Quick Review

1. Here are the row reductions of five $m \times n$ matrices into reduced row echelon form.

$$A \to \begin{bmatrix} 1 & 0 & 5 & -3 & 0 \\ 0 & 1 & -2 & 8 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad B \to \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad C \to \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad D \to \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad E \to \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Let $T_A : \mathbb{R}^n \to \mathbb{R}^m$ be the linear transformation whose matrix is A. That is, $T_A(x) = Ax$. Determine the appropriate values for n and m, and decide whether T_A is one-to-one and/or onto and whether the columns span $\mathbb{R}^m$ and are linearly independent. Do the same for B, C, D, E.

transformation	n	m	one-to-one?	onto?	columns span $\mathbb{R}^m$	columns are linearly independent
T_A						
T_B						
T_C						
T_D						
T_E						

2. $T : \mathbb{R}^3 \to \mathbb{R}^2$ is the linear transformation defined by the formula below. Find its matrix. .

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{3}(x_1 + x_2 + x_3) \\ x_3 - (x_1 + x_2) \end{bmatrix}$$

Matrix Multiplication

1. Fill in the missing entry:

$$\mathsf{AB} = \underbrace{\begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 4 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}}_{\mathsf{A}} \underbrace{\begin{bmatrix} 1 & 1 \\ 2 & -1 \\ 0 & -1 \\ 1 & 1 \end{bmatrix}}_{\mathsf{B}} = \underbrace{\begin{bmatrix} 2 & -2 \\ 6 & \\ 2 & -2 \end{bmatrix}}_{\mathsf{AB}} = \begin{bmatrix} \mathsf{A}\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} & \mathsf{A}\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \end{bmatrix}$$

2. Matrix multiplication corresponds to composition of functions. Draw a picture:

3. Suppose that a transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ first rotates by $\pi/6$ counter-clockwise and then reflects over $y = x$ (i.e., exchanges the x and y coordinates). Find the matrix of T in two ways.

   (a) First follow the standard basis vectors through this process:

   (b) Then make the rotation matrix and the reflection matrix and multiply them.