraints and acts as a simple economic dispatch problem. **case 3** has a higher cost than case 1 as it has constraints added to it making it a optimal power flow problem. **Case 3** has the highest operating cost, as it has the most number of constraints including line limits. This observations shows that, generally optimal power flow problems has cost equal to or higher than that of a simple economic dispatch problem. The cost will only be higher if any of the constraints are violated in the simple economic dispatch problem.

[**B**] The daily energy generation, cost and daily commitment cost for different types of generators are as shown in Table:3 for Case-1.

Table 3: Generation output and energy costs for II.1 (Case-1)

Generation Type	**Total Daily Generation [MWhr]**	**Total Energy Cost [€/day]**	**Total Commitment Cost [€/day]**
U100	2792.6	479534.6	15432.0
U12	0.0	0.00	751.5
U155	14880.0	3107926	11470.0
U197	6175.4	1076367	20156.5
U20	0.0	0	47.0
U350	8318.4	1783953	0.0
U400	19200.0	3964301	0.0
U50	7200.0	0	0.0
U76	6333.6	1661906	0.0

This is the unconstrained case which will reveal the ideal situation in which the most efficient operating units are used. It can be noticed that generating units **U12** and **U20** which represent Oil/Steam and Oil/Combustion Turbine respectively are not generating any power. This can be directly seen from the input data, since these units have the highest cost of energy of the power blocks. However, these units are still incurred by a shut down commitment cost due to some of them

Table 4: Generation output and energy costs for II.1 (Case-2)

Type	Total Daily Generation [MWhr]	Total Energy Cost [€/day]	Total Commitment Cost [€/Day]
U100	5337.249446	947379.7	7716.0
U12	0.000000	0.000000	751.5
U155	14562.0	303770.3	11470.0
U197	8300.872048	1453849	11518.0
U20	63.200000	14723.20	188.0
U350	8361.000000	179396.1	0.0
U400	13780.164306	282365.5	0.0
U50	7200.000000	0.000	0.0
U76	7295.514200	193615.9	0.0

Table 5: Generation output and energy costs for II.1 (Case-3)

Type	Total Daily Generation [MWhr]	Total Energy Cost [€/day]	Total Commitment Cost [€/Day]
U100	2842.6	489218.7	15432.0
U12	29.0	6323.3210	751.5
U155	14810.0	309222.5	11470.0
U197	6189.4	1078875	20156.5
U20	16.0	372.8715	47.0
U350	8279.4	177479	0.0
U400	19200.0	396430	0.0
U50	7200.0	0	0.0
U76	6333.6	166190.6	0

being initially operated. All the units which have 0 total commitment cost were initially on and were not shut down. The most inefficient active operating unit is **U100** which is the Oil/Steam generator. This is the most expensive unit of the operating units and is the last in the merit order. Finally, the most cost efficient generating unit is the **U50** which is the Hydro power. This is expected since Hydro is not subjected to operating costs.

Different situation is expected in the non-ideal constraint cases 2 and 3 as shown in Table:4 and Table:5 respectively. As expected, due to violation of network constraints, also the more costly and inefficient power generating units have to be operated.

II.2 The heat maps demonstrating line loading percentage for case-2 and case-3 are shown in Fig:1 and Fig:2 respectively. As defined in the cases, case-2 has transmission limit constraints added to it. This constraint makes sure that, non of the transmission lines among the buses are overloaded. By overloading it means thee line loading is less than 100%. This can be evidently seen in Fig:1, where non of the lines are shown red in colour for any given hour. Especially the lines 25, 23 and 22 which are near their maximum capacity but are not exceeding the maximum limits.

In the other hands, for case-3 as there are no constraints attached with the transmission lines, the lines 25, 23 and 22 which previously were near their maximum limits are overloaded now. This happens because the optimization solver chooses the option which gives the system cost as minimum without checking the line loading. This is also evident from the total system operating costs reported in the previous question.

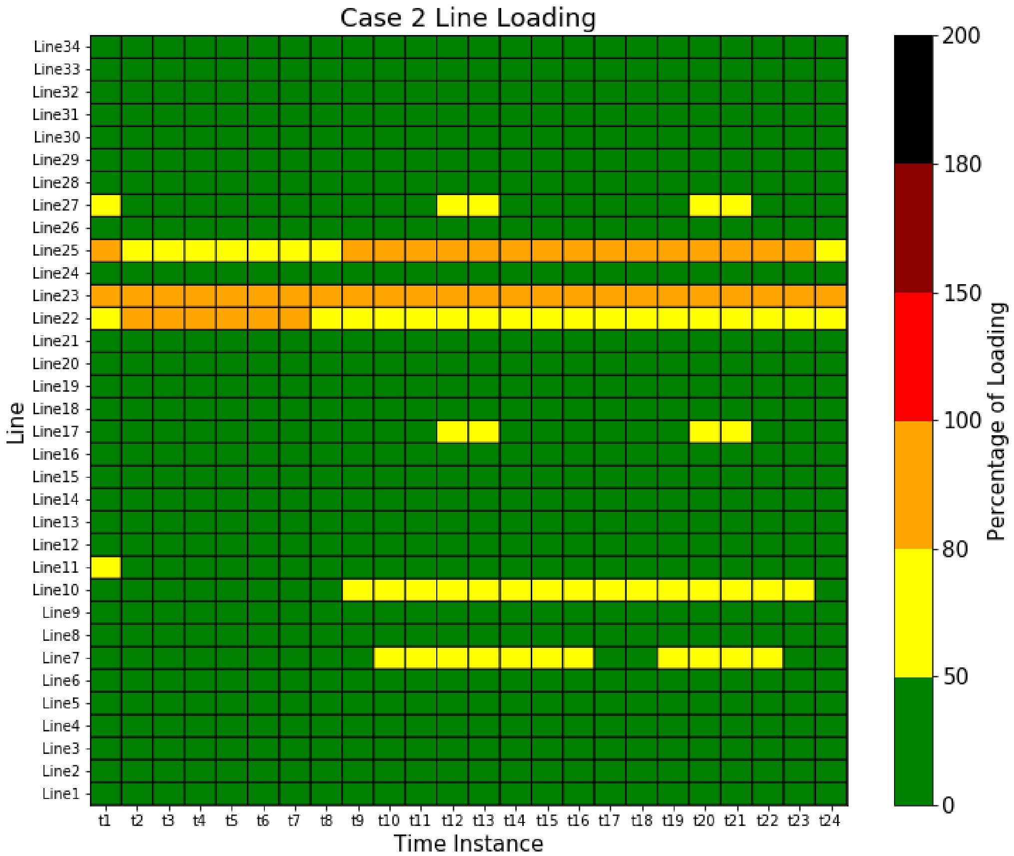

Figure 1: Heat-map showing line loading for **case-2**

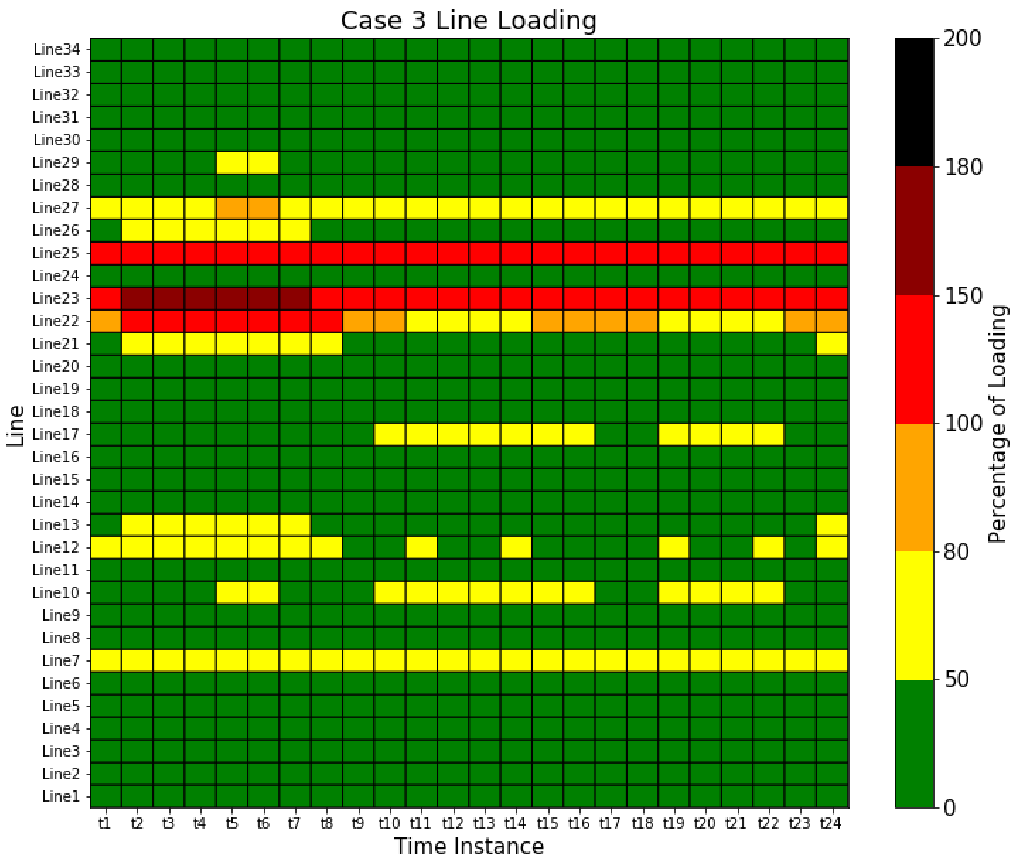

Figure 2: Heat-map showing line loading for **case-3**

III The effect of renewable generation and the ESS

III.1 [**A**] The variation of state-of-energy of the ESS together with the original total system load demand for case-3 is as shown in Fig:3. For the case with no transmission line limits, the energy storage is in **charging mode** whenever the total generation exceeds the system demand at that period of time. This **charging regime** is denoted in the picture by **increase in state of charge**. As it can be visualised, during first 5 hours, when the system demand ramps down, the battery is charged and there is a ramping of the state of energy as the stat of charge is increasing. Similarly when the system demand keeps on increasing the storage is drained and the state of energy falls.

However there are clear differences between the scenarios with only PV and Wind respectively for the same case-3. This can be explained by the generation profile given in the assignment. It can be seen that wind somewhat has an uniform generation profile, while PV has a u-shaped generation which peaks during the noon. So we can see in Fig:3, that during first four hours, though the system load falls, due to no generation of solar the battery is yet to charge. But this is not the case for wind only as there is some generation during the first four hours.

[**B**] The variation of state-of-energy of the ESS together with the original total system load demand for case-2 is as shown in Fig:3. In this part, along with the constraints of case-3, there is an additional constraint on the transmission line limits. For this scenario, the. characteristics of the ESS remain similar. However due to the new configuration of generator power dispatch, there is additional power availability which charges the energy storage system in spite of no generation of PV during first four hours. Hence we could see the state of energy of the ESS ramping up for both PV and wind during the period with decreasing system demand. However the charging rate is faster in case of only wind scenario due to the addition of wind power with the already available power. However during the noon time, despite much availability of power from PV, the battery is in discharge regime. This is because due to the line loading constraints of the transmission lines feeding power to the bus-14, the battery has to contribute to part of the demand of this particular bus as other generators cannon safely send power here.

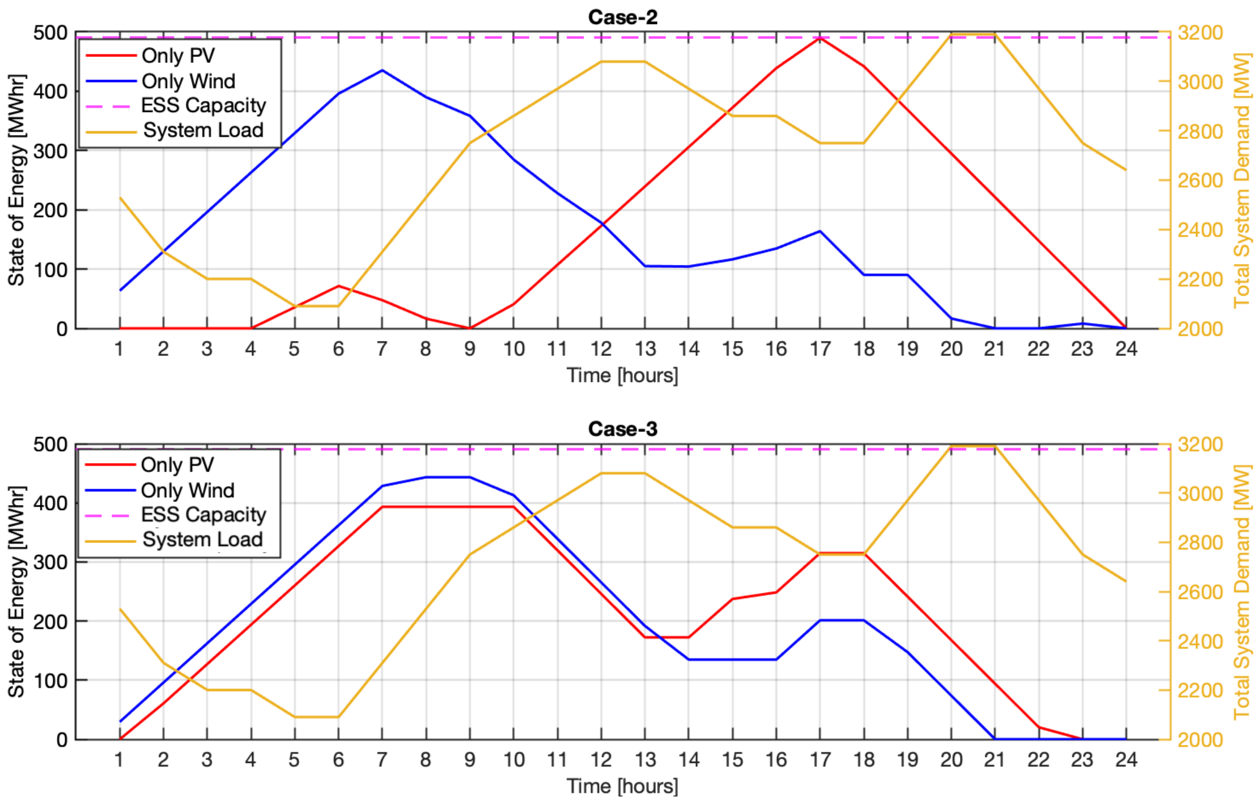

Figure 3: State of energy of storage for different configuration of energy sources.

III.2 Table:6 shows the system cost comparison for all the different scenarios. It can be easily seen that for each case addition of **PV park** or **Wind park** reduces the total system cost. The addition of **energy storage** further brings down the costs of the system. Finally, increase in power rating of the energy storage facility yields further cost reduction due to increased flexibility. It should be also noted that the influence of Wind park on the cost reduction is higher compared to the Solar park it contributes more given day and injects significantly more power to the system. Finally, as expected, the total system costs are higher in Case 2 for all the situations. This is because all the network constraints are present. However, for wind generation in combination with energy storage this difference is not as significant. This is because the distribution of the wind generation is equally distributed throughout the day. Pure solar generation creates **Duck Curve** which causes congestion in the network, driving prices higher.

Table 6: Cost analysis for cases two and three taking into account different scenarios.

Cases	**Total System Costs [Euror]**			
	No Renewable	**Only PV**	**Only PV with ESS (7h discharge)**	**Only PV with ESS (1h discharge)**
2	3406743.352	2714416.944	2628670.513	2621953.933
3	2675324.913	2225935.791	2155231.547	2135396.990
	No Renewable	**Only Wind**	**Only Wind with ESS (7h discharge)**	**Only Wind with ESS (1h discharge)**
2	3406743.352	2251186.642	2159255.453	2140127.551
3	2675324.913	2084572.144	2006599.888	1991522.549