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Finite-Dimensional Variational Inequalities and Complementarity Problems

Volume I

With 18 Figures



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