1. Below is a subspace of $\mathbb{R}^5$. Find a basis for S. And find the dimension of S.

$$S = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} \;\middle|\; c = \frac{1}{2}(a+b), d = a - b, e = a \right\} \subseteq \mathbb{R}^5. \qquad \vec{v} = \begin{bmatrix} 5 \\ 3 \\ 4 \\ 2 \\ 5 \end{bmatrix}$$

The vector $\vec{v}$ is in S. Give its coordinates with respect to your basis.

2. Below a matrix A is row reduced, and so is its transpose. Compute $\mathsf{rank}(A), \mathsf{nullity}(A)$ and find bases for $\mathsf{Col}(A), \mathsf{Row}(A), \mathsf{Nul}(A)$.

$$A = \begin{bmatrix} 1 & 3 & 2 & 3 & 7 \\ 1 & -2 & 0 & 0 & -5 \\ 0 & 4 & -1 & 5 & 7 \\ -1 & -5 & -1 & -6 & -10 \\ -1 & 1 & 2 & -3 & 5 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 2 & -1 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, A^T = \begin{bmatrix} 1 & 1 & 0 & -1 & -1 \\ 3 & -2 & 4 & -5 & 1 \\ 2 & 0 & -1 & -1 & 2 \\ 3 & 0 & 5 & -6 & -3 \\ 7 & -5 & 7 & -10 & 5 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & -\frac{11}{13} & \frac{7}{13} \\ 0 & 1 & 0 & -\frac{2}{13} & -\frac{20}{13} \\ 0 & 0 & 1 & -\frac{9}{13} & -\frac{12}{13} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The vectors below are in $\mathsf{Col}(A), \mathsf{Row}(A), \mathsf{Nul}(A)$. There is one vector in each. Which one is in which subspace?

$$\vec{u} = \begin{bmatrix} 8 \\ -1 \\ -5 \\ -3 \\ 2 \end{bmatrix}, \qquad \vec{v} = \begin{bmatrix} 13 \\ 26 \\ 0 \\ -15 \\ -33 \end{bmatrix}, \qquad \vec{\mathbf{w}} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \\ 2 \end{bmatrix},$$

3. If A is 3×3 matrix, are either of the subsets below, subspaces of $\mathbb{R}^3$? Justify your answers.

$$S_1 = \{\vec{v} \in \mathbb{R}^3 \mid A\vec{v} = -\vec{v}\} \qquad\qquad S_2 = \left\{\vec{v} \in \mathbb{R}^3 \mid A\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right\}.$$

4. Below is a basis of $\mathbb{R}^3$. Let $\mathcal{S} = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ denote the standard basis. Find the missing change of coordinates for one of the examples at the right. For the other, describe *two* ways of finding the missing entries.

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} \right\}. \qquad\qquad \begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix}_{\mathcal{B}} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}_{\mathcal{S}} \qquad \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}_{\mathcal{S}} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}_{\mathcal{B}}$$

5. See also: E2 review problems.

6. See also: PS6 problems.