① 常见级数展开和麦克劳林公式:

$$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!} = 1+x+\frac{x^2}{2}+o(x^2) \quad x\in(-\infty,+\infty)$$

$$\ln(1+x)= \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}x^n = x-\frac{x^2}{2}+\frac{x^3}{3}+o(x^3) \quad x\in(-1,1]$$

$$\ln(1-x)= -\sum_{n=1}^{\infty}\frac{x^n}{n} = -(x+\frac{x^2}{2}+\frac{x^3}{3}+o(x^3)) \quad x\in[-1,1)$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty}x^n = 1+x+x^2+x^3+o(x^3) \quad x\in(-1,1)$$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty}(-1)^n x^n = 1-x+x^2-x^3+o(x^3) \quad x\in(-1,1)$$

$$(1+x)^\alpha = 1+\sum_{n=1}^{\infty}\frac{\prod_{i=0}^{n-1}(\alpha-i)}{n!}x^n = 1+\alpha x+\frac{\alpha(\alpha-1)}{2!}x^2+o(x^2) \quad x\in(-1,1)$$

$$(1+x)^{\frac{1}{x}} = \text{可由(2)推得} = e-\frac{e}{2}x+\frac{11e}{24}x^2-\frac{7e}{16}x^3+o(x^3)$$

$$\sin x = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{2n+1} = x-\frac{x^3}{6}+\frac{x^5}{120}+o(x^5) \quad x\in(-\infty,+\infty)$$

$$\cos x = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n} = 1-\frac{x^2}{2}+\frac{x^4}{24}+o(x^4) \quad x\in(-\infty,+\infty)$$

$$\tan x = \sum_{n=1}^{\infty}\frac{(4^n-1)\zeta(2n)}{\pi^{2n}}x^{2n-1} = x+\frac{1}{3}x^3+\frac{2}{15}x^5+o(x^5) \quad x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$$

$$\arcsin x = \sum_{n=0}^{\infty}\frac{C_{2n}^n}{4^n(2n+1)}x^{2n+1} = x+\frac{1}{6}x^3+\frac{3}{40}x^5+o(x^5) \quad x\in(-1,1)$$

$$\arccos x = \frac{2}{\pi}-\arcsin x = \frac{2}{\pi}-x-\frac{1}{6}x^3-\frac{3}{40}x^5+o(x^5) \quad x\in(-1,1)$$

$$\arctan x = \sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}x^{2n+1} = x-\frac{1}{3}x^3+\frac{1}{5}x^5+o(x^5) \quad x\in[-1,1]$$

$$f(x)\sim\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx) \quad \begin{cases} a_n=\frac{1}{l}\int_{-l}^{l}f(x)\cos\frac{n\pi x}{l}\mathrm{d}x \\ b_n=\frac{1}{l}\int_{-l}^{l}f(x)\sin\frac{n\pi x}{l}\mathrm{d}x \end{cases}$$

② 常用积分:

$\int x^\alpha \mathrm{d}x = \frac{1}{\alpha+1}x^{\alpha+1}+C, \quad (\alpha\neq -1)$

$\int a^x \mathrm{d}x = \frac{a^x}{\ln a}+C, \quad (a>0, a\neq 1)$

$\int e^x \mathrm{d}x = e^x+C$

$\int \sin x\mathrm{d}x = -\cos x+C$

$\int \cos x\mathrm{d}x = \sin x+C$

$\int \tan x\mathrm{d}x = -\ln|\cos x|+C$

$\int \cot x\mathrm{d}x = \ln|\sin x|+C$

$\int \sec x\mathrm{d}x = \ln|\sec x+\tan x|+C$

$\int \csc x\mathrm{d}x = \ln|\csc x-\cot x|+C$

$\int \sec^2 x\mathrm{d}x = \tan x+C$

$\int \csc^2 x\mathrm{d}x = -\cot x+C$

$\int \frac{1}{x}\mathrm{d}x = \ln|x|+C$

$\int \frac{1}{a^2+x^2}\mathrm{d}x = \frac{1}{a}\arctan\frac{x}{a}$

$\int \frac{1}{a^2-x^2}\mathrm{d}x = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|+C$

$\int \frac{1}{\sqrt{a^2+x^2}}\mathrm{d}x = \arcsin\frac{x}{a}+C$

$\int \frac{1}{\sqrt{x^2\pm a^2}}\mathrm{d}x = \ln\left|x+\sqrt{x^2\pm a^2}\right|+C$

$\int\sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\arcsin\frac{x}{a}+C$

$\int\sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\ln|x+\sqrt{x^2-a^2}|+C$

$\int\sqrt{x^2+a^2}dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\ln(x+\sqrt{x^2+a^2})+C$

③ 积分公式:

$$\begin{cases}
\int R\left(x,\sqrt{a^2-x^2}\right)\mathrm{d}x \xlongequal{x=a\sin t} \int R(a\sin t, a\cos t)\,a\cos t\mathrm{d}t \\
\int R\left(x,\sqrt{x^2+a^2}\right)\mathrm{d}x \xlongequal{x=a\tan t} \int R(a\tan t, a\sec t)\,a\sec^2 t\mathrm{d}t \\
\int R\left(x,\sqrt{x^2-a^2}\right)\mathrm{d}x \xlongequal{x=a\sec t} \int R(a\sec t, a\tan t)\,a\sec t\tan t\mathrm{d}t \\
\int R\left(x,\sqrt[n]{ax+b},\sqrt[m]{ax+b}\right)\mathrm{d}x \xlongequal{\sqrt[mn]{ax+b}=t} \int R\left(\frac{t^{mn}-b}{a},t^m,t^n\right)\frac{mn}{a}t^{mn-1}\mathrm{d}t \\
\int R\left(x,\sqrt{\frac{ax+b}{cx+d}}\right)\mathrm{d}x \xlongequal{\sqrt{\frac{ax+b}{cx+d}}=t} \int R\left(\frac{dt^2-b}{a-ct^2},t\right)\frac{2(ad-bc)t}{(a-ct^2)^2}\mathrm{d}t \quad (ad-bc\neq 0) \\
\int R(\sin x,\cos x)\mathrm{d}x \xlongequal[\tan\frac{x}{2}=t]{\sin x=\frac{2t}{1+t^2},\cos x=\frac{1-t^2}{1+t^2}} \int R\left(\frac{2t}{1+t^2},\frac{1-t^2}{1+t^2}\right)\frac{2}{1+t^2}\mathrm{d}t
\end{cases}$$

$$\int_0^{\frac{\pi}{2}}\sin^n x\mathrm{d}x = \int_0^{\frac{\pi}{2}}\cos^n x\mathrm{d}x = \begin{cases} \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{1}{2}\cdot\frac{\pi}{2}, & (n>0, n\text{为偶}) \\ \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{2}{3}\cdot 1, & (n>1, n\text{为奇}) \end{cases}$$

$$\int_0^{\pi}\sin^n x\mathrm{d}x = 2\cdot\int_0^{\frac{\pi}{2}}\sin^n x\mathrm{d}x, (n>0)$$

$$\int_0^{\pi}\cos^n x\mathrm{d}x = \begin{cases} 2\cdot\int_0^{\frac{\pi}{2}}\cos^n x\mathrm{d}x, & (n>0, n\text{为偶}) \\ 0, & (n>0, n\text{为奇}) \end{cases}$$

$$\int_0^{2\pi}\sin^n x\mathrm{d}x = \int_0^{2\pi}\cos^n x\mathrm{d}x = \begin{cases} 4\cdot\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{1}{2}\cdot\frac{\pi}{2}, & (n>0, n\text{为偶}) \\ 0, & (n>0, n\text{为奇}) \end{cases}$$

$$\int x^n e^{-x}\mathrm{d}x = -e^{-x}\sum_{i=0}^{n}(x^n)^{(i)}+C \qquad \int x^n e^x\mathrm{d}x = e^x\sum_{i=0}^{n}(-1)^i(x^n)^{(i)}+C$$

$$\int_0^{+\infty}x^n e^{-x}\mathrm{d}x = n! = \Gamma(n+1)$$

$$\int_{-\infty}^{+\infty}e^{-x^2}\mathrm{d}x = \sqrt{\pi}$$

④ 微分导数相关:

$$(uv)^{(n)} = \sum_{i=0}^{n}C_n^i u^{(n-i)}v^{(i)}$$

$(e^{ax})^{(n)} = a^n e^{ax}$

$(\sin ax)^{(n)} = a^n\sin\left(\frac{n\pi}{2}+ax\right)$

$(\cos ax)^{(n)} = a^n\cos\left(\frac{n\pi}{2}+ax\right)$

$(\ln(1+x))^{(n)} = \frac{(-1)^{n-1}(n-1)!}{(1+x)^n}$

$((1+x)^\alpha)^{(n)} = \prod_{i=0}^{n-1}(a-i)(1+x)^{\alpha-n}$

$((1+x)^n)^{(n)} = n! \quad ((1+x)^n)^{(n+j)} = 0$

$$R=\frac{|y''|}{(1+y'^2)^{3/2}} \quad \text{曲率}k=\frac{1}{R} \quad \text{曲率中心}(\alpha,\beta)=\left(x-\frac{y'(1+y'^2)}{y''},y+\frac{1+y'^2}{y''}\right)$$

⑤ 解析几何:

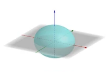
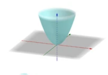
$$d_{\text{点面}} = \frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}} \qquad d_{\text{线线}} = \frac{\left|(\mathbb{s}_1\mathbb{s}_2\vec{AB})\right|}{|\mathbb{s}_1\times\mathbb{s}_1|}$$

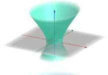
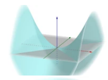
$$d_{\text{点线}} = \frac{|\{x_1-x_0,y_1-y_0,z_1-z_0\}\times\{l,m,n\}|}{\sqrt{l^2+m^2+n^2}}$$

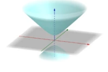
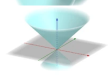
直线 $\begin{cases} A_1x+B_1y+C_1z+D_1=0 \\ A_2x+B_2y+C_2z+D_2=0 \end{cases}$

的平面束：$\lambda(A_1x+B_1y+C_1z+D_1)+\mu(A_2x+B_2y+C_2z+D_2)=0$

或：$(A_1x+B_1y+C_1z+D_1)+k(A_2x+B_2y+C_2z+D_2)=0 \quad \left(k=\frac{\mu}{\lambda}\right)$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$  $\frac{x^2}{a^2}+\frac{y^2}{b^2}-2pz=0$ 

$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$  $\frac{x^2}{a^2}-\frac{y^2}{b^2}-2pz=0$ 

$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$  $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$ 

⑥ 多元微积分:

$$f''_{xx}(x_0,y_0)=A, f''_{xy}(x_0,y_0)=B, f''_{yy}(x_0,y_0)=C$$

$$\begin{cases} AC-B^2>0 & A>0\text{极小值}, A<0\text{极大值} \\ AC-B^2<0 & \text{非极值} \\ AC-B^2=0 & \text{无法判断} \end{cases}$$

⑦ 常微分方程:

$\boxed{y'=f(x)g(y)}$ 同除$g(y)$积分

$\boxed{y'+p(x)y=q(x)}$ 同乘$e^{\int p(x)dx}$积得$y=e^{-\int p(x)dx}\left[C+\int q(x)e^{\int p(x)dx}dx\right]$

$\boxed{P(x,y)dx+Q(x,y)dy=0}$ 若$\frac{\partial Q}{\partial x}=\frac{\partial P}{\partial y}$,求$u(x,y)=C$使$du=Pdx+Qdy$

$\boxed{y'=f(\frac{y}{x})}$ 令$u=\frac{y}{x},u'=\frac{y'x-y}{x^2}=\frac{y'-u}{x}$,代得$xu'+u=f(u)$(可分)

$\boxed{y'=f(ax+by+c)}$ 令$u=ax+by+c,u'=a+by'=a+bf(u)$(可分)

$\boxed{y'=f(\frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2})}$ 令$x=X+h,y=Y+k$消去c_1c_2化为齐次

$\boxed{y'+p(x)y=q(x)y^\alpha,(\alpha\neq 0,1)}$ 令$u=y^{1-\alpha},u'=(1-\alpha)y^{-\alpha}y'$,代得一阶

$\boxed{y'=\frac{h(y)}{p(y)x+q(y)}}$ 以y为自变量,得$\frac{dx}{dy}=\frac{p(y)}{h(y)}x+\frac{q(y)}{h(y)}$(一阶)

$\boxed{y'=\frac{h(y)}{p(y)x+q(y)x^\alpha},(\alpha\neq 0,1)}$ 以y为自变量,得$\frac{dx}{dy}=\frac{p(y)}{h(y)}x+\frac{q(y)}{h(y)}x^\alpha$(伯)

$\boxed{y^{(n)}=f(x)}$ n次积分,结果有n个任意常数

$\boxed{y''=f(x,y')}$ 令$u=y'$,代得$u'=f(x,u)$(一阶),解得u后积分得y

$\boxed{y''=f(y,y')}$ 令$u=y',y''=\frac{du}{dx}=\frac{du}{dy}\frac{du}{dx}=u\frac{du}{dy}$,代得$u\frac{u}{y}=f(y,u)$

$\boxed{y''+p(x)y'+q(x)y=f(x)\text{及高阶线性微分方程}}$ 解性质&通解结构

$\boxed{y''+py'+qy=0}$ 解$\lambda^2+p\lambda+q=0$,根的3种情对应3种通解

$\boxed{y^{(n)}+p_1y^{(n-1)}+\cdots+p_ny=0}$ 解$\lambda^n+p_1\lambda^{n-1}+\cdots+p_n=0$,$k$重根3种情况

$$\boxed{y''+py'+qy=f(x)} \begin{cases} =P_n(x) & \text{特解}y^*=x^kH_n(x) \\ =P_n(x)e^{\alpha x} & \text{特解}y^*=x^kH_n(x)e^{\alpha x} \\ =e^{\alpha x}\left[P_n(x)sin\beta x+Q_m(x)cos\beta x\right] & \\ \quad\text{特解}y^*=x^ke^{\alpha x}\left[R_l(x)sin\beta x+S_l(x)cos\beta x\right]_{l=max\{n,m\}} & \\ (k\text{为}\alpha\text{或}\alpha\pm i\beta\text{在特征方程中根重数}) & \end{cases}$$

$\boxed{x^2y''+pxy'+qy=f(x)}$ 令$u=\pm e^t,\frac{dy}{dx}=\frac{dy}{dt}\frac{1}{x},\frac{d^2y}{dx^2}=\left(\frac{d^2y}{dx^2}-\frac{dy}{dx}\right)\frac{1}{x^2}$

⑧ 极限相关:

$$\lim_{f(x)\to\infty}(1+\frac{1}{f(x)})^{f(x)}=e \qquad \lim_{f(x)\to 0}(1+f(x))^{\frac{1}{f(x)}}=e$$

⑨ 初等数学:

· 三角函数

$\sin(a+b)=\sin a\cos b+\cos a\sin b \quad \cos(a+b)=\cos a\cos b-\sin a\sin b$

$\tan(a+b)=\frac{\tan a+\tan b}{1-\tan a\tan b} \quad \alpha\sin a+\beta\cos a=\sqrt{\alpha^2+\beta^2}\sin(a+\varphi)$

· 因式分解

$(a+b)^3=a^3+3a^2b+3ab^2+b^3 \quad (a-b)^3=a^3-3a^2b+3ab^2-b^3$

$a^3-b^3=(a-b)(a^2+ab+b^2) \quad a^n-b^n=(a-b)(\sum_{i=0}^{n-1}a^{n-1-i}b^i)$

求根：$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ 韦达定理：$x_1+x_2=-\frac{b}{a},x_1x_2=\frac{c}{a}$

· 不等式

$\boxed{||a|-|b||\le|a\pm b|\le|a|+|b|} \quad \frac{2}{a^{-1}+b^{-1}}\le\sqrt{ab}\le\frac{a+b}{2}\le\sqrt{\frac{a^2+b^2}{2}}_{(a,b>0)}$

$ab\le\left(\frac{a+b}{2}\right)^2\le\frac{a^2+b^2}{2}_{(a,b>0)} \quad \frac{(a+b)^2}{2}\le a^2+b^2_{(a,b>0)} \quad 2ab\le a^2+b^2_{(a,b\in R)}$

$$\boxed{\frac{n}{\sum_{i=1}^n a_i^{-1}}\le\sqrt[n]{\prod_{i=1}^n a_i}\le\frac{\sum_{i=1}^n a_i}{n}\le\sqrt{\frac{\sum_{i=1}^n a_i^2}{n}}_{(a_i>0)}}$$

$$\boxed{\sqrt[\alpha]{\frac{\sum_{i=1}^n a_i^\alpha}{n}}\ge\sqrt[\beta]{\frac{\sum_{i=1}^n a_i^\beta}{n}}_{(\alpha>\beta,a_i>0)}} \quad a^3+b^3+c^3\ge 3abc_{(a,b,c>0)}$$

$$\boxed{(a_1^2+a_2^2+\cdots+a_n^2)(b_1^2+b_2^2+\cdots+b_n^2)\ge(a_1b_1+a_2b_2+\cdots+a_nb_n)^2_{(a_i,b_i\in R)}}$$

$(a_1^2+a_2^2+\cdots+a_n^2)\ge\frac{1}{n}(a_1+a_2+\cdots+a_n)^2 \quad a^2+b^2+c^2\ge ab+bc+ca$

$ab>0\Rightarrow\frac{a}{b}+\frac{b}{a}\ge 2 \quad ab<0\Rightarrow\frac{a}{b}+\frac{b}{a}\le-2 \quad \boxed{\frac{b}{a}<\frac{b+m}{a+m}<1_{(a>b>0,m>0)}}$

$\sin x<x<\tan x_{(0<x<\frac{\pi}{2})} \quad \sin x<x_{(x>0)} \quad \arcsin x\ge x\arctan x_{(a\le x\le 1)}$

$e^{f(x)}\ge f(x)+1,e^{f(x)}\ge ef(x) \quad \ln x\le x-1_{(x>0)} \quad \frac{x}{1+x}\le\ln(1+x)\le x_{(x\ge 0)}$

· 数列

$a_n=a_1+(n-1)d\Rightarrow S_n=\frac{n}{2}(a_1+a_n)=na_1+\frac{1}{2}n(n-1)d$

$a_n=a_1q^{n-1}(q\ne 1)\Rightarrow S_n=\frac{a_1(1-q^n)}{1-q}=\frac{a_1-a_nq}{1-q}$

$\sum_{k=1}^n k=\frac{n(n+1)}{2} \quad \sum_{k=1}^n k^2=\frac{n(n+1)(2n+1)}{6}$

⑩ 放缩和不等式:

$$(uv)^{(n)}=\sum_{i=0}^n C_n^i u^{(n-i)}v^{(i)}$$

① 行列式

② 矩阵

③ 向量

④ 线性方程组

⑤ 秩

⑥ 等价相似合同

① 事件和概率运算:

$A\cup B=B\cup A$ $\qquad A\cap B=B\cap A$

$A\cup(B\cup C)=(A\cup B)\cup C$ $\qquad A\cap(B\cap C)=(A\cap B)\cap C$

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ $\qquad A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

$\overline{\bigcup_{i=1}^{n}A_i}=\bigcap_{i=1}^{n}\overline{A_i}$ $\quad \overline{\bigcap_{i=1}^{n}A_i}=\bigcup_{i=1}^{n}\overline{A_i}$ $\qquad \overline{A-B}=\overline{A}\cup B$

$P(A\cup B)=P(A)+P(B)-P(AB)$

$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(AB)-P(BC)-P(AC)+P(ABC)$

$P(A-B)=P(A)-P(AB)$

$P(A)>0$时, $P(AB)=P(A)P(B|A)$

$P(A_1\cdots A_{n-1})>0$时, $P(A_1\cdots A_n)=P(A_1)P(A_2|A_1)\cdots P(A_n|A_1\cdots A_{n-1})$

$\bigcup_{i=1}^{n}B_i=\Omega, B_iB_j=\varnothing(i\neq j), P(B_k)>0$时：

$P(A)=\sum_{i=1}^{n}P(B_i)P(A|B_i)$

$\bigcup_{i=1}^{n}B_i=\Omega, B_iB_j=\varnothing(i\neq j), P(A)>0, P(B_k)>0$时：

$P(B_j|A)=\frac{P(B_j)P(A|B_j)}{\sum_{i=1}^{n}P(B_i)P(A|B_i)}$

② 分布和性质:

$\boxed{X\sim B(n,p)\quad P\{X=k\}=C_n^kp^kq^{n-k}}$

· $E(X)=np\quad D(X)=np(1-p)$

· p_n与n有关, 若 $\lim\limits_{n\to\infty}np_n=\lambda$, 则 $\lim\limits_{n\to\infty}C_n^kp_n^k(1-p_n)^{n-k}=\frac{\lambda^k}{k!}e^{-\lambda}$

· $n\geq 100, p\leq 0.1, C_n^kp^k(1-p)^{n-k}\approx\frac{(np)^k}{k!}e^{-np}$

$\boxed{X\sim G(p)\quad P\{X=k\}=pq^{k-1}}$

· $E(X)=\frac{1}{p}\quad D(X)=\frac{1-p}{p^2}$

$\boxed{X\sim H(n,N,M)\quad P\{X=k\}=\frac{C_M^kC_{N-M}^{n-k}}{C_N^n}}$

$\boxed{X\sim P(\lambda)\quad P\{X=k\}=\frac{\lambda^k}{k!}e^{-\lambda}}$

· $E(X)=\lambda\quad D(X)=\lambda$

· $X_1\sim P(\lambda_1), X_2\sim P(\lambda_2), X_1X_2$独立 $\Rightarrow X_1+X_2=P(\lambda_1+\lambda_2)$

$\boxed{X\sim U(a,b)\quad f(x)=\begin{cases}\frac{1}{b-a}, & x\leq x\leq b\\ 0, & \text{其他}\end{cases}}$

· $E(X)=\frac{a+b}{2}\quad D(X)=\frac{(b-a)^2}{12}$

$\boxed{X\sim E(\lambda)\quad f(x)=\begin{cases}\lambda e^{-\lambda x}, & x>0\\ 0, & x\leq 0\end{cases}}$

· $E(X)=\frac{1}{\lambda}\quad D(X)=\frac{1}{\lambda^2}$

· $P\{X>t+s|X>s\}=\frac{e^{-\lambda(t+s)}}{e^{-\lambda s}}=e^{-\lambda t}=P\{X>t\},\quad (t,s>0)$

$\boxed{X\sim N(\mu,\sigma^2)\quad f(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$

· $E(X)=\mu\quad D(X)=\sigma^2$

$\boxed{(X,Y)\sim N_{(\mu_1,\mu_2;\sigma_1^2,\sigma_2^2;\rho)}\quad f(x,y)=\frac{e^{-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_1)^2}{\sigma_1^2}-\frac{2\rho(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2}+\frac{(x-\mu_2)^2}{\sigma_2^2}\right]}}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}}$

· $(X,Y)\sim N_{(\mu_1,\mu_2;\sigma_1^2,\sigma_2^2;\rho)}\Rightarrow X\sim N(\mu_1,\sigma_1^2), Y\sim N(\mu_2,\sigma_2^2)$

· $(X,Y)\sim N_{(\mu_1,\mu_2;\sigma_1^2,\sigma_2^2;\rho)}$：$XY$不相关 $\Leftrightarrow XY$相互独立

· $(X,Y)\sim N_{(\mu_1,\mu_2;\sigma_1^2,\sigma_2^2;\rho)}, \begin{vmatrix}a & b\\ c & d\end{vmatrix}\neq 0\Rightarrow(aX+bY,cX+dY)\sim N(\cdot)$

· $X\sim N_{(\mu_1,\sigma_1^2)}, Y\sim N_{(\mu_2,\sigma_2^2)}, XY$独立 $\Rightarrow aX+bY\sim N_{(a\mu_1+b\mu_2,a^2\sigma_1^2+b^2\sigma_2^2)}$

$\boxed{X\sim\chi^2(n)\quad X=\sum_{i=1}^{n}X_i^2,\ X_i\sim N(0,1),\ X_i\text{相互独立}}$

· $E(X)=n\quad D(X)=2n$

· $A\sim\chi^2(n_1), B\sim\chi^2(n_2), AB$独立 $\Rightarrow A+B\sim\chi^2(n_1+n_2)$

$\boxed{T\sim t(n)\quad X=\frac{X}{\sqrt{Y/n}},\ X\sim N(0,1),\ Y\sim\chi^2(n),\ XY\text{相互独立}}$

· $E(T)=0$ 概率密度函数为偶函数

$\boxed{F\sim F(n_1,n_2)\quad F=\frac{X/n_1}{Y/n_2},\ X\sim\chi^2(n_1),\ Y\sim\chi^2(n_2),\ XY\text{相互独立}}$

· $F\sim F(n_1,n_2)\Rightarrow\frac{1}{F}\sim F(n_2,n_1)$, 且$F_{1-\alpha}(n_1,n_2)=\frac{1}{F_\alpha(n_2,n_1)}$

③ 数字特征:

$EX=\sum_{i=1}^{\infty}x_ip_i\qquad EX=\int_{-\infty}^{+\infty}xf(x)dx$

$E[g(X)]=\sum_{i=1}^{\infty}g(x_i)p_i\quad E[g(X)]=\int_{-\infty}^{+\infty}g(x)f(x)dx$

$E[g(X,Y)]=\sum_{j=1}^{\infty}\sum_{i=1}^{\infty}g(x_i,y_j)p_{ij}\quad E[g(X,Y)]=\iint g(x,y)f(x,y)dxdy$

$D(X)=E[(X-EX)^2]=EX^2-(EX)^2=[\sigma(X)]^2\quad \rho_{XY}=\frac{Cov(X,Y)}{\sqrt{DX}\sqrt{DY}}$

$Cov(X,Y)=E[(X-EX)(Y-EY)]=EXY-EXEY=Cov(Y,X)$

原点矩$E(X^k),(k=1,\cdots,n)$ 中心矩$E(X-EX)^k,(k=2,\cdots,n)$

混合原点矩$E(X^kY^l)$ 混合中心矩$E[(X-EX)^k(Y-EY)^l]$

$E(c)=c\quad E(cX)=cEX\quad E(X\pm Y)=EX\pm EY$

$D(c)=0\quad D(X+c)=DX\quad D(cX)=c^2D(X)$

$D(X\pm Y)=DX+DY\pm 2Cov(X,Y)\quad Cov(aX,bY)=abCov(X,Y)$

$Cov(X_1+X_2,Y)=Cov(X_1,Y)+Cov(X_2,Y)$

不相关 $\Leftrightarrow Cov(X,Y)=0\Leftrightarrow EXY=EXEY\Leftrightarrow D(X+Y)=DX+DY$

$D(X)=0\nRightarrow X=c\quad D(X)=0\Leftrightarrow P(X=c)=1$

$|\rho_{XY}|\leq 1\quad |\rho_{XY}|=0\Leftrightarrow$ 不相关 $\quad|\rho_{XY}|=1\Leftrightarrow P\{Y=aX+b\}=1$

独立 $\Rightarrow$ 不相关, 不相关 $\nRightarrow$ 独立

$X\sim B(1,p), Y\sim B(1,p)$：$XY$独立 $\Leftrightarrow XY$不相关

$(X,Y)\sim N(\mu_1,\mu_2;\sigma_1^2,\sigma_2^2;\rho)$：$XY$独立 $\Leftrightarrow XY$不相关

$\{X_i|i=1,2,\cdots,n\}$为X样本：$\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i\quad S^2=\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2$

样本原点矩$\frac{1}{n}\sum_{i=1}^{n}X_i^k,(k=1,\cdots,n)$

样本中心矩$\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^k,(k=2,\cdots,n)$

④ 正态总体的样本分布:

$\{X_i|i=1,2,\cdots,n\}$为$X\sim N(\mu,\sigma^2)$总体的样本：

· $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}=\frac{\sqrt{n}(\bar{X}-\mu)}{\sigma}\sim N(0,1)\qquad\frac{(n-1)S^2}{\sigma^2}\sim\chi^2(n-1)$

· $\frac{\bar{X}-\mu}{S/\sqrt{n}}=\frac{\sqrt{n}(\bar{X}-\mu)}{S}\sim t(n-1)$

· $\bar{X}$与S^2相互独立

$X\sim N(\mu_1,\sigma_1^2), Y\sim N(\mu_2,\sigma_2^2), XY$相互独立,

$\{X_i|i=1,2,\cdots,m\}$为X的m个样本, $\{X_i|i=1,2,\cdots,n\}$为Y的n个样本：

· $\frac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{m}+\frac{\sigma_2^2}{n}}}\sim N(0,1)$

· $\frac{(m-1)S_1^2}{\sigma_1^2}+\frac{(n-1)S_2^2}{\sigma_2^2}\sim\chi^2(m+n-2)\qquad\frac{S_1^2\cdot\sigma_2^2}{S_2^2\cdot\sigma_1^2}\sim F(m-1,n-1)$

· $\sigma_1^2=\sigma_2^2=\sigma^2$时, $\frac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)}{S_w\sqrt{\frac{1}{m}+\frac{1}{n}}}\sim t(m+n-2)$, 其中$S_w^2=\frac{(m-1)S_1^2+(n-1)S_2^2}{m+n-2}$

⑤ 大数定律和中心极限定理:

· 切比雪夫不等式:

随机变量X, EX和DX存在 $\Rightarrow\forall\epsilon>0,\quad P\{|X-EX|\geq\epsilon\}\leq\frac{DX}{\epsilon^2}$

$\Rightarrow\forall\epsilon>0,\quad P\{|X-EX|<\epsilon\}\geq 1-\frac{DX}{\epsilon^2}$

· 切比雪夫大数定律：

随机变量序列$X_1,\cdots,X_n$相互独立, EX_i和DX_i存在, DX_i一致有上界

$\Rightarrow\forall\epsilon>0,\lim\limits_{n\to\infty}P\left\{\left|\frac{1}{n}\sum_{i=1}^{n}X_i-\frac{1}{n}\sum_{i=1}^{n}EX_i\right|<\epsilon\right\}=1$

· 辛钦大数定律：

随机变量序列$X_1,\cdots,X_n$相互独立, 同分布, EX_i存在为μ

$\Rightarrow\forall\epsilon>0,\lim\limits_{n\to\infty}P\left\{\left|\frac{1}{n}\sum_{i=1}^{n}X_i-\mu\right|<\epsilon\right\}=1$

· 伯努利大数定律：

随机变量$X\sim B(n,p)$

$\Rightarrow\forall\epsilon>0,\lim\limits_{n\to\infty}P\left\{\left|\frac{X}{n}-p\right|<\epsilon\right\}=1$

· 列维-林德伯格中心极限定理：

随机变量序列$X_1,\cdots,X_n$相互独立, 同分布, EX_i和DX_i存在为μ和σ^2

$\Rightarrow\lim\limits_{n\to\infty}P\left\{\frac{\sum_{i=1}^{n}X_i-n\mu}{\sqrt{n}\sigma}\leq x\right\}=\Phi(x)$

· 棣莫弗-拉普拉斯定理：

随机变量$X\sim B(n,p)$

$\Rightarrow\lim\limits_{n\to\infty}P\left\{\frac{X-np}{\sqrt{np(1-p)}}\leq x\right\}=\Phi(x)$