

# Section 1.1 (Continued)

Notation for derivatives

$y', y'', y^{(n)}$   $\leftarrow$  the  $n^{\text{th}}$  derivative

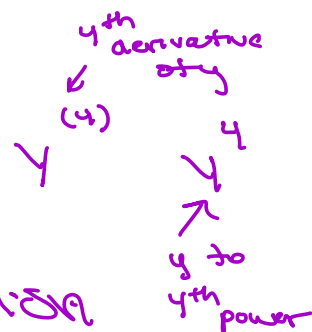
$$\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^n y}{dx^n}$$

Lagrange's notation

Leibniz' notation

$$\dot{y}, \ddot{y}, \dddot{y}$$

$\leftarrow$  Newton's notation



Solutions to DE's

- $y = e^{4x^2}$  is a solution to  $\frac{dy}{dx} = 8xy$
- $y = x^2 e^{5x}$  is a solution to  $y'' - 10y' + 25y = 2e^{5x}$

- $y = \frac{1}{x}$  is a solution to  $x \frac{dy}{dx} + y = 0$  on  $(0, \infty)$  or on  $(-\infty, 0)$

DEs do NOT have unique solutions!

Example:  $y = e^{5x}$  and  $y = 4e^{5x}$  are solutions

$$y' - 5y = 0$$

In fact,  $y = \underline{A}e^{5x}$  is a solution for any real number  $A$ .

$$y' = 5Ae^{5x}$$

$$y' - 5y = 5Ae^{5x} - 5(Ae^{5x}) = 0 \quad \checkmark$$

$$y = Ae^{5x} \quad \text{is a solution.}$$

$y = Ae^{5x}$  is called a one-parameter family of solutions to  $y' - 5y = 0$

Given an  $n^{\text{th}}$  order differential equation, we find a  $n$ -parameter family of solutions

Example:  $y = Ae^x + Bxe^x$  is a 2-parameter family of solutions to  $y'' - 2y' + y = 0$ .

A solution without any parameters is a particular solution to the DE.

Example:  $y = 2e^x - 7xe^x$  is a particular solution to

$$y'' - 2y' + y = 0.$$

$$y'(t) = \boxed{-6e^{-t}} + \boxed{7te^{-t}} - 7te^{-t} \quad \boxed{(A)}$$

$$= e^{-t} - 7te^{-t}$$

$$y'' = -e^{-t} + 7te^{-t} - 7e^{-t}$$

$$y''(t) + 2y'(t) + y(t) = 0$$

$$y = 6e^{-t} + 7te^{-t}$$