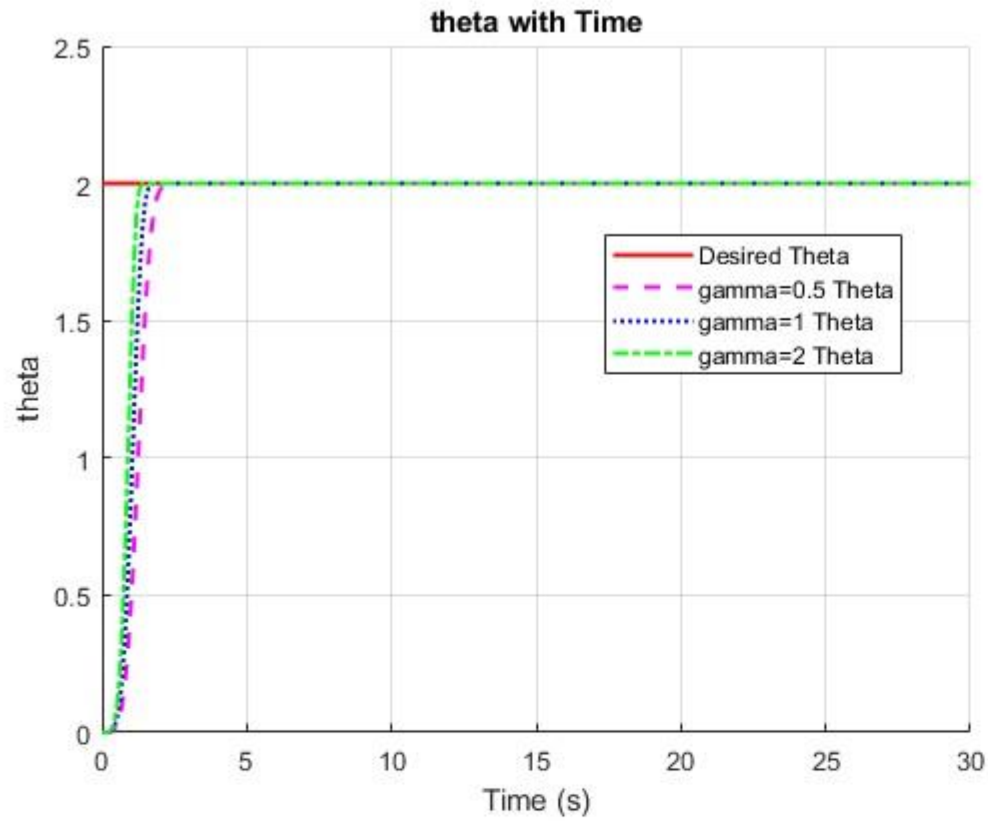
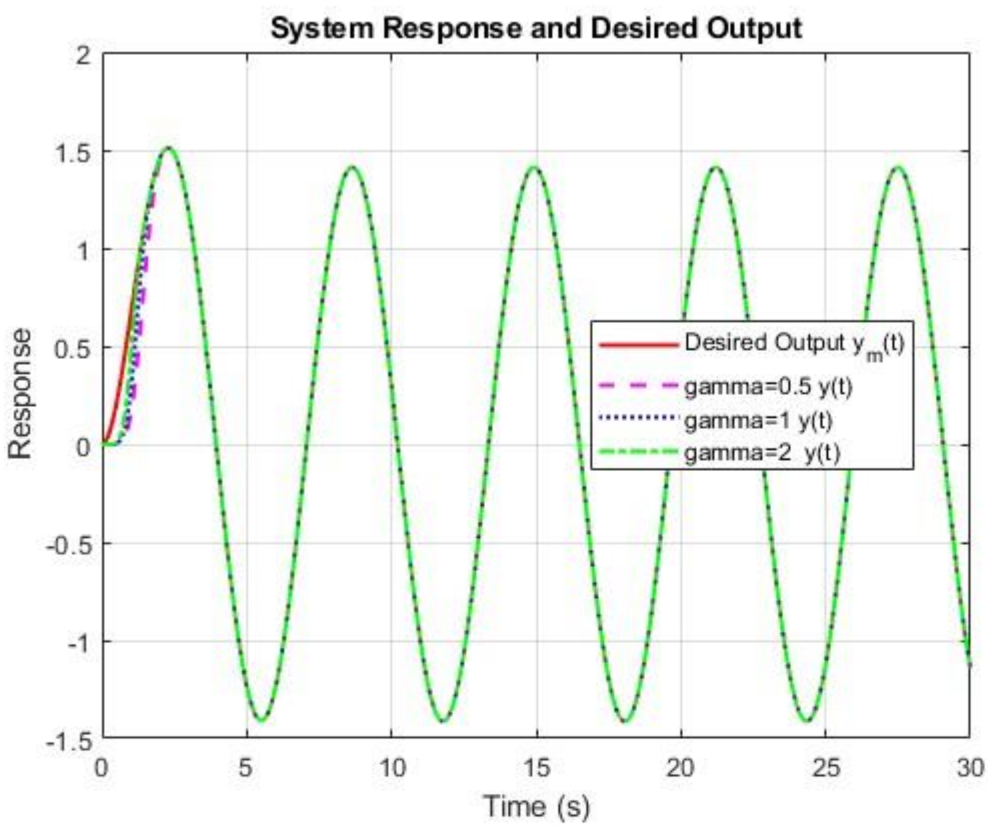
Question 1

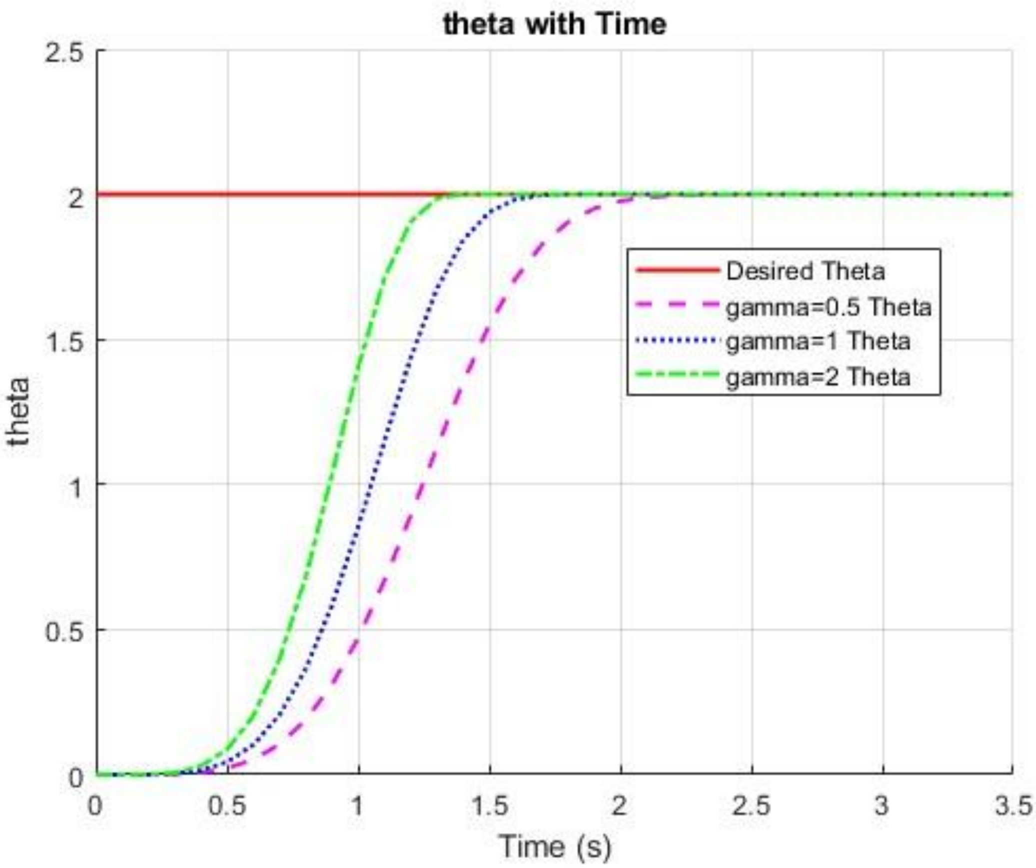
Results

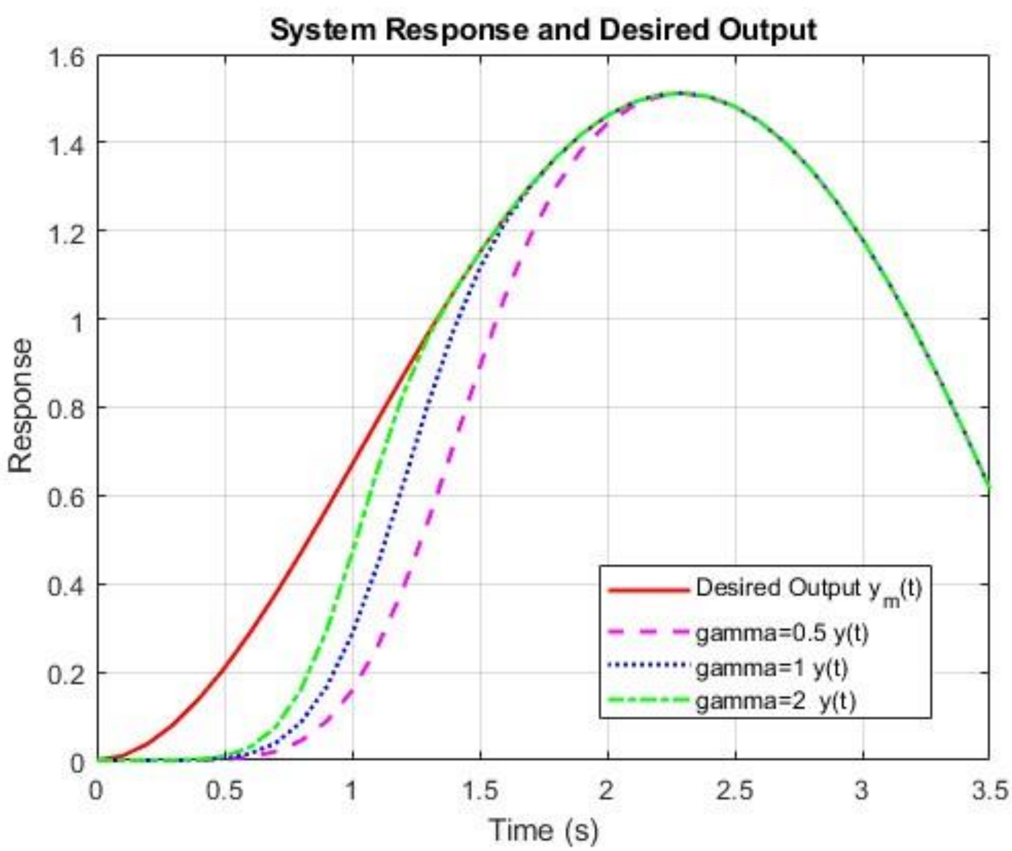
Full Scale





Zoomed at first 3 Seconds





Equations Used

Q1)

$G(s) = \frac{1}{s+1}$

$K_0 = 2$

$K = 1$

$gamma = [0.5, 1, 2]$

$u_c(t) = \sin(t)$

$Uc(s) = \mathcal{L}\{u_c\}$

$YM(s) = Uc(s) * K_0 * G(s)$

$y_m(t) = \mathcal{L}^{-1}\{YM(s)\}$

$\rightarrow$ $error = y(t) - y_m(t)$

$\rightarrow$ $theta = theta + y_m(t) * error * -gamma$

$\rightarrow$ $Y(s) = UC(s) * K * G(s) * theta$

$\rightarrow$ $y(t) = \mathcal{L}^{-1}\{Y(s)\}$

Proof

Q1)

$u = \theta u_c$ Feed forward controller

$y = \theta K G(s) u_c$ actual output

$y_m = K_0 G(s) u_c$ desired output

$\therefore \theta K = K_0$

$y = y_m$

$\theta = \frac{K_0}{K}$

error $(e) = y - y_m = (\theta K - K_0)(G(s) u_c)$

MIT rule

$$\frac{d\theta}{dt} = -\gamma' e \frac{\partial e}{\partial \theta} = -\gamma' e \frac{K}{K_0} K_0 G(s) u_c$$

$$= -\gamma' e \frac{K}{K_0} y_m \quad \rightarrow \gamma = \gamma' \frac{K}{K_0}$$

$$\therefore \frac{d\theta}{dt} = -\gamma y_m e = \theta_{new} - \theta_{old}$$

$$\theta_{new} = \theta_{old} - \gamma y_m e$$

Question 2

Proof

Q2)

$$\frac{dy}{dt} = -ay + bu$$
$$= -y + 0.5u$$

$$\frac{dy_m}{dt} = -a_m y_m + b_m u_c$$
$$= -2y_m + 2u_c$$

$$u(t) = \theta_1 u_c(t) - \theta_2 y(t)$$

Subs. u in $\frac{dy}{dt}$

$$\frac{dy}{dt} = -y + 0.5[\theta_1 u_c(t) - \theta_2 y(t)$$

$\therefore$ $0.5\theta_2 - 1 = -2 \Rightarrow \theta_2 = -2$

$\therefore$ $\theta_1 = 4$

for MIT

$$\theta_1 \rightarrow \frac{d\theta_1}{dt} = -\gamma e \frac{\partial e}{\partial \theta_1}$$

$$\theta_2 \rightarrow \frac{d\theta_2}{dt} = -\gamma e \frac{\partial e}{\partial \theta_2}$$

$$e = y - y_m$$

$$\therefore \frac{dy}{dt} + y = 0.5(\theta_1 u_c(t) + \theta_2 y(t))$$

Q2)

Laplace transform

$$sY + Y = 0.5\,[\theta_1 u_c + \theta_2 Y]$$

$$Y = \frac{0.5\,\theta_1}{s - 0.5\theta_2 + 1}\,U_c$$

$$\frac{\partial e}{\partial \theta_1} = \frac{0.5}{s - 0.5\theta_2 + 1}\,U_c$$

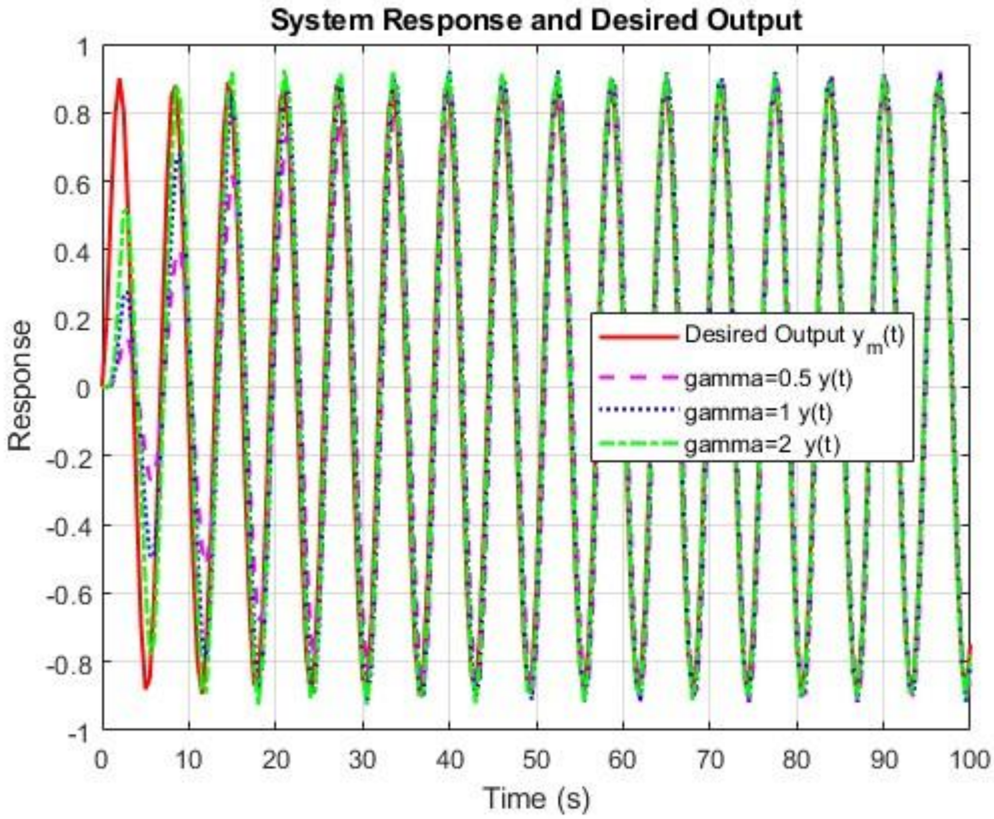
$$\frac{\partial e}{\partial \theta_2} = \frac{0.5 * 0.5\,\theta_1}{(s - 0.5\theta_2 + 1)^2}\,U_c$$

$$\therefore \frac{d\theta_1}{dt} = \frac{-0.5\,\partial e}{s - 0.5\theta_2 + 1}\,U_c$$

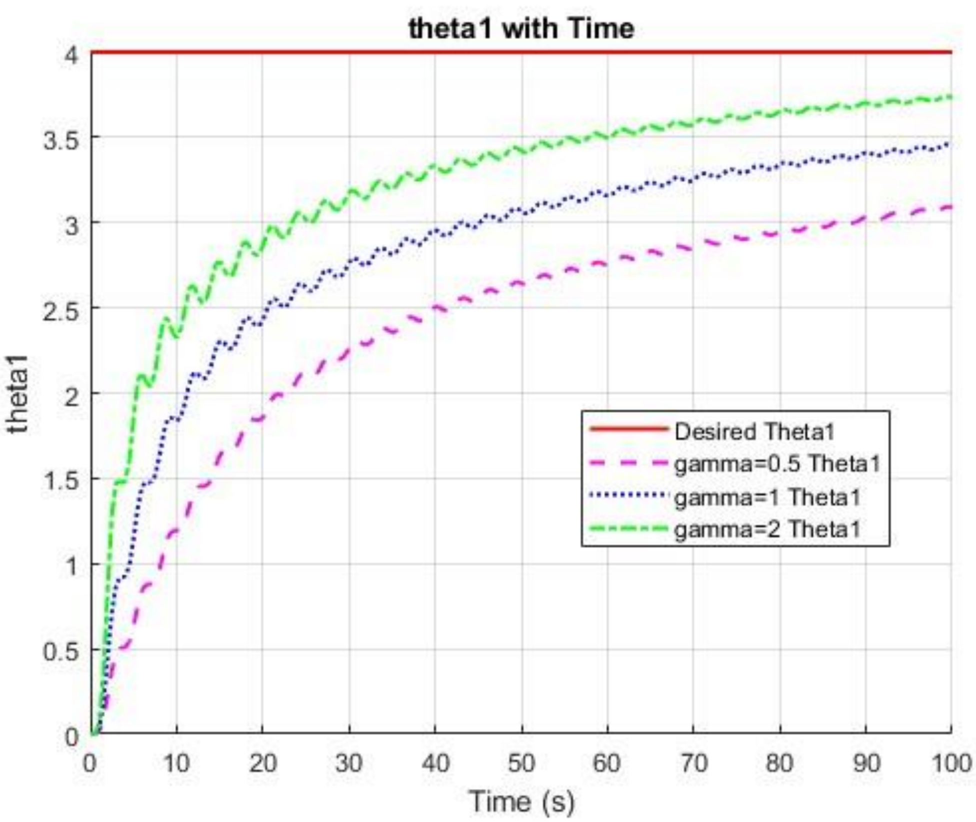
$$\frac{d\theta_2}{dt} = \frac{-0.5\,\partial e}{s - 0.5\theta_2 + 1}\cdot Y$$

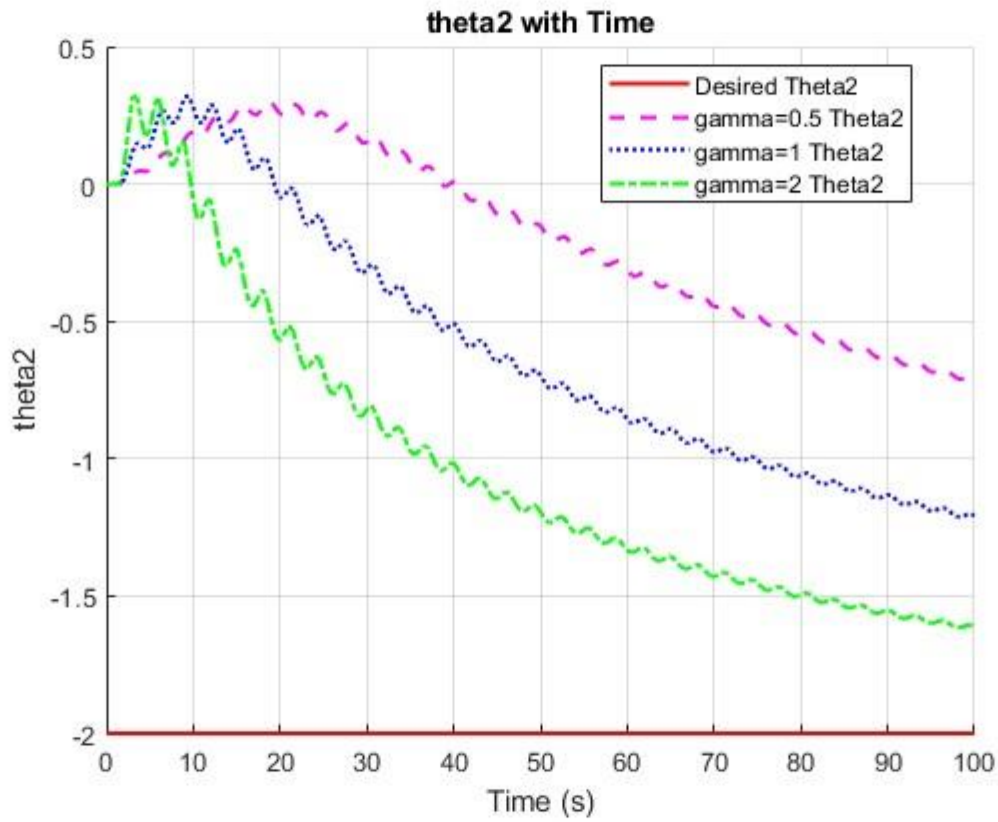
$$Y_m = \frac{2}{s+2}\,U_c$$

Results

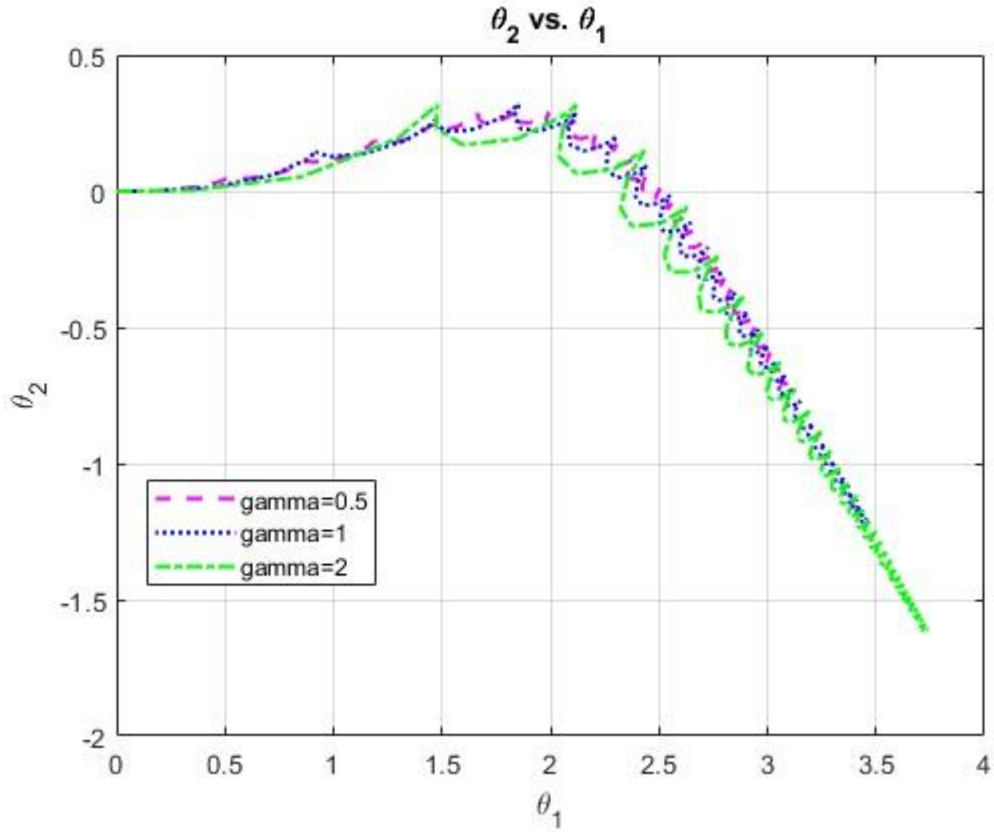


The next graph illustrates the behavior of theta 1 over time with a target value of 4. However, the system did not reach the desired value within 100 seconds unless the learning rate was increased to 5 or the duration was extended to 150 seconds. Similarly, for theta 2, the target value was -2, and adjustments like those for theta 1 were necessary to achieve the desired result.



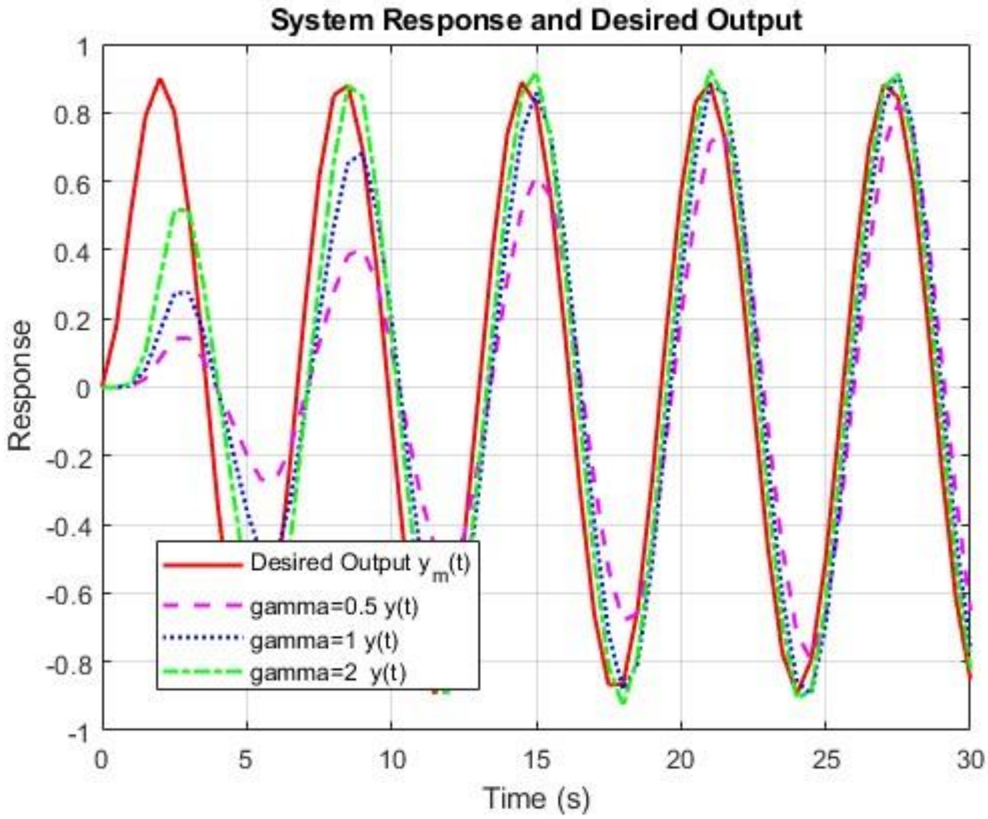


theta2 with Time
Desired Theta2
gamma=0.5 Theta2
gamma=1 Theta2
gamma=2 Theta2
theta2
Time (s)
0.5
0
-0.5
-1
-1.5
-2
0
10
20
30
40
50
60
70
80
90
100



θ_2 vs. θ_1
gamma=0.5
gamma=1
gamma=2
θ_2
θ_1
0.5
0
-0.5
-1
-1.5
-2
0
0.5
1
1.5
2
2.5
3
3.5
4

Zoomed at first 30 seconds



Conclusion

The learning rate (γ) affects how quickly the controller adapts its parameters to minimize the error between the reference model output (ym) and the actual plant output (y). Here are some observations based on the plots:

Higher learning rates (e.g., γ = 2.0) cause faster parameter updates, resulting in quicker convergence of the controller. However, very high learning rates can lead to instability or overshooting behavior.

Lower learning rates (e.g., γ = 0.5) cause slower parameter updates, leading to slower convergence. It can smooth out oscillations, but the response may be sluggish.

The middle ground (e.g., γ = 1.0) generally provides a good balance between convergence speed and stability.

The parameter evolution plots show how the parameters θ1 and θ2 change over time. Higher learning rates lead to larger and more rapid changes in the parameters, while lower learning rates result in smaller and smoother changes. The parameter values converge to different steady-state values depending on the learning rate, representing the controller's final behavior.