Matrices and Vectors
a basic guide

Notation, Vocabulary: Matrices

$$C = \begin{pmatrix} 2 & 5 & 5 & 6 \\ 0 & 2 & -8 & 3 \\ -1 & -1 & 6 & -3 \end{pmatrix}$$

$$I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(I is the identity matrix)

Is C a 4×3 matrix or a 3×4 matrix?

$c_{23} =$ or " *(how can we describe c_{23} in words?)* "

$c_2 =$ or "."

The third entry on the diagonal of C is .

The transpose of C is .

Notation, Vocabulary: Matrices

$$C = \begin{pmatrix} 2 & 5 & 5 & 6 \\ 0 & 2 & -8 & 3 \\ -1 & -1 & 6 & -3 \end{pmatrix}$$

$$I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(I is the identity matrix)

Is C ~~a 4×3 matrix~~ or a 3×4 matrix?

$c_{23} = -8$ or "the entry in the second row and third column of C is -8."

$c_2 = \begin{pmatrix} 5 \\ -8 \\ 6 \end{pmatrix}$ or "the second column of C is $\begin{pmatrix} 5 \\ -8 \\ 6 \end{pmatrix}$."

The third entry on the diagonal of C is 6.

The transpose of C is $C^T = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 2 & -1 \\ 5 & -8 & 6 \\ 6 & 3 & -3 \end{pmatrix}$.

Notation, Vocabulary: Vectors

$$v = \begin{pmatrix} \sqrt{6} \\ -1 \\ 3 \end{pmatrix}$$

In $\mathbb{R}^4$, $e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

e_3 is a "standard basis vector"

Strong convention: "never name a row."

Named vectors are always columns: view v as a 3×1 matrix.

If you want to talk about the row vector $\begin{pmatrix} \sqrt{6} & -1 & 3 \end{pmatrix}$, define v as above (a column) and then refer to v^T (a row).

The second entry of v is written v_2 or -1 in this case.

Notation clash: v_2 could be the second entry in a vector v, or it could be the second column of a matrix V. It's the writer's job to make this clear. Use words in addition to symbols!

Notation, vocabulary: Ax

Develop a "column attitude"

Beginners see nine numbers; experts see three columns

$$\begin{pmatrix} -1 & 2 & 1 \\ 2 & 7 & -3 \\ 1 & -3 & 4 \end{pmatrix}$$

Notation, vocabulary: Ax

Develop a “column attitude”

$$\begin{pmatrix} -1 & 2 & 1 \\ 2 & 7 & -3 \\ 1 & -3 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} =$$

$$A \qquad x \qquad =$$

Notation, vocabulary: Ax

Develop a "column attitude"

$$\begin{pmatrix} -1 & 2 & 1 \\ 2 & 7 & -3 \\ 1 & -3 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} = 1 \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} - 2 \begin{pmatrix} 2 \\ 7 \\ -3 \end{pmatrix} + 5 \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ -29 \\ \end{pmatrix}$$

$$A \quad x \quad = \quad x_1\vec{a}_1 \quad + x_2\vec{a}_2 \quad + x_3\vec{a}_3$$

Notation, vocabulary: Ax

Develop a "column attitude"

$$\begin{pmatrix} -1 & 2 & 1 \\ 2 & 7 & -3 \\ 1 & -3 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} = 1 \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} - 2 \begin{pmatrix} 2 \\ 7 \\ -3 \end{pmatrix} + 5 \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ -29 \\ 27 \end{pmatrix}$$

$$A \quad x \quad = \quad x_1\vec{a}_1 \quad + x_2\vec{a}_2 \quad + x_3\vec{a}_3$$

Notation, vocabulary: AB

Develop a "column attitude"

To multiply two matrices AB:

see columns in the second matrix,

then do that many matrix-vector multiplications.

$$AB = A\begin{pmatrix} | & | & | \\ b_1 & b_2 & b_3 \\ | & | & | \end{pmatrix} = \begin{pmatrix} | & | & | \\ Ab_1 & Ab_2 & Ab_3 \\ | & | & | \end{pmatrix}$$

"column k in matrix (AB) is the same as A times column k in matrix B"

Avoid matrix-matrix multiplication if you can: it's expensive / slow!

Example: $(AB)v = A(Bv)$ and one of these is much better.

Transpose of a sum, transpose of a product

Suppose matrices A and B have the same shape (we can add them). Then:

$$(A+B)^T =$$

"the transpose of the sum is ."

Suppose that matrices A and B have compatible dimensions (we can multiply AB, that is, the number of columns of A matches the number of rows of B). Then:

$$(AB)^T =$$

"the transpose of the product is ."

Transpose of a sum, transpose of a product

Suppose matrices A and B have the same shape (we can add them). Then:

$$(A+B)^T = A^T + B^T$$

"the transpose of the sum is the sum of the transposes."

Suppose that matrices A and B have compatible dimensions (we can multiply AB, that is, the number of columns of A matches the number of rows of B). Then:

$$(AB)^T = B^T A^T$$

"the transpose of the product is the reversed product of the transposes."

Inner product, outer product

$$v = \begin{pmatrix} 2 \\ 0 \\ -1 \\ 0 \end{pmatrix} \qquad w = \begin{pmatrix} 0 \\ 3 \\ -4 \\ 1 \end{pmatrix}$$

Which of $v^T w$ and $v w^T$ is a matrix and which is a scalar?

$v^T w$ is a ______ and $v w^T$ is a ______

Inner product, outer product

$$v = \begin{pmatrix} 2 \\ 0 \\ -1 \\ 0 \end{pmatrix} \qquad w = \begin{pmatrix} 0 \\ 3 \\ -4 \\ 1 \end{pmatrix}$$

Which of $v^T w$ and $v w^T$ is a matrix and which is a scalar?

$v^T w$ is a scalar and $v w^T$ is a matrix.

Inner product

$v^T w = 4$

Carries geometric info

(lengths, angles)

Outer product

$$v w^T = \begin{pmatrix} 0 & 6 & -8 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & -3 & 4 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

creates a *rank-one* matrix