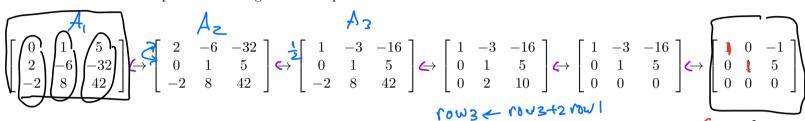
Row Space. The row space, Row(A), is the span of the rows of A.

1. Consider the row space:

$$S = \operatorname{Row}\left(\left[\begin{array}{ccc} 0 & 1 & 5 \\ 2 & -6 & -32 \\ -2 & 8 & 42 \end{array}\right]\right) = \operatorname{span}\left\{\left[\begin{matrix} 0 \\ 1 \\ 5 \end{matrix}\right], \left[\begin{matrix} \mathbf{2} \\ \mathbf{-6} \\ \mathbf{-32} \end{matrix}\right], \left[\begin{matrix} \mathbf{-2} \\ \mathbf{8} \\ \mathbf{42} \end{matrix}\right]\right\} \subseteq \mathbb{R}^3.$$

How do row operations change the row space?



Rey Points:

Now operations do not change the now space

Row (A) = Row (RREF (A))

Besis for Row (A) is the set of nonzer nows in RREF(A)

Alm (Row (A)) = # pivots (A) = rank (A)

row operations mess up the column space
The tell us about relations a mong the columns  $C_{2} = (-1)C_{1} + 5C_{2}$ 

2. Here is the transpose of the matrix above row reduced. How does this compare to the row space above?

$$\begin{bmatrix} 0 & 2 & -2 \\ 1 & -6 & 8 \\ 5 & -32 & 42 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

3. Let  $S \subseteq \mathbb{R}^4$  be the span of the vectors below. Find the dimension of S and find a basis for S.

$$S = \operatorname{span} \left\{ \begin{bmatrix} 1\\1\\2\\4 \end{bmatrix}, \begin{bmatrix} 1\\-1\\1\\-1 \end{bmatrix}, \begin{bmatrix} -1\\-3\\-3\\-9 \end{bmatrix}, \begin{bmatrix} 2\\4\\5\\13 \end{bmatrix}, \begin{bmatrix} 1\\0\\0\\1 \end{bmatrix} \right\}.$$

(a) Method 1: put the vectors of S into the columns of a matrix A and row reduce:

$$A = \begin{bmatrix} 1 & 1 & -1 & 2 & 1 \\ 1 & -1 & -3 & 4 & 0 \\ 2 & 1 & -3 & 5 & 0 \\ 4 & -1 & -9 & 13 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & -2 & 3 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) Method 2: put the vectors of S into the  $\mathcal{M}$  of a matrix A and row reduce:

$$A^{T} = \begin{bmatrix} 1 & 1 & 2 & 4 \\ 1 & -1 & 1 & -1 \\ -1 & -3 & -3 & -9 \\ 2 & 4 & 5 & 13 \\ 1 & 0 & 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & \frac{7}{3} \\ 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(c) Is the vector  $\vec{\mathbf{v}}$  in S?

$$\vec{\mathsf{v}} = \begin{bmatrix} 5\\ -9\\ 12\\ -12 \end{bmatrix}$$

(d)  $\dim(S) = \operatorname{rank}(A) = \operatorname{rank}(A^T)$ .

## You Try!

Fill in the blanks:

a) 
$$rank(\mathbf{A}) = 3$$

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 i)  $Col(\mathbf{A})$  is a  $\frac{3}{2}$  dimensional subspace of  $\frac{\mathbb{R}^6}{2}$ 

b) 
$$nullity(A) = 2$$

b) 
$$\text{nullity}(\mathbf{A}) = 2$$
 ii)  $\text{Nul}(\mathbf{A})$  is a  $2$  dimensional subspace of  $\mathbb{R}^5$  c)  $\text{rank}(\mathbf{A}^T) = 3$  iii)  $\text{Row}(\mathbf{A})$  is a  $3$  dimensional subspace of  $\mathbb{R}^5$ 

c) 
$$rank(\mathbf{A}^T) = 3$$

iii) Row(**A**) is a
$$\textcircled{3}$$
 dimensional subspace of R

$$\bullet$$
d) nullity( $\mathbf{A}^{\mathrm{T}}$ ) =  $\mathbf{3}$ 

(a) nullity(
$$\mathbf{A}^{T}$$
) = 3 iv) Null( $\mathbf{A}^{T}$ ) is a  $\underline{3}$  dimensional subspace of