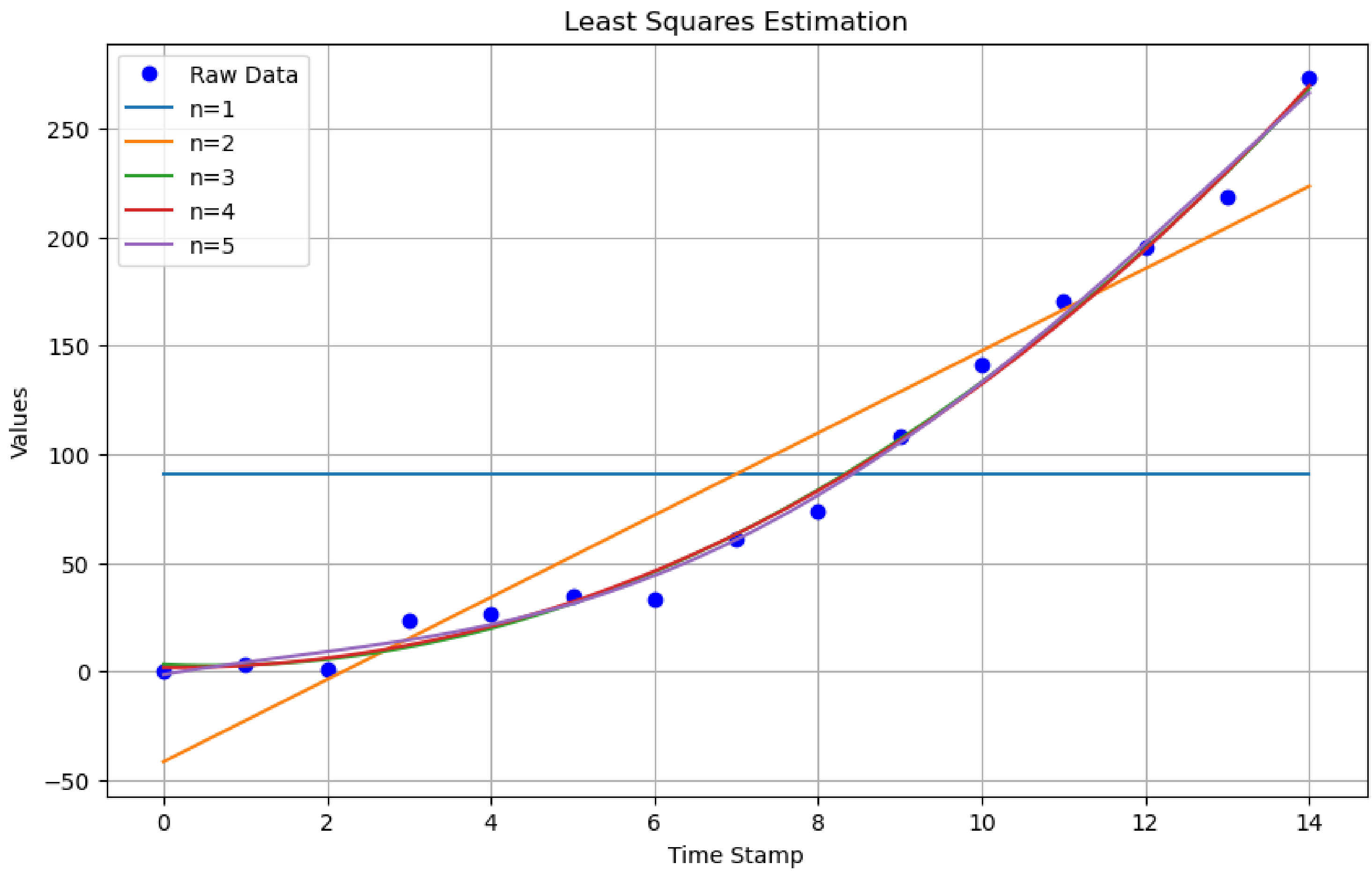
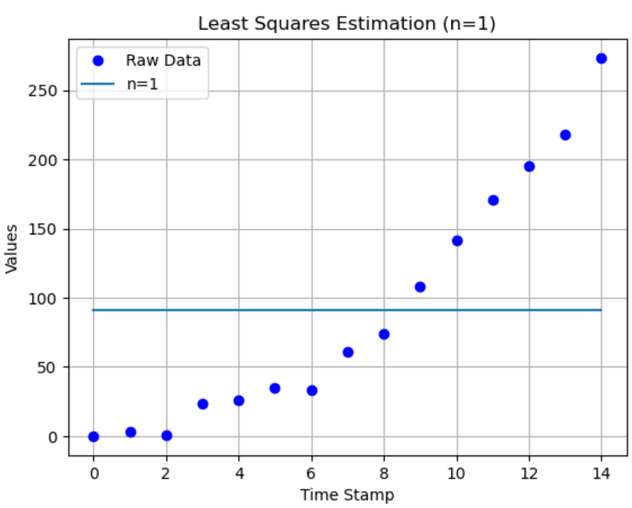
Question 1

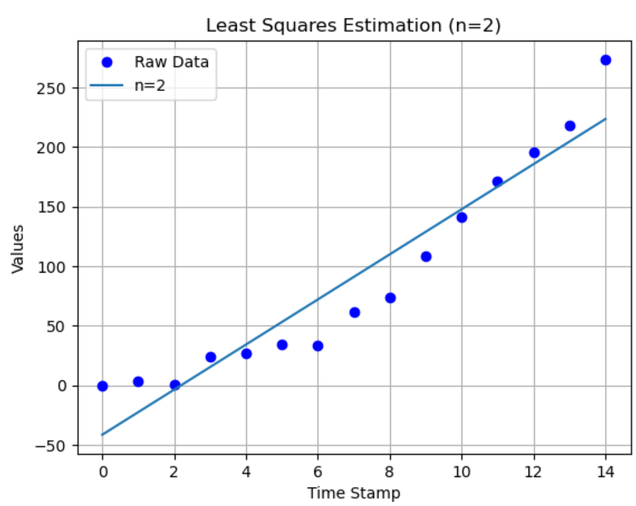
a) Plot the raw data along with the 5 curves developed.



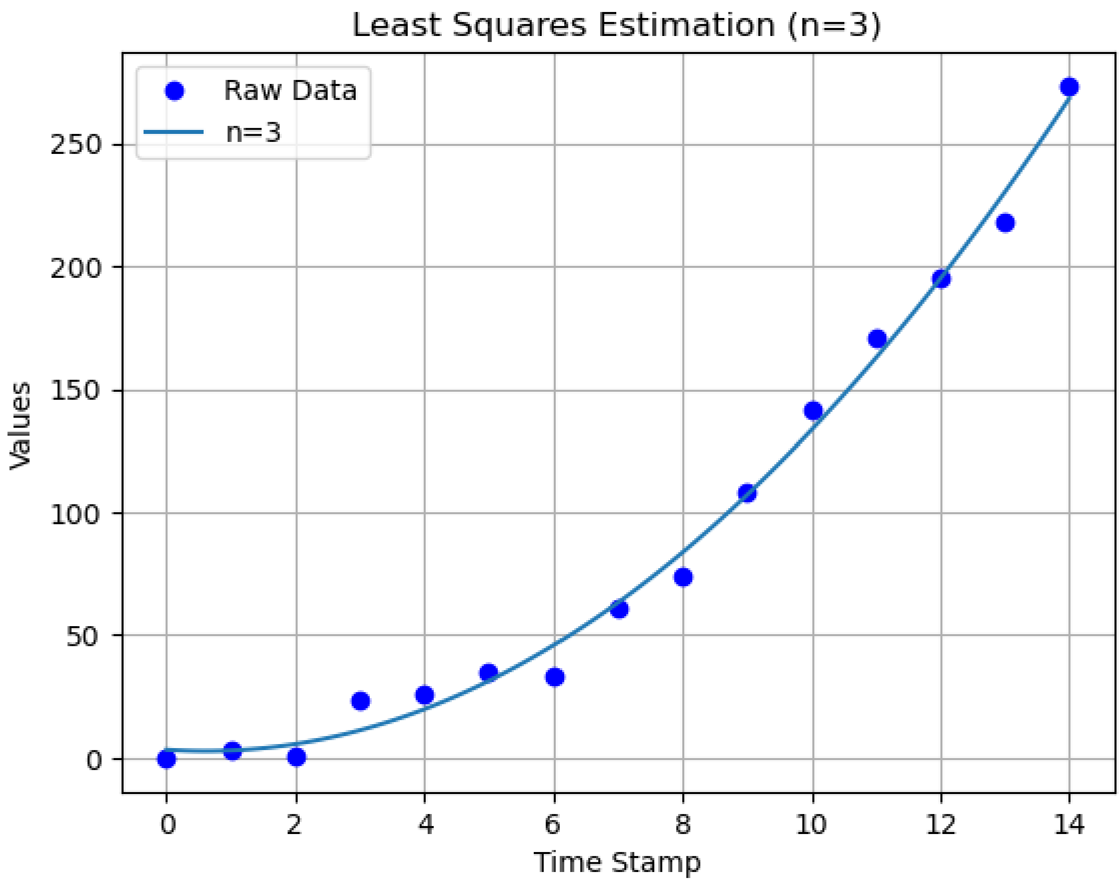
Estimated parameters for n=1: [90.89224526]



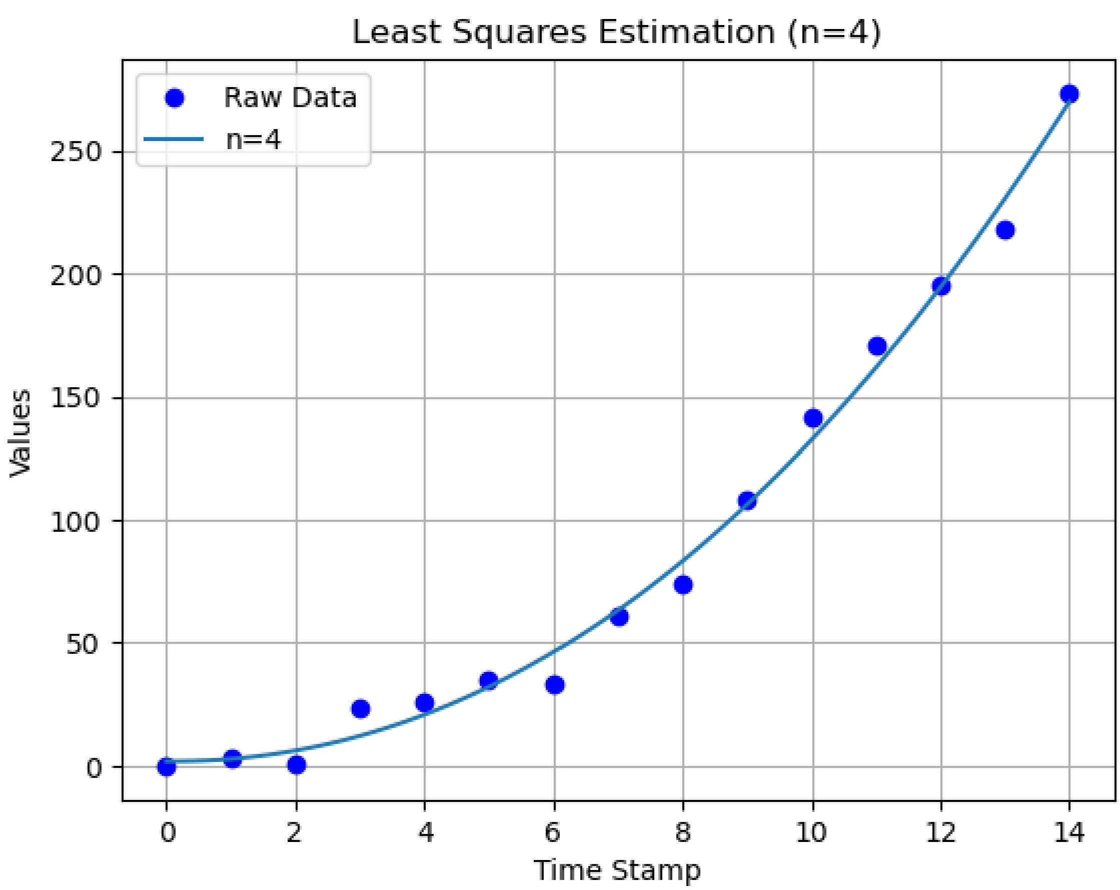
Estimated parameters for n=2: [-41.56671445 18.92270853]



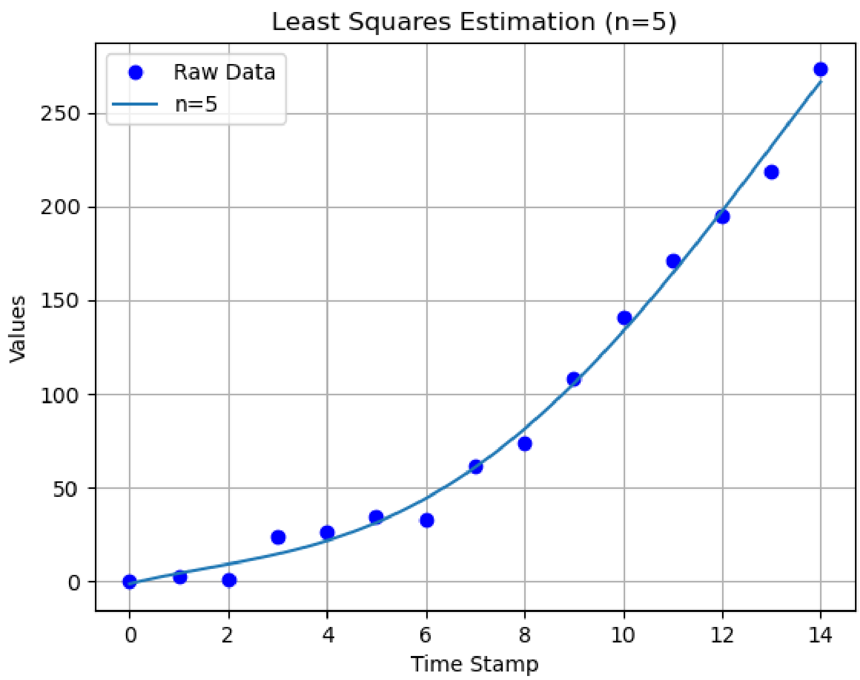
Estimated parameters for n=3: [3.34292721 -1.80481839 1.48053764]



Estimated parameters for n=4: [1.88449686 -0.28762345 1.20007026 0.01335559]



Estimated parameters for n=5: [-1.47608676 6.66463289 -1.2157339   0.28752908 -0.00979191]



b) Configure a table

Model	b0	b1	b2	b3	b4	Loss
Model 1	90.8922	-	-	-	-	55050
Model 2	-41.5667	18.922708530378575	-	-	-	4920.33
Model 3	3.34293	-1.804818388284308	1.48053763704735	-	-	398.984
Model 4	1.8845	-0.2876234540058155	1.2000702636331906	0.013355589210197968	-	393.875
Model 5	-1.47609	6.664632886791497	-1.2157339014692568	0.28752907870648853	-0.00979191033915468	357.433

c) Determine the appropriate number of parameters for the data

From the table we can find that the best number of parameters is for model 3 where estimated parameters for n=3 is
[3.34292721 -1.80481839 1.48053764]
and the loss is 398.875

My investigation revealed that Model 3 exhibited the best performance compared to Models 1, 2, 4, and 5. Not just for the loss value as model 3 still not the least loss but on overall performace. Where Models 4 and 5 demonstrated slightly lower loss values than Model 3, the differences were relatively small. In contrast, Models 1 and 2 exhibited significantly higher loss values compared to Model 3. This suggests that the improvement in loss achieved by increasing the number of parameters from 3 to 4 or 5 is not substantial enough to warrant the added complexity. I opted for the simpler Model 3 over the more complex Models 4 and 5, considering that the marginal decrease in loss for the latter models was not significant. Choosing Model 3 strikes a balance between achieving a reasonably low loss and avoiding overfitting, as excessive parameters can lead to overfitting the model to the training data.

By choosing a model with three parameters, we achieve a satisfactory level of accuracy without unnecessarily increasing the model's complexity. This decision aligns with the principle of selecting the simplest model that adequately captures the underlying patterns in the data, avoiding the pitfalls of overfitting and ensuring better generalizability.

d) find the loss function using least mean square method

for model 3 (Most appropriate number of parameters) the loss calculated to be 377 as the adapting rate is 0.00019 and the theta matrix filled with zeros as initial filling.

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Model 3 Loss: 377.66291319639697
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Question 2

Find a results graphs after answering the questions.
Having 15 graphs (5 for each part) to see the signal, compare for each parameter (total 4 parameters).
Using t=100 to see the results clear and using t=500 to check the conversion of the parameters results.

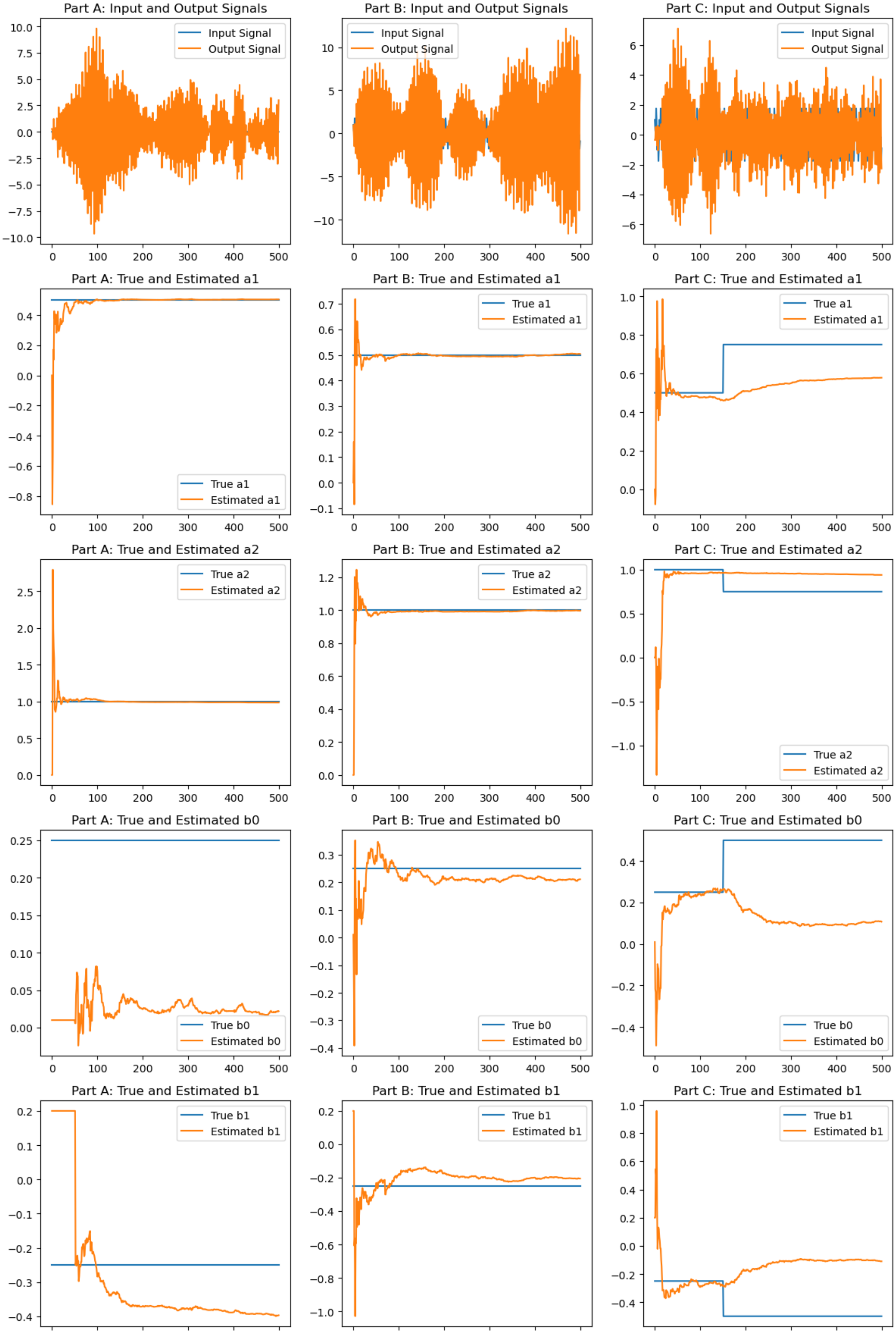
a) Discuss the accuracy of the results.

In Part A, the accuracy of the parameter estimation is observed to be relatively low. This can be attributed to the nature of the input signal, which is a delayed delta-dirac function. Since the impulse function doesn't provide sufficient excitation to fully explore the dynamics of the system, it becomes hard for the recursive least squares algorithm to accurately estimate the parameters. As a result, the estimated parameters a1 and a2 partially converge, indicating some degree of accuracy in capturing the dynamics of the system. However, the estimated parameters b0 and b1 do not converge effectively, leading to lower accuracy in their estimation. To improve the accuracy of parameter estimation in this case, it would be beneficial to utilize input signals that provide more diverse and comprehensive coverage of the system's dynamics. By introducing input signals with a wider range of frequencies and amplitudes, the recursive least squares algorithm would have more information to accurately estimate the parameters of the system.

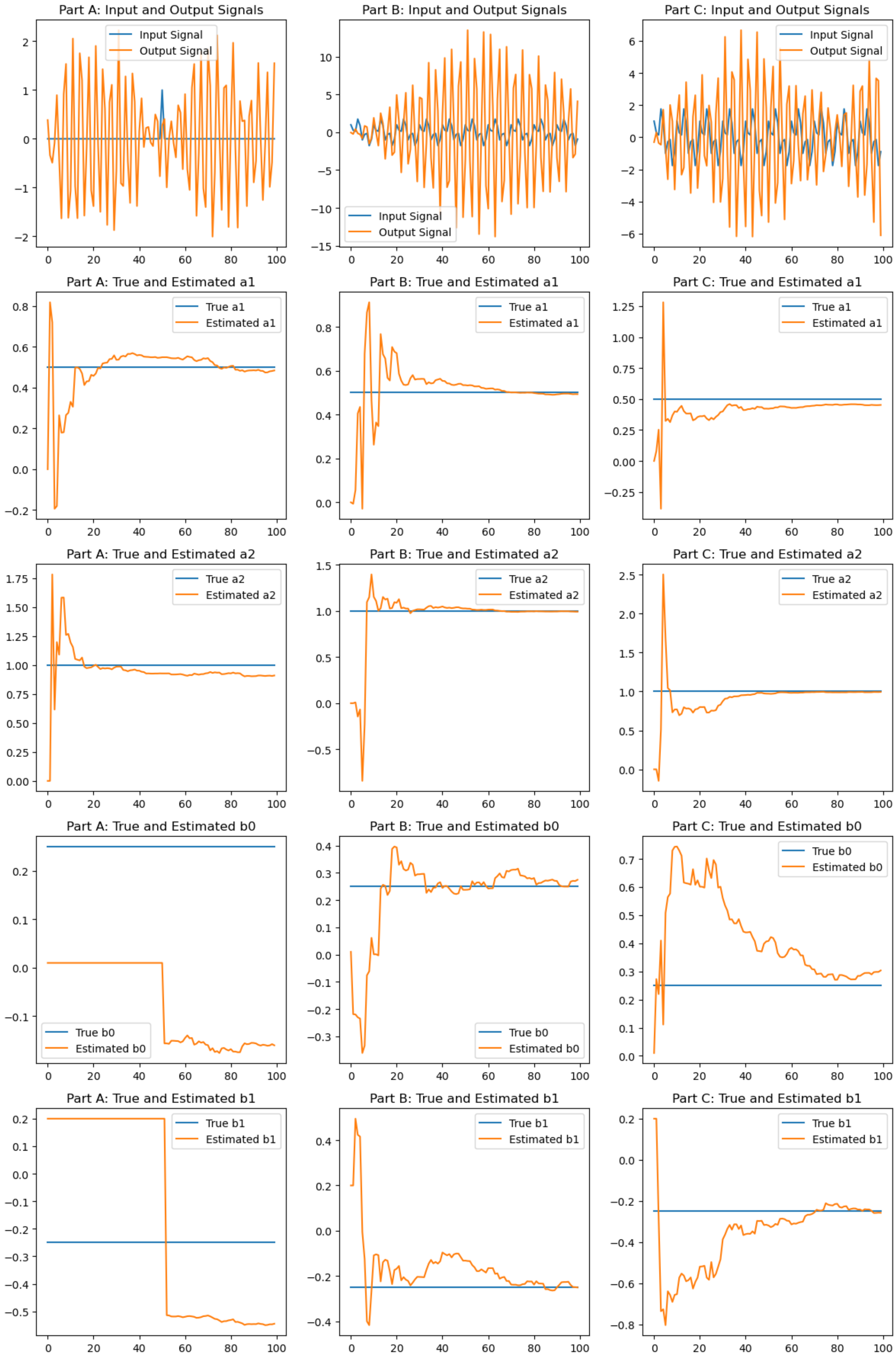
b) Which input was most appropriate and why?

In comparison to the delayed delta-dirac function used in Part A, the input signal consisting of sine and cosine functions in Part B was more appropriate for parameter estimation. The sin+cos input signal provided a more comprehensive and diverse excitation to the system, allowing for better exploration of its dynamics. The sin+cos input signal covers a wider range of frequencies and amplitudes compared to the step function. This enables the recursive least squares algorithm to capture a broader spectrum of the system's behavior, leading to improved accuracy in parameter estimation. By incorporating both the sine and cosine components, the input signal introduces a rich harmonic content that facilitates a more thorough examination of the system's response. This allows the algorithm to adaptively adjust the parameter estimates to better fit the observed output, resulting in more accurate convergence of the estimated parameters. Therefore, the sin+cos input signal is considered more appropriate for parameter estimation in this scenario due to its ability to provide a more comprehensive and informative excitation to the system, leading to improved accuracy in estimating the system's parameters.

At t= 500


Part A: Input and Output Signals
Input Signal
Output Signal
Part B: Input and Output Signals
Input Signal
Output Signal
Part C: Input and Output Signals
Input Signal
Output Signal
Part A: True and Estimated a1
True a1
Estimated a1
Part B: True and Estimated a1
True a1
Estimated a1
Part C: True and Estimated a1
True a1
Estimated a1
Part A: True and Estimated a2
True a2
Estimated a2
Part B: True and Estimated a2
True a2
Estimated a2
Part C: True and Estimated a2
True a2
Estimated a2
Part A: True and Estimated b0
True b0
Estimated b0
Part B: True and Estimated b0
True b0
Estimated b0
Part C: True and Estimated b0
True b0
Estimated b0
Part A: True and Estimated b1
True b1
Estimated b1
Part B: True and Estimated b1
True b1
Estimated b1
Part C: True and Estimated b1
True b1
Estimated b1

At t=100


Part A: Input and Output Signals
Input Signal
Output Signal
Part B: Input and Output Signals
Input Signal
Output Signal
Part C: Input and Output Signals
Input Signal
Output Signal
Part A: True and Estimated a1
True a1
Estimated a1
Part B: True and Estimated a1
True a1
Estimated a1
Part C: True and Estimated a1
True a1
Estimated a1
Part A: True and Estimated a2
True a2
Estimated a2
Part B: True and Estimated a2
True a2
Estimated a2
Part C: True and Estimated a2
True a2
Estimated a2
Part A: True and Estimated b0
True b0
Estimated b0
Part B: True and Estimated b0
True b0
Estimated b0
Part C: True and Estimated b0
True b0
Estimated b0
Part A: True and Estimated b1
True b1
Estimated b1
Part B: True and Estimated b1
True b1
Estimated b1
Part C: True and Estimated b1
True b1
Estimated b1