# 哈尔滨工业大学

## 2018年硕士研究生《控制原理》考研试题 试题参考答案

**报考专业：控制科学与工程、控制工程** 考试科目代码：【 801 】

一、（1）$u+mg\sin\theta_0 l=\left(ml^2\right)\ddot{\theta}_0$

（2）小球在$\theta_0=0$点附近摆动

根据泰勒级数展开：

$$\sin\theta_0=\theta_0-\frac{\theta_0^3}{3!}+\frac{\theta_0^5}{5!}-\cdots$$

$\theta_0$很小时，高阶项可忽略。

$$\sin\theta_0\approx\theta_0$$

$$\therefore mg\theta_0 l+u=ml^2\ddot{\theta}_0$$

（3）拉氏变换得：

$$mgl\theta_0(s)+U(s)=ml^2s^2\theta_0(s)$$

$$\frac{\theta_0(s)}{U(s)}=\frac{1}{ml^2s^2-mgl}$$

极点：$ml^2s^2-mgl=0$，无零点

$$S_{1.2}=\pm\sqrt{\frac{g}{l}}$$

二、开环传函$G(S)=\dfrac{KS+K_1}{S(S+1)(S+2)}$

闭环传函$\Phi(S)=\dfrac{G(S)}{1+G(S)}=\dfrac{KS+K_1}{S^3+3S^2(2+K)S+K_1}$

特征方程$D(S)=S^3+3S^2+(2+K)S+K_1=0$

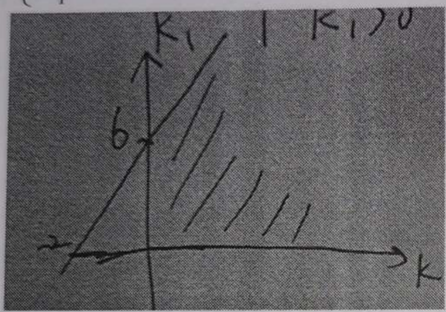
$S^3$ 1 （2+K）

$S^2$ 3 $K_1$

$S^1$ $\dfrac{3(2+K)K_1}{3}$

$S^0$ $K_1$

$$\begin{cases}\dfrac{6+3K-K_1}{3}>0\\ K_1>0\end{cases}$$




三、 开环传函

$$G(S)=\frac{K\left(S^2+S+1\right)}{S^3}$$

极点$P_1=P_2=P_3=0$零点：$Z_{l2}=-\dfrac{1}{2}\pm\dfrac{\sqrt{3}}{2}j$

n=3 m=2

实轴上根轨迹：$(-\infty,0)$

渐进线$\Phi=\pi$

出射角：$3\theta_P=\pm(2l+1)\pi+\sum_{i=1}^{2}\angle(P-Z_i)$

$$=\pm(2l+1)\pi$$

$\therefore \angle\theta P_1=60^{\text{O}}$ $\angle\theta P_2=180^{\text{O}}$ $\angle\theta P_3=-60^{\text{O}}$

入射角 $\angle\theta Z_1=\pm(2l+1)\pi+\sum_{j=1}^{3}\angle(Z_1-P_j)-(Z_1-Z_2)=90^{\text{O}}$

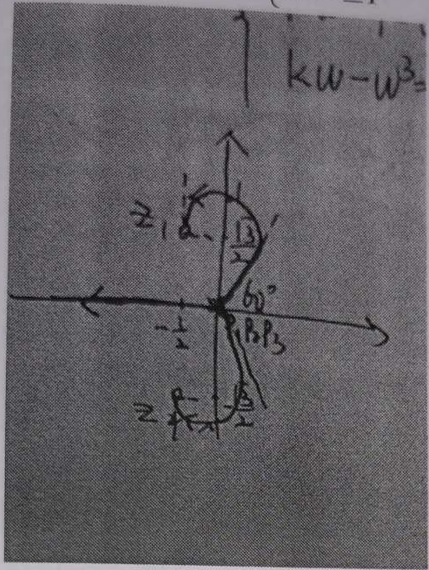
$\angle\theta Z_2=-90^{\text{O}}$

与虚轴交点$S=jw$代入 $1+G(S)=0$中

$$S^3+KS^2+KS+K=0$$

$$-W^3 j - KW^2 + Kj W + K = 0$$

$$\begin{cases} kw^2 - 1 = 0 \\ kw - w^3 = 0 \end{cases} \Rightarrow \begin{cases} k = 1 \\ w = \pm 1 \end{cases} \therefore \pm j$$




四、 零极点对消，取 Z=3

$$\therefore G_0(S) = \frac{K}{S(S+P)},\ 为\ 2\ 阶系统$$

$$闭环\ \Phi(S) = \frac{K}{S^2 + PS + K}$$

$$\left.\begin{aligned} \sigma_p &= e^{-\frac{\pi\varsigma}{\sqrt{1-\varsigma^2}}} \times 100\% = 5\% \\ t_s &= \frac{4}{\xi\omega_n} \le 1.15 \end{aligned}\right\} \Rightarrow 解得 \begin{cases} \xi = 0.69 \\ w_n \ge 5.04 \end{cases}$$

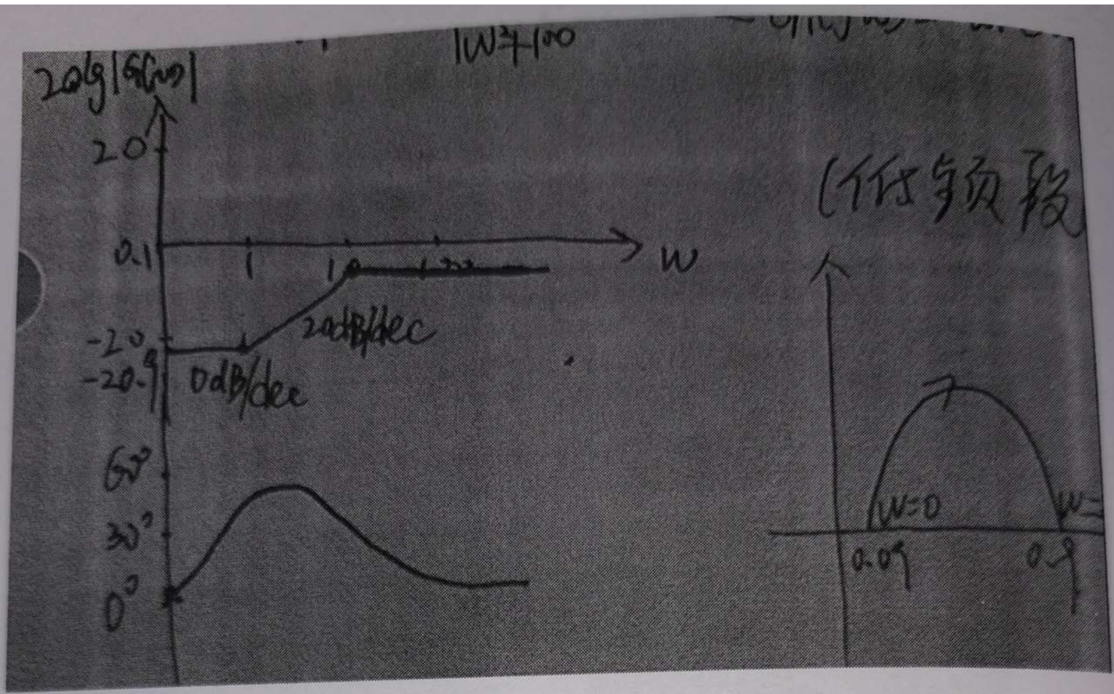
$\therefore$ 可取 $w_n = 5.2$

$$\begin{cases} 2\xi w_n = p \\ w_n^2 = k \end{cases} \therefore \begin{cases} k = 27.04 \\ p = 7.176 \end{cases}$$

五、 $G_1(S) = \dfrac{0.9(S+1)}{S+10}$

$$|G_1(jw)| = \frac{0.9\sqrt{w^2+1}}{\sqrt{w^2+100}} \qquad \angle G_1(jw) = \arctan w - \arctan\frac{w}{10}$$

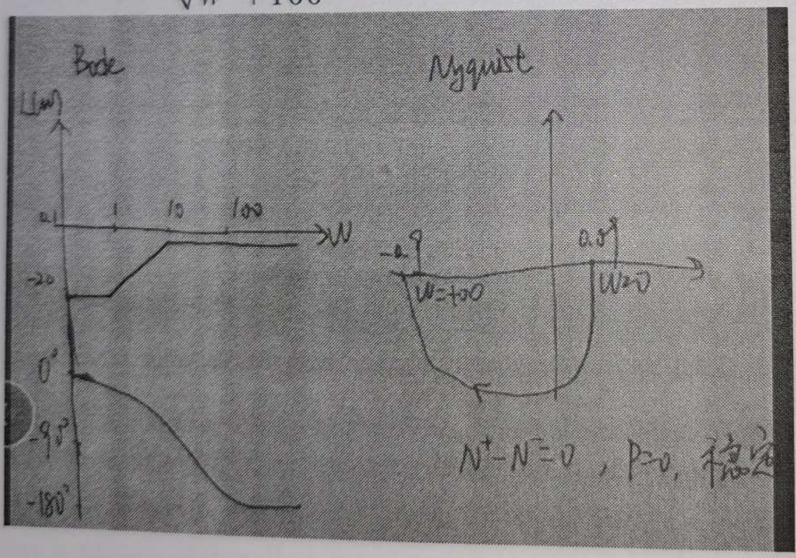
（低频段或其延长线经过$(1, 20\lg k)$）




$N^{+}-N^{-}=0,P=0$，稳定

$$G_2(S)=-\frac{0.9(S-1)}{S+10}$$

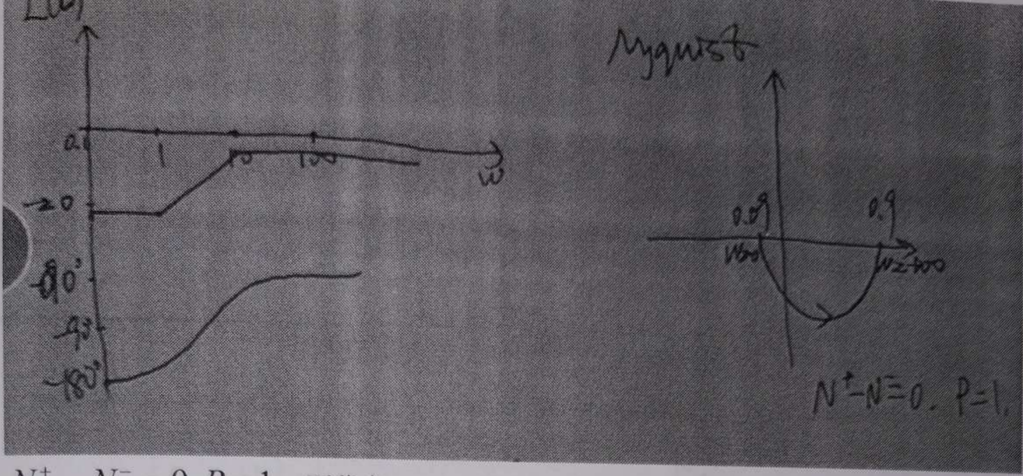
$$|G_2(jw)|=\frac{0.9\sqrt{w^2+1}}{\sqrt{w^2+100}} \qquad \angle G_2(jw)=-\arctan w-\arctan\frac{w}{10}$$




$N^{+}-N^{-}=0,P=0$　稳定

$$G_3(S)=-\frac{0.9(S+1)}{S-10} \qquad \angle G_3(jw)=\arctan w-\left(180^{\circ}-\arctan\frac{w}{10}\right)$$

$$=\arctan w+\arctan\frac{w}{10}-180^{\circ}$$




$N^+ - N^- = 0, P = 1$，不稳定

六、（1）$\dfrac{d\dot{x}}{dx} = -\dfrac{0.6\dot{x} + 3x + x^2}{\dot{x}}$

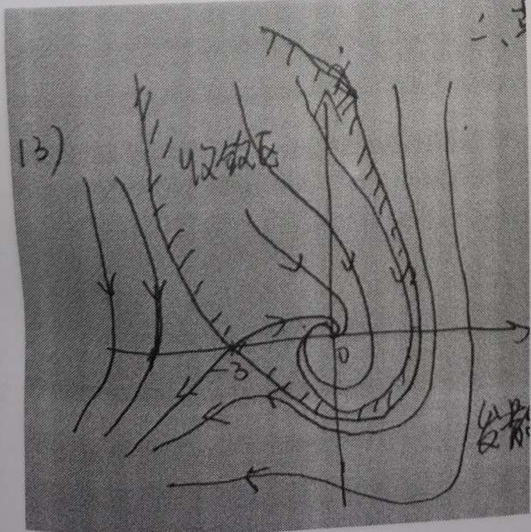
分子=0，分母=哦，得 $\begin{cases} x = 0 \\ \dot{x} = 0 \end{cases}$ 或 $\begin{cases} x = -3 \\ \dot{x} = 0 \end{cases}$

（2）$\dfrac{\partial f}{\partial \ddot{x}} = 1$ $\dfrac{\partial f}{\partial \dot{x}} = 0.6$ $\dfrac{\partial f}{\partial x} = 3 + 2x$

在（0, 0）处，$\ddot{x} + 0.6\dot{x} + 3x = 0, \lambda_{1,2} = \dfrac{-0.6 \pm \sqrt{0.6^2 - 12}}{2} = \dfrac{-0.6 \pm 3.4j}{2}$，稳定焦点

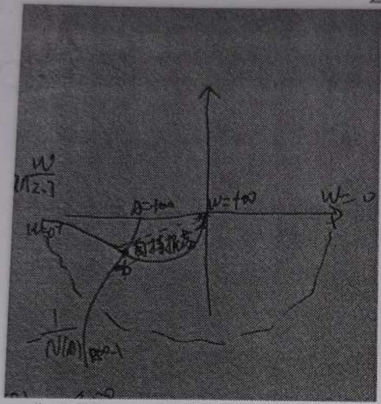
在（-3, 0）处 $\ddot{x} + 0.6\dot{x} - 3x = 0$ $\lambda_{1,2} = \dfrac{-0.6 \pm \sqrt{12.36}}{2}$

$\lambda_1 = 1.46$ $\lambda_2 = -2.06$ 鞍点



七、（1）$G(jw)=\dfrac{10(jw+2.7)}{2.7(jw)^2}$

$\angle G(jw)=-180^{\circ}+\arctan\dfrac{w}{2.7}$




∴会出现
自持振荡点 b

（2）$\dfrac{a}{A}=0.7$ 时，$\angle N(A)=-45^{\circ}$

当 $\lambda_1=\angle G(jw)=-135^{\circ}$ 时，$w=2.7$ $\left|G(jw)=1.93,N(A)\right|=0.35$

$-\dfrac{1}{N(A)}$ 在 $G(jw)$ 外侧

$\dfrac{a}{A}=0.8$ 时，$\angle N(A)=-85^{\circ}$

当 $\angle G(jw)=-95^{\circ}$ 时，$\lambda w=30.86$ $\quad \left|G(jw)\right|=0.12 \quad N(A)=0.25$

$-\dfrac{1}{N(A)}$ 仍在 $G(jw)$ 外侧

$\dfrac{a}{A}=0.6$ 时 $\quad \angle N(A)=-40^{\circ}$

$\angle G(jw)=-135^{\circ}$ 时 $\quad w=2.266 \quad \left|G(jw)\right|=2.54 \quad N(A)=0.47$

$G(jw)$ 在 $-\dfrac{1}{N(A)}$ 外侧

$\therefore \dfrac{a}{A}\approx 0.65$，$A=0.15, W\approx 2.52$

八、 $G(Z)=\left(1-Z^{-1}\right)Z\left[\dfrac{G(S)}{S}\right]$

$$=\left(1-Z^{-1}\right)Z\left[\frac{K}{S^2(S+1)}\right]$$

$$=K\left(1-Z^{-1}\right)\left(\frac{TZ}{(Z-1)^2}-\frac{Z}{Z-1}+\frac{Z}{Z-e^{-T}}\right)=\frac{0.1K(Z+1)}{(Z-1)(Z-0.6)}$$

$$D(Z)=Z^2+(0.1K-1.6)Z+0.1K+0.6=0$$

$$\begin{cases}D(1)>0\\ D(-1)>0\\ \left|D(0)\right|<1\end{cases}$$ ∴0<k<4 时稳定，临界值为 k=4

九、（1）$\begin{bmatrix}B & AB\end{bmatrix}=\begin{bmatrix}0 & 1\\ 1 & -\alpha-\beta\end{bmatrix}$ $\quad rank\begin{bmatrix}B & AB\end{bmatrix}=2$，可控

$$\begin{bmatrix}C\\ CA\end{bmatrix}=\begin{bmatrix}\gamma & 1\\ -\alpha\beta & \gamma-\alpha-\beta\end{bmatrix}\qquad \begin{bmatrix}C\\ CA\end{bmatrix}=\gamma^2-\gamma\alpha-\gamma\beta+\alpha\beta=0\text{时，不可观}$$

$$=\gamma^2-\gamma\alpha-\gamma\beta+\alpha\beta\neq 0\text{时，可观}$$

（2）$\gamma=\alpha$，可控性不变：可控

可观性：$rank\begin{bmatrix}C\\ CA\end{bmatrix}=rank\begin{bmatrix}\alpha & 1\\ -\alpha\beta & -\beta\end{bmatrix}=1$，不可观

（3）$\gamma\neq\alpha$ 时

$$G(S)=C(SI-A)^{-1}B=\begin{bmatrix}\gamma & 1\end{bmatrix}\begin{bmatrix}S & -1\\ \alpha\beta & S+\alpha+\beta\end{bmatrix}^{-1}\begin{bmatrix}0\\ 1\end{bmatrix}$$

$$=\frac{S+\gamma}{(S+\alpha)(S+\beta)},\quad \begin{matrix}\text{零点}:-\gamma\\ \text{极点}:-\alpha,-\beta\end{matrix}$$

$$\gamma=\alpha\text{时}\quad G(S)=\frac{1}{S+\beta}\quad \begin{matrix}\text{极点}:-\beta\\ \text{无零点}\end{matrix}$$

十、（1）能控性：$rank\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}=2$，能控

$$\dot{x}=Ax+B(-kx)$$
$$=(A-Bk)x$$

设 $k=\begin{bmatrix}k_1 & k_2\end{bmatrix}$

$$A-BK=\begin{bmatrix}0 & 1\\ -w_0^2-k_1 & -k_2\end{bmatrix}\quad \left|\lambda I-(A-Bk)\right|=\begin{vmatrix}\lambda & -1\\ w_0^2+k_1 & \lambda+k_2\end{vmatrix}$$

$$=\lambda^2+K_2\lambda+W_0^2+K_1$$

期望 $f^*(\lambda)=(\lambda+2w_0)^2=\lambda^2+4w_0\lambda+4w_0^2$

$\therefore \begin{matrix} K_1 = 3w_0^2 \\ K_2 = 4w_0 \end{matrix}$

$\therefore K = \begin{bmatrix} 3w_0^2 & 4w_0 \end{bmatrix}$

（2）能观性

$rank\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 2$，能观

$$\dot{\hat{x}} = (A - LC)\hat{x} + Bu + Ly$$

设 $L:\begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$

$$A - LC = \begin{bmatrix} -L_1 & 1 \\ -W_0^2 - L_2 & 0 \end{bmatrix} \quad |\lambda I - (A - LC)| = \begin{vmatrix} \lambda + l_1 & -1 \\ w_0^2 + l_2 & \lambda \end{vmatrix}$$

$$= \lambda^2 + l_1\lambda + W_0^2 + l_2$$

期望 $f^*(\lambda) = (\lambda + 10w_0)^2 = \lambda^2 + 20w_0\lambda + 100w_0^2$

$\therefore \begin{matrix} l_1 = 20w_0 \\ l_2 = 99w_0^2 \end{matrix}$

$\therefore L:\begin{bmatrix} 20w_0 \\ 99w_0^2 \end{bmatrix}$

(3)