300389386 Ibrahim Elshenhory Sub. Date

Ass. 2

1) $Y(s) = \underbrace{Y_d(s)}_{\to 0} + Y_r(s)$

$$\frac{Y(s)}{R(s)} = \frac{K(s)\,H(s)}{1+K(s)\,H(s)} = \frac{(K_p + K_d s)\,^{a}/_{s^2}}{1+\left((K_p+K_d s)\,^{a}/_{s^2}\right)} \times \frac{s^2}{s^2} \quad (a=5)$$

$$= \frac{5K_d s + 5K_p}{s^2 + 5K_d s + 5K_p} \quad \#$$

2) $5K_d s + 5K_p = 0$

$$s = \frac{-K_p}{K_d} \quad \#$$

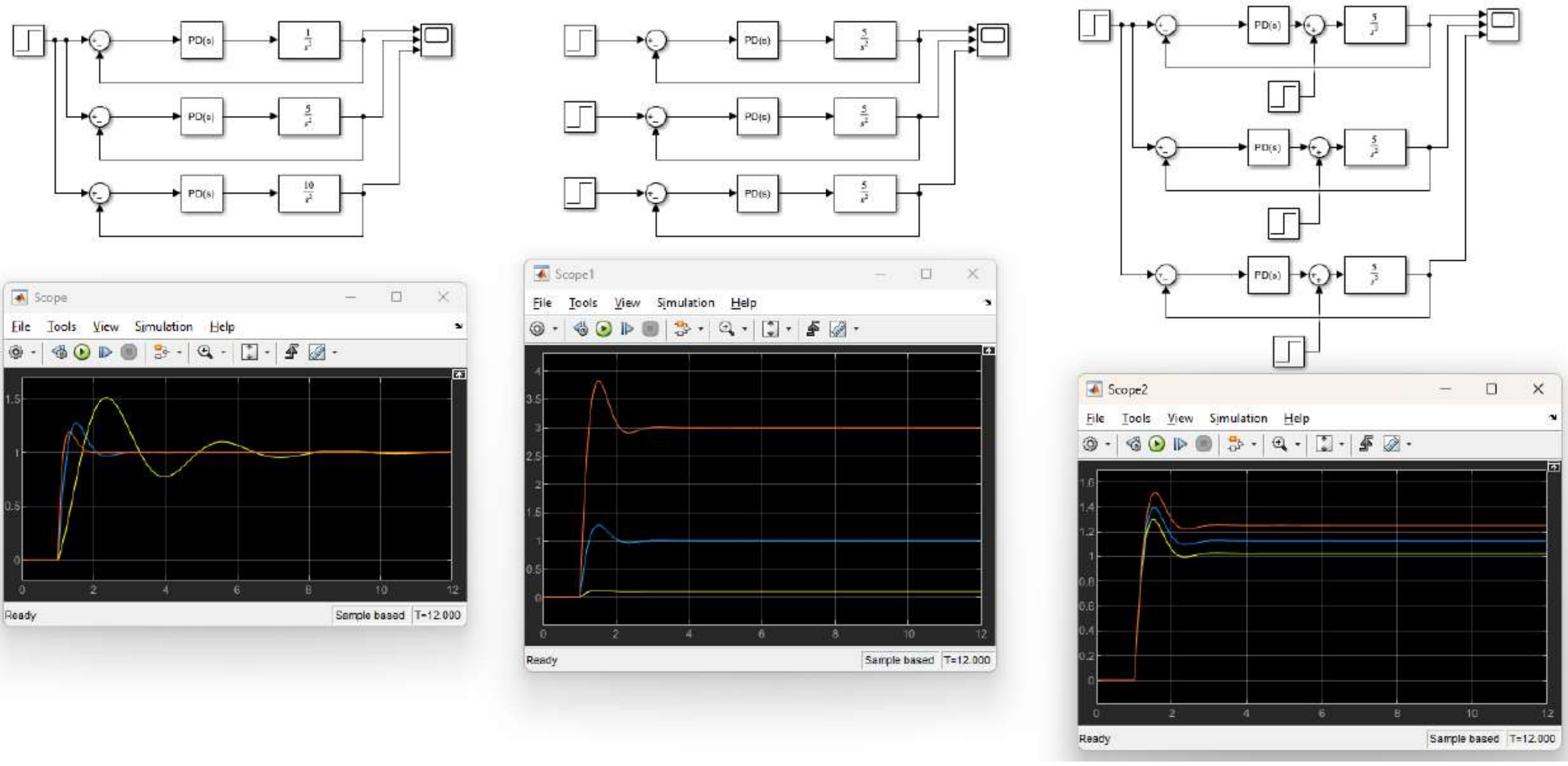
3) $s^2 + 5K_d s + 5K_p \longrightarrow s^2 + 2\xi\omega_n s + \omega_n^2$

$2\xi\omega_n = 2\alpha = 2\times 4 = 5K_d$

$\therefore K_d = 8/5$ #

$\omega_n^2 = 5^2 = 5K_p$

$\therefore K_p = 5$ #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Q)

$$\phi(t) = \mathcal{L}^{-1}[sI - A]^{-1}$$

$$= \mathcal{L}^{-1}\begin{bmatrix} s-0 & -1 \\ 8 & s+6 \end{bmatrix}^{-1} = \mathcal{L}^{-1}\begin{bmatrix} \frac{s+6}{s^2+6s+8} & \frac{1}{s^2+6s+8} \\ \frac{-8}{s^2+6s+8} & \frac{s}{s^2+6s+8} \end{bmatrix}$$

$$\mathcal{L}^{-1}\left\{\frac{s+6}{(s+2)(s+4)}\right\} = \frac{2}{s+2} + \frac{-1}{s+4} = 2e^{-2t} - e^{-4t}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+2)(s+4)}\right\} = \mathcal{L}^{-1}\left\{\frac{0.5}{s+2} - \frac{0.5}{s+4}\right\} = e^{-2t}/2 - e^{-4t}/2$$

$$\mathcal{L}^{-1}\left\{\frac{-8}{(s+2)(s+4)}\right\} = \mathcal{L}^{-1}\left\{\frac{-4}{s+2} + \frac{4}{s+4}\right\} = -4e^{-2t} + 4e^{-4t}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s+2)(s+4)}\right\} = \mathcal{L}^{-1}\left\{\frac{-1}{s+2} + \frac{2}{s+4}\right\} = -e^{-2t} + 2e^{-4t}$$

$$A_d = \phi(t) = \begin{bmatrix} 2e^{-2t} - e^{-4t} & e^{-2t}/2 - e^{-4t}/2 \\ -4e^{-2t} + 4e^{-4t} & 2e^{-4t} - e^{-2t} \end{bmatrix}_{t=0.1} = \begin{bmatrix} 0.967 & 0.074 \\ -0.59 & 0.52 \end{bmatrix}$$

$$B_d = (A_d - I)A^{-1}B = \begin{bmatrix} -0.033 & 0.074 \\ -0.59 & -0.48 \end{bmatrix}\begin{bmatrix} -0.75 & -1/8 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4.125 \times 10^{-3} \\ 0.074 \end{bmatrix}$$

$$C_d = C = [3 \quad 1] \qquad D_d = 0$$

$$T_f = C_d[zI - A_d]^{-1}B_d + D_d$$

$$= [3 \quad 1]\begin{bmatrix} z-0.967 & -0.074 \\ 0.59 & z-0.52 \end{bmatrix}^{-1}\begin{bmatrix} 4.125 \times 10^{-3} \\ 0.074 \end{bmatrix}$$

$$= [3 \quad 1]\begin{bmatrix} \frac{z-0.52}{det} & \frac{0.074}{det} \\ \frac{-0.59}{det} & \frac{z-0.967}{det} \end{bmatrix}\begin{bmatrix} 4.125 \times 10^{-3} \\ 0.074 \end{bmatrix}$$

Sub. .... Date ....

$$det = Z^2 - 1.5Z + 0.55$$

$$TF = \begin{bmatrix} \frac{3Z - 2.15}{det} & \frac{Z - 0.745}{det} \end{bmatrix} \begin{bmatrix} 4.125 \times 10^{-3} \\ 0.074 \end{bmatrix}$$

$$TF = \frac{0.086Z - 0.064}{Z^2 - 1.5Z + 0.55}$$

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>> Ass2Q2

ans =

   0.08653 z - 0.06412
  ---------------------
  z^2 - 1.489 z + 0.5488

Sample time: 0.1 seconds
Discrete-time transfer function.
Model Properties
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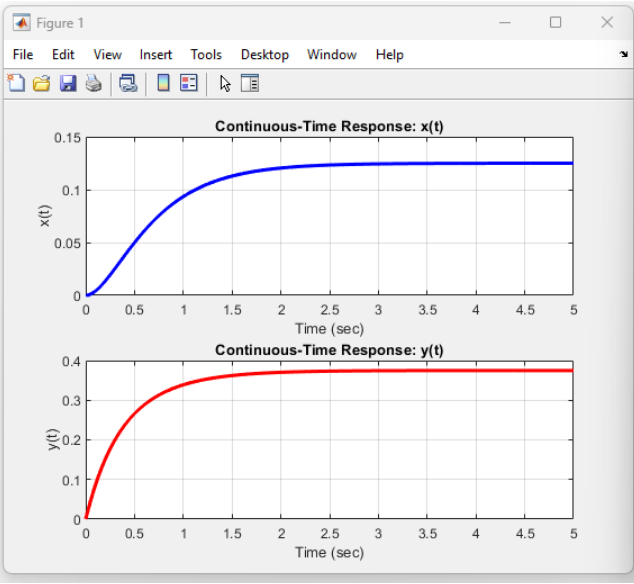

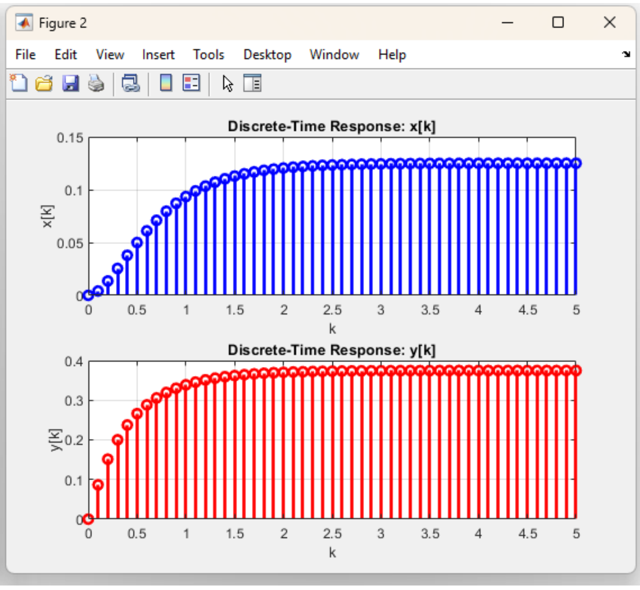

Q3) $t_d = 0.497 = \frac{1+0.7\xi}{\omega_n}$, $t_p = 1.05 = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$

$M_p = \frac{0.405-0.3}{0.3} = e^{-\left(\frac{\xi\pi}{\sqrt{1-\xi^2}}\right)}$

$\therefore \xi = 0.317$

$\therefore \omega_n = 3.15$

$$\therefore G(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\therefore G(s) = \frac{9.92}{s^2+2s+9.92}$$ #