Fourier Series

Write down the first four non-zero terms in the Fourier series for the following functions; when a domain is listed, that is the domain for a single period.

1. $f(t) = 2 - t$ for $t \in (-2, 2]$

Solution:

$$2 + \sum_{n=1}^{\infty} \frac{4(-1)^n}{\pi n} \sin\left(\frac{\pi n}{2} t\right)$$

2. $f(t) = t^2$ for $t \in (-1, 1]$

Solution:

$$\frac{1}{3} + \sum_{n=1}^{\infty} \frac{4}{n\pi^2} (-1)^n \cos(\pi t)$$

3. $f(t) = 3t$ for $t \in (-3, 3]$

Solution:

$$\frac{18}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin(\frac{n\pi}{3} t)$$

4. $f(t) = \begin{cases} 2 & -\pi \leq t \leq 0 \\ 2 & 0 \leq t \leq \pi \end{cases}$

Solution:

$$\frac{8}{\pi} \sum_{n \text{ odd}} \frac{\sin(nt)}{n}$$