LMI Methods For Satellite Control

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***Abstract*— In this project, the issue of satellite attitude control is examined. After presenting the model of the system, we study H_2 and H_∞ control. Later, we consider the scenario of attitude stabilization in which disturbance attenuation and desired-loop eigenvalue location is achieved using LMI methods. Furthermore, a robust H_∞ controller is designed in presence of interval uncertainties.**

I. INTRODUCTION

Satellite motion in orbit has an important aspect in military and scientific activities. In order to achieve various objectives, attitude control and stabilization is essential.

A satellite attitude system is exposed to different type of uncertainties and disturbances. Aerodynamic moments, pressure torques due to sunlight pressure, gravity gradient torques are among external disturbances. Internal disturbances consist of of parameter uncertainties, fuel-related influence and due to carried facilities such as 2-D scanners. [1]

A. System Modeling

Taking inspiration from the well-known moment of momentum theorem, satellite attitude dynamics in the inertial co-ordinate system is expressed as

$$\dot{H} = T_c + T_g + T_d, \tag{1}$$

where ,

- T_c, T_g, T_d are the torques with respect to flywheel, gravity and disturbance respectively.
- H is the total momentum acting on the satellite.

The total momentum H is given by [1], [3], [4]

$$H = I_b \omega \tag{2}$$

where ,

- I_b is the inertia matrix.
- ω is the angular velocity.

It follows from eq.2 [2], [1] :

$$\dot{H} = I_b \dot{\omega} + \omega \times (I_b \omega) \tag{3}$$

Combining 1 and 3 :

$$I_b \dot{\omega} + \omega \times (I_b \omega) = T_c + T_g + T_d, \tag{4}$$

Choose,

$$I_b = diag(I_x, I_y, I_z) \tag{5}$$

and ,

$$T_c = \begin{bmatrix} T_{cx} \\ T_{cy} \\ T_{cz} \end{bmatrix} T_g = \begin{bmatrix} T_{gx} \\ T_{gy} \\ T_{gz} \end{bmatrix} T_d = \begin{bmatrix} T_{dx} \\ T_{dy} \\ T_{dz} \end{bmatrix} \tag{6}$$

Using eq.4 and eq.5 we get,

$$I_x \dot{\omega}_x + (I_z - I_y)\omega_y \omega z = T_{cx} + T_{gx} + T_{dx} \tag{7}$$

$$I_y \dot{\omega}_y + (I_x - I_z)\omega_z \omega x = T_{cy} + T_{gy} + T_{dy} \tag{8}$$

$$I_z \dot{\omega}_z + (I_y - I_x)\omega_x \omega y = T_{cz} + T_{gz} + T_{dz} \tag{9}$$

Using small-angle approximation, the angular velocity of the satellite in the inertial co-ordinate system can be denoted in the body co-ordinate system [4],

$$\omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \dot{\phi} - \omega_0 \psi \\ \dot{\theta} - \omega_0 \\ \dot{\psi} + \omega_0 \phi \end{bmatrix} \tag{10}$$

where ω_0 = 7.292115 $\times$ 10^{-5} rad/sec , is the rotational angular velocity of the earth. ϕ, θ, ψ are the three Euler angles.

Putting eq.10 in eqs.7,8,9, satellite attitude dynamics can be found as shown below.

$$I_x \ddot{\phi} + (I_y - I_z)\omega_0^2 \phi + (I_y - I_z - I_x)\omega_0 \dot{\psi} = T_{cx} + T_{gx} + T_{dx} \tag{11}$$

$$I_y \ddot{\theta} = T_{cy} + T_{gy} + T_{dy} \tag{12}$$

$$I_z \ddot{\psi} + (I_y - I_x)\omega_0^2 \dot{\psi} + (I_x + I_z - I_y)\omega_0 \dot{\psi} = T_{cz} + T_{gz} + T_{dz} \tag{13}$$

where the gravitational torques are :

$$T_{gx} = -3\omega_0^2 (I_y - I_z)\phi \tag{14}$$

$$T_{gy} = -3\omega_0^2 (I_x - I_z)\theta \tag{15}$$

$$T_{gz} = 0 \tag{16}$$

II. SECOND-ORDER SYSTEM FORM

The vectors are defined as follows:

$$q = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} u = \begin{bmatrix} T_{cx} \\ T_{cy} \\ T_{cz} \end{bmatrix} d = \begin{bmatrix} T_{dx} \\ T_{dy} \\ T_{dz} \end{bmatrix} \tag{17}$$

The following second order equation can be found from eq.11-16 :

$$M\ddot{q} + H\dot{q} + Gq = L_1 u + L_2 d \tag{18}$$

where ,

M = diag(I_x, I_y, I_z), $L_1, L_2 = I_{3\times3}$

$$\mathrm{H} = \omega_0(I_y - I_x - I_z)\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\mathrm{G} = \mathrm{diag}(4\omega_0^2(I_y - I_z), 3\omega_0^2(I_x - I_z), \omega_0^2(I_y - I_x)$$

A. State-Space Form

Define $x = \begin{bmatrix} q & \dot{q} \end{bmatrix}^T$,$z_\infty = 10^{-3} M\ddot{q}$, $z_2 = q$ Therefore, the problem at hand can be formulated as the state-space form below:

$$\dot{x} = Ax + B_1 u + B_2 d \tag{19}$$

$$z_\infty = C_1 x + D_1 u + D_2 d \tag{20}$$

$$z_2 = C_2 x \tag{21}$$

Using the $A, B_1, B_2, C_1, C_2, D_1, D_2$ as given in from eq.(12.11) - eq.(12.15) in [1], H_2,H_∞,mixed $H_2 \backslash H_\infty$ and a robust H_∞ controllers are designed.

III. RESULTS

The parameter values used are :

$I_x = 1030.17 kgm^2, I_y = 3015.65 kgm^2, I_z = 3030.43 kgm^2,$

Using the LMI stated in eq.(12.16) [1], H_∞ controller with a state-feedback gain matrix :

$$K = \begin{bmatrix} -25.9 & 0 & -0.05 & -284 & 0 & -0.73 \\ 0 & -17.6 & 0 & 0-416.7 & 0 & \\ 0.06 & 0 & -17.5 & 0.77 & 0 & -417.5 \end{bmatrix}$$

H_2 controller is achieved using eq.(12.19) in [1],

$$K = 10^4 \begin{bmatrix} -1.64 & 0 & 0 & -0.14 & 0 & 0 \\ 0 & -4.82 & 0 & 0-0.43 & 0 & \\ 0 & 0 & -4.84 & 0 & 0 & -0.4339 \end{bmatrix}$$

The first robust controller designed is the mixed $H_2 \backslash H_\infty$ controller using eq.(12.24) in [1]. This LMI is based on placing the poles in a desired region. In this case, the desired strip region is specified by the characteristic function :

$$F_{1,3} = L + sM + \bar{s}M^T$$

where,

$$L = \begin{bmatrix} 2 & 0 \\ 0 & -6 \end{bmatrix} M = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$K = 10^4 \begin{bmatrix} -1.93 & 0 & -0.006 & -0.62 & 0 & -0.002 \\ 0 & -5.65 & 0 & 0-1.8 & 0 & \\ 0.001 & 0 & -5.67 & 0 & 0 & -1.82 \end{bmatrix}$$

Introducing interval uncertainties into the system to design a robust H_∞ controller using *hinfsys* command on MATLAB gives us the following controller.

I_x is made to vary about 90%, while I_y and I_z is made to vary about 10%.

The impulse response of the closed-loop system after robust H_∞ closed loop system is plotted in Fig.4.

```
Kint =

  A =
              x1          x2          x3          x4          x5          x6
   x1     -493.7   7.182e-14      0.3377  -5.077e+06   4.054e-10  -3.616e+07
   x2 -7.889e-15   3.051e-06   5.361e-18   -8.11e-11       1.013  -5.778e-10
   x3      133.1  -1.936e-14    -0.09103   1.369e+06  -1.093e-10   9.748e+06
   x4   -0.01041   1.507e-18    7.12e-06      -107.1   8.516e-15      -762.4
   x5  1.148e-19  -7.836e-08  -7.849e-23   1.181e-15  -7.765e-09   8.408e-15
   x6   0.002018  -2.963e-19  -1.356e-06       20.76  -1.669e-15       147.8

  B =
              u1          u2
   x1  1.177e-07  -1.118e+07
   x2        5.6  -1.786e-10
   x3 -3.173e-08   3.013e+06
   x4  2.468e-12      -235.6
   x5     0.3187   2.598e-15
   x6 -4.861e-13          46

  C =
              x1          x2          x3          x4          x5          x6
   y1 -1.309e-17   -0.000818   6.261e-18  -5.403e-12      -2.266   4.529e-14
   y2   -0.04417   6.899e-19   0.0001176      -453.2   1.241e-14       -3236

  D =
       u1  u2
   y1   0   0
   y2   0   0

Continuous-time state-space model.
```

Fig. 1. Robust H_∞ controller

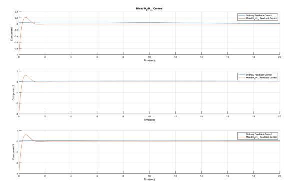

Fig. 2. mixed $H_2 \backslash H_\infty$ v/s H_∞ control

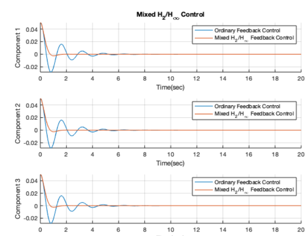

Fig. 3. mixed $H_2 \backslash H_\infty$ v/s H_2 control

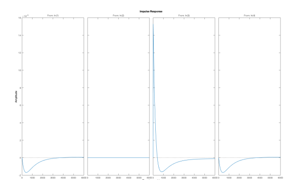

Fig. 4. Impulse Response of Closed Loop System

IV. CONCLUSIONS

It is noticed that in H_∞ feedback control, the optimum gain matrix gives very slow response transfer property,H_2 feedback control provided optimum gain matrix which is of a large magnitude.The resulting feedback controller with mixed $H_2 \backslash H_\infty$ has better ability to attenuate disturbance while also maintaining stability. Robust H_∞ control in presence of interval uncertainties has larger attenuation bound but it is still much less than that of the mixed $H_2 \backslash H_\infty$.

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