

Francisco Facchinei Jong-Shi Pang

Finite-Dimensional Variational Inequalities and Complementarity Problems

Volume I

With 18 Figures



Springer

Contents

Preface	v
Contents	xvii
Contents of Volume II	xxi
Acronyms	xxiii
Glossary of Notation	xxv
Numbering System	xxxiii
1 Introduction	1
1.1 Problem Description	2
1.1.1 Affine problems	7
1.2 Relations Between Problem Classes	8
1.3 Integrability and the KKT System	12
1.3.1 Constrained optimization problems	13
1.3.2 The Karush-Kuhn-Tucker system	18
1.4 Source Problems	20
1.4.1 Saddle problems	21
1.4.2 Nash equilibrium problems	24
1.4.3 Nash-Cournot production/distribution	26
1.4.4 Economic equilibrium problems	36
1.4.5 Traffic equilibrium models	41
1.4.6 Frictional contact problems	46
1.4.7 Elastoplastic structural analysis	51
1.4.8 Nonlinear obstacle problems	55
1.4.9 Pricing American options	58
1.4.10 Optimization with equilibrium constraints	65
1.4.11 CPs in SPSD matrices	67
1.5 Equivalent Formulations	71

1.5.1	Equation reformulations of the NCP	71
1.5.2	Equation reformulations of the VI	76
1.5.3	Merit functions	87
1.6	Generalizations	95
1.7	Concluding Remarks.	98
1.8	Exercises	98
1.9	Notes and Comments	113
2	Solution Analysis I	125
2.1	Degree Theory and Nonlinear Analysis	126
2.1.1	Degree theory	126
2.1.2	Global and local homeomorphisms	134
2.1.3	Elementary set-valued analysis	138
2.1.4	Fixed-point theorems	141
2.1.5	Contractive mappings	143
2.2	Existence Results	145
2.2.1	Applications to source problems	150
2.3	Monotonicity	154
2.3.1	Plus properties and F-uniqueness	162
2.3.2	The dual gap function	166
2.3.3	Boundedness of solutions	168
2.4	Monotone CPs and AVIs	170
2.4.1	Properties of cones	171
2.4.2	Existence results	175
2.4.3	Polyhedrality of the solution set	180
2.5	The VI (K, q, M) and Copositivity	185
2.5.1	The CP (K, q, M)	192
2.5.2	The AVI (K, q, M)	199
2.5.3	Solvability in terms of feasibility	202
2.6	Further Existence Results for CPs	208
2.7	A Frictional Contact Problem	213
2.8	Extended Problems	220
2.9	Exercises	226
2.10	Notes and Comments	235
3	Solution Analysis II	243
3.1	Bouligand Differentiable Functions	244
3.2	Constraint Qualifications	252
3.3	Local Uniqueness of Solutions	266
3.3.1	The critical cone	267

3.3.2	Conditions for local uniqueness	271
3.3.3	Local uniqueness in terms of KKT triples	279
3.3.4	Local uniqueness theory in NLP	283
3.3.5	A nonsmooth-equation approach	287
3.4	Nondegenerate Solutions	289
3.5	VIIs on Cartesian Products	292
3.5.1	Semicopositive matrices	294
3.5.2	P properties	298
3.6	Connectedness of Solutions	309
3.6.1	Weakly univalent functions	310
3.7	Exercises	317
3.8	Notes and Comments	330
4	The Euclidean Projector and Piecewise Functions	339
4.1	Polyhedral Projection	340
4.1.1	The normal manifold	345
4.2	Piecewise Affine Maps	352
4.2.1	Coherent orientation	356
4.3	Unique Solvability of AVIs	371
4.3.1	Inverse of $\mathbf{M}_K^{\text{nor}}$	374
4.4	B-Differentiability under SBCQ	376
4.5	Piecewise Smoothness under CRCQ	384
4.6	Local Properties of PC^1 Functions	392
4.7	Projection onto a Parametric Set	401
4.8	Exercises	407
4.9	Notes and Comments	414
5	Sensitivity and Stability	419
5.1	Sensitivity of an Isolated Solution	420
5.2	Solution Stability of B-Differentiable Equations	427
5.2.1	Characterizations in terms of the B-derivative	439
5.2.2	Extensions to locally Lipschitz functions	443
5.3	Solution Stability: The Case of a Fixed Set	445
5.3.1	The case of a finitely representable set	452
5.3.2	The NCP and the KKT system	462
5.3.3	Strong stability under CRCQ	469
5.4	Parametric Problems	472
5.4.1	Directional differentiability	482
5.4.2	The strong coherent orientation condition	489
5.4.3	PC^1 multipliers and more on SCOC	496

5.5	Solution Set Stability	500
5.5.1	Semistability	503
5.5.2	Solvability of perturbed problems and stability . .	509
5.5.3	Partitioned VIs with P_0 pairs	512
5.6	Exercises	516
5.7	Notes and Comments	525
6	Theory of Error Bounds	531
6.1	General Discussion	531
6.2	Pointwise and Local Error Bounds	539
6.2.1	Semistability and error bounds	539
6.2.2	Local error bounds for KKT triples	544
6.2.3	Linearly constrained monotone composite VIs . .	548
6.3	Global Error Bounds for VIs/CPs	554
6.3.1	Without Lipschitz continuity	557
6.3.2	Affine problems	564
6.4	Monotone AVIs	575
6.4.1	Convex quadratic programs	586
6.5	Global Bounds via a Variational Principle	589
6.6	Analytic Problems	596
6.7	Identification of Active Constraints	600
6.8	Exact Penalization and Some Applications	605
6.9	Exercises	610
6.10	Notes and Comments	616
Bibliography for Volume I		I-1
Index of Definitions and Results		I-51
Subject Index		I-57