Section 1.1 (Continued)

Notation for derivatives

$y', y'', y^{(n)}$ ← the n^{th} derivative

Lagrange's notation

$y^{(4)}$ ↙ 4th derivative of y

y^4 ↖ y to 4th power

$\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^ny}{dx^n}$

Leibniz' notation

$\dot{y}, \ddot{y}, \dddot{y}$ ← Newton's notation

Solutions to DE's

- $y = e^{4x^2}$ is a solution to $\frac{dy}{dx} = 8xy$
- $y = x^2e^{5x}$ is a solution to $y'' - 10y' + 25y = 2e^{5x}$
- $y = \frac{1}{x}$ is a solution to
  $x\frac{dy}{dx} + y = 0$ on $(0, \infty)$
  or on $(-\infty, 0)$

DEs do <u>NOT</u> have unique solutions!

Example: $y = e^{5x}$ and $y = 4e^{5x}$ are solutions

$$y' - 5y = 0$$

In fact, $y = \underline{A}e^{5x}$ is a solution for any real number A.

$$y' = 5Ae^{5x}$$

$$y' - 5y = 5Ae^{5x} - 5(Ae^{5x}) = 0 \checkmark$$

$y = \pi e^{5x}$ is a solution.

$\underline{y = Ae^{5x}}$ is called a one-parameter family of solutions to

$$y' - 5y = 0$$

Given an n^{th} order differential equation, we find a n-parameter family of solutions

Example: $y = Ae^{x} + Bxe^{x}$ is a 2-parameter family of solutions to

$$y'' - 2y' + y = 0.$$

A solution without any parameters is a <u>particular solution</u> to the DE.

Example: $y = 2e^{x} - 7xe^{x}$ is a particular solution to

$$y'' - 2y' + y = 0.$$

$$y'(t) = \boxed{-6e^{-t}} + \boxed{7e^{-t}} - 7te^{-t} \qquad \boxed{(A)}$$
$$= e^{-t} - 7te^{-t}$$

$$y'' = -e^{-t} + 7te^{-t} - 7e^{-t}$$

$$y''(t) + 2y'(t) + y(t) = 0$$

$$y = 6e^{-t} + 7te^{-t}$$