Numerical Experiments for “OPTIMAL BATCHING UNDER COMPUTATION BUDGET”

We set $\psi(P)$ as the 0.7 quantile of $X \sim P$ where the true distribution is standard log-normal. We set the sample size as $n = 3000$, nominal level $\alpha = 90\%$, and try the following methods under different computational budgets ($K = 6, 12, 17$) measured by the number of times we can compute ψ:

1. Standard batching: CI_B as introduced in Section 3.
2. Batching with general batch sizes: $CI_B^{(\gamma)}$ as introduced in Theorem 2. We use $\gamma_i = \frac{i}{K(K+1)/2}$.
3. Overlapping batching: CI_{OB} as introduced in Theorem 4. The K batched estimates $\psi_1, \ldots, \psi_K$ are constructed as follows. $\psi_1 = \psi(X_1, \ldots, X_n)$ is the estimation using the entire empirical distribution.
$$\psi_j = \psi(X_{(j-2)n(1-\gamma)/(K-2)+1}, \ldots, X_{(j-2)n(1-\gamma)/(K-2)+\gamma n}), 2 \leq j \leq K.$$
Here γ is set as 0.3. Note that γ measures the batch size (the size of each batch is γn).
4. Cheap bootstrap: CI_{CB} introduced in Section 6 with $K-1$ resampled estimates (subtracting one since the estimation using the entire empirical distribution already used one budget)
5. Overlapping batching (OB-I) as in Su et al. [2023]. The batched estimates are the same as our overlapping batching outlined in item 3 above. The CI in their construction is
$$CI_{OB-I} := \psi_1 \pm c_{\beta,b,\alpha_1} \sqrt{\frac{\beta}{1-\beta} \frac{1}{K-1} \sum_{j=2}^{K} (\psi_j - \psi_1)^2}.$$
Here, the critical values c_{β,b,α_1} can be checked from Fig 3 from Su et al. [2023] and in correspondance with our settings in 3, we use $\beta = 0.3, b = K-1, \alpha_1 = 0.95$.

For each method, we replicate 10^5 times to find its empirical coverages and half lengths. The results are shown in Table 1. We can check that the empirical coverages are all close to the nominal coverage: the largest gap is observed for CI_{OB} when $K = 17$ where there is an overcoverage of about 1.3%. This suggests that the sample size is large enough for our asymptotic argument to be true. If we compare the half lengths of different methods, we find that 1) the half lengths of $CI_B, CI_B^{(\gamma)}$, and CI_{CB} are very close, which agrees with our theoretical findings. 2) the half length of CI_{OB} is longer than $CI_B, CI_B^{(\gamma)}$, and CI_{CB}. For example, when $K = 6$, the half length of CI_{OB} is 0.082 where the half lengths of $CI_B, CI_B^{(\gamma)}$, and CI_{CB} are all 0.078. This difference can be explained by its higher coverage, which is a result of finite-sample errors as we discussed. 3) The half length of CI_{OB-I} is even longer than CI_{OB}. For example, when $K = 12$, the half length of CI_{OB-I} is longer than CI_{OB} even though its coverage is smaller than CI_{OB}. This is expected as the construction in Su et al. [2023] does not have the optimality guarantee in terms of CI accuracy as we do.

method	$K=6$		$K=12$		$K=17$	
	coverage	half length	coverage	half length	coverage	half length
CI_B	90.0%	0.078	90.0%	0.071	90.0%	0.070
$CI_B^{(\gamma)}$	90.0%	0.078	89.9%	0.071	90.0%	0.070
CI_{OB}	90.5%	0.082	91.1%	0.076	91.3%	0.075
CI_{CB}	89.4%	0.078	89.6%	0.072	89.9%	0.071
CI_{OB-I}	90.7%	0.086	90.3%	0.080	90.3%	0.079

Table 1: Empirical coverages and half lengths for each batching method. The uncertainty of these empirical estimates (measured by the 95% CI half length) is less than 0.2% for the empirical coverage and is less than 3×10^{-4} for the estimation of the half length.

References

Ziwei Su, Raghu Pasupathy, Yingchieh Yeh, and Peter W. Glynn. Overlapping batch confidence intervals on statistical functionals constructed from time series: Application to quantiles, optimization, and estimation, 2023.