教材162页1,2; 丘老师书上册43-44页1(2,4),2,4,6.

162-1

Use the classical adjoint formula to compute the inverse of each of the following 3×3 real matrices.

$$A = \begin{pmatrix} -2 & 3 & 2 \\ 6 & 0 & 3 \\ 4 & 1 & -1 \end{pmatrix}, \; B = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}$$

Solution: $\det A = 72$. $A^* = \begin{pmatrix} -3 & 18 & 6 \\ 5 & -6 & 14 \\ 9 & 18 & -18 \end{pmatrix}$, so $A^{-1} = \frac{1}{72}A^*$.

$\det B = 1$, $B^* = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} = B$ so $B^{-1} = B$.

162-2

Use Cramer's rule to solve each of the following systems of linear questions over $\mathbb{Q}$:

$$(a): \begin{cases} x + y + z = 11 \\ 2x - 6y - z = 0 \\ 3x + 4y + 2z = 0 \end{cases}, \; (b): \begin{cases} 3x - 2y = 7 \\ 3y - 2z = 6 \\ 3z - 2x = -1 \end{cases}$$

Solution: (a): $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -6 & -1 \\ 3 & 4 & 2 \end{pmatrix}$, $\det A = 11$. The solutions are $(-8, -7, 26)$.

(b) $B = \begin{pmatrix} 3 & -2 & 0 \\ 0 & 3 & -2 \\ -2 & 0 & 3 \end{pmatrix}$, $\det B = 19$. The solutions are $(63, 76, 57)$.

43-1(2)

Calculate

$$\begin{vmatrix} 2 & -4 & -3 & 5 \\ -3 & 1 & 4 & -2 \\ 7 & 2 & 5 & 3 \\ 4 & -3 & -2 & 6 \end{vmatrix}$$

Solution: The answer is -100.

43-1(4)

Calculate

$$\begin{vmatrix} \lambda - 2 & -3 & -2 \\ -1 & \lambda - 8 & -2 \\ 2 & 14 & \lambda + 3 \end{vmatrix} = \lambda^3 - 7\lambda^2 + 15\lambda - 9.$$

43-2

Calculate

$$\begin{vmatrix} a_1 & a_2 & a_3 & \cdots & a_{n-1} & a_n \\ 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 2 & -2 & \cdots & 0 & 0 \\ & & & \cdots & & \\ 0 & 0 & 0 & \cdots & n-1 & 1-n \end{vmatrix}$$

Solution:

$$= \begin{vmatrix} a_1 & a_2+a_1 & a_3 & \cdots & a_{n-1} & a_n \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 2 & -2 & \cdots & 0 & 0 \\ & & & \cdots & & \\ 0 & 0 & 0 & \cdots & n-1 & 1-n \end{vmatrix} = \cdots = (-1)^{n-1}(a_1+\cdots+a_n)(n-1)!.$$

43-4

Prove that for $n \geqslant 2$,

$$\begin{vmatrix} x & 0 & 0 & \cdots & 0 & 0 & a_0 \\ -1 & x & 0 & \cdots & 0 & 0 & a_1 \\ 0 & -1 & x & \cdots & 0 & 0 & a_2 \\ & & & \cdots & & & \\ 0 & 0 & 0 & \cdots & -1 & x & a_{n-2} \\ 0 & 0 & 0 & \cdots & 0 & -1 & x+a_{n-1} \end{vmatrix} = x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0.$$

Solution: Expand the determinant along the n^{th} column, we obtain the right hand side.

44-6

Calculate

$$\begin{vmatrix} 2a & a^2 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 2a & a^2 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 2a & a^2 & \cdots & 0 & 0 & 0 \\ & & & & \cdots & & & \\ 0 & 0 & 0 & 0 & \cdots & 1 & 2a & a^2 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 2a \end{vmatrix}$$

Solution: Let the result be A_n, then $A_n = 2aA_{n-1} - a^2 A_{n-2}$. Hence $A_n = a^n(An+B)$ while $A_1 = 2a$, $A_0 = 1$, so $A_n = a^n(n+1)$.